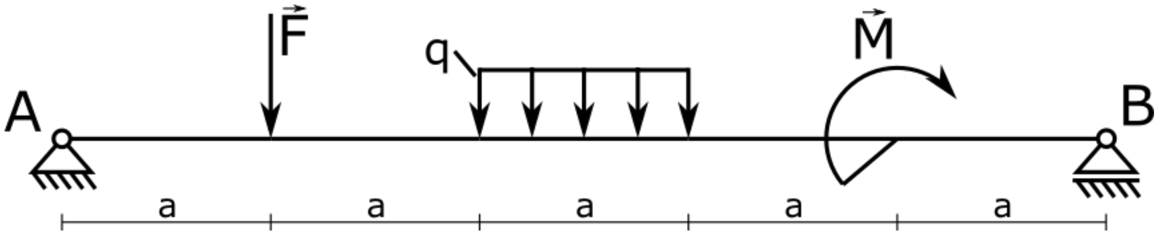
Beams – all types of loads

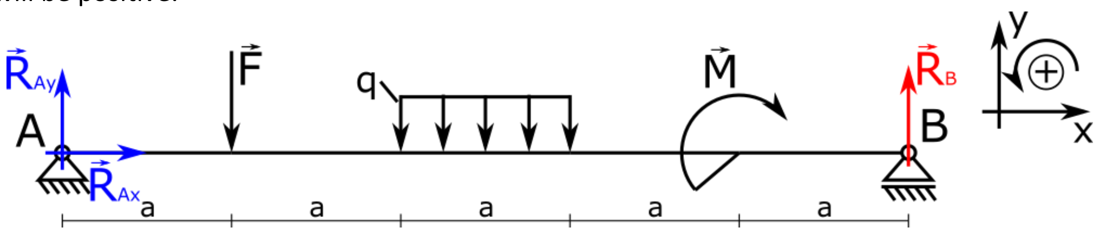
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam with all types of loads. An important information, the beam solution also includes drawing internal force diagrams.

Ex.4

For the beam shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a = 1 [m], F=2[kN], q=1[kN/m], M = 20 [kNm].



1. The first thing to do when solving beams is to determine the number of supporting unknowns, as was the case with trusses. It is clear that in this case we have three unknown supports. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, anticlockwise turning moments will be positive.



2. Now we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

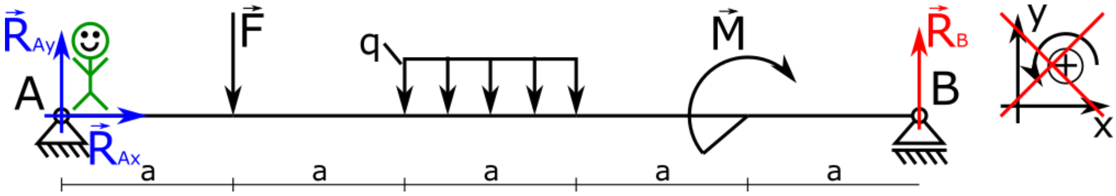
$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \to R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - F - q * a + R_B \to R_{Ay} = F + q * a - R_B = 2 + 1 * 1 - 1{,}3 = 1{,}7kN$$

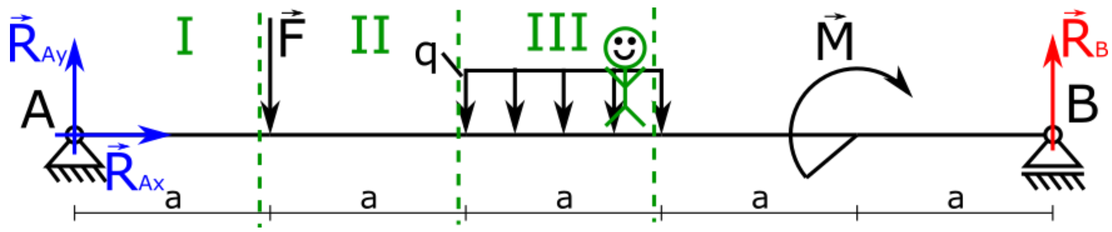
$$\sum_{i=1}^{n} M_A = 0 = -F * a - q * a * 2{,}5a - M + R_B * 5a \to R_B = \frac{F * a + q * 2{,}5a^2 + M}{5a} =$$

$$= \frac{2 * 1 + 1 * 2{,}5 * 1^2 + 2}{5} = \frac{6{,}5}{5} = 1{,}3kN$$

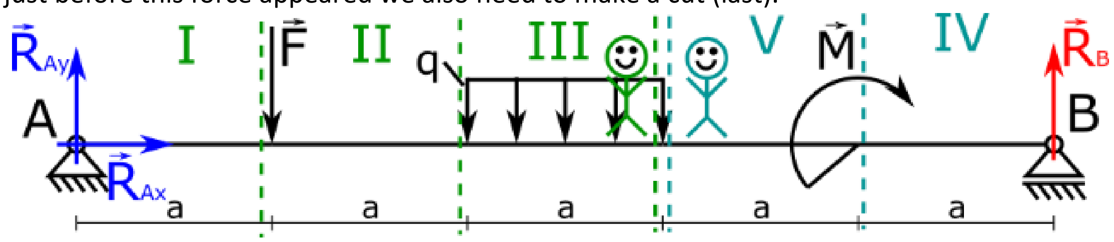
3. After determining the reaction in the supports, in the next step we need to determine how many cuts the beam needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces that was introduced at the beginning. We enter the beam from its left (you can also enter the beam from the right).



4. Cuts will be made on the beam in places where something happens on the beam (something appears or disappears). The important information is that we always cut before something on the beam has happened.
We see that when we enter the beam from its left, forces R_{Ax} and R_{Ay} appear. However, we cannot make the cut before these forces, because then we are not yet on the beam.
We go further along the beam and come across force F. Something happened, force appeared. So we know that in this place just before the appearance of force F should be first cut. Going further we reach the place where distributed force appears. So in this place just before this force appeared we also need to make a cut. Continuing the journey we reach the place where the distributed force ends, i.e. something has happened (the force has disappeared). This means that in this place just before the distributed force ends we have to make another cut. In this place we end our journey on the left and continue on the right.



5. We see that when we enter the beam from its right, force R_B appears. However, we cannot make the cut before this force, because then we are not yet on the beam. We go further along the beam and come across moment M. Something happened, moment appeared. So we know that in this place just before the appearance of moment M should be next cut (4th section). Going further we reach the place where distributed force appears. So in this place just before this force appeared we also need to make a cut (last).



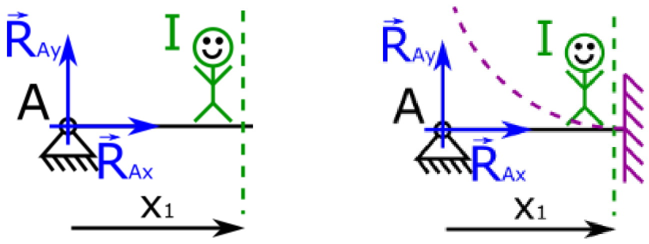
6. In this way, we divided the beam into five sections I, II and III from the left side and IV and V from the right side. The first section within $0 \leq x < a$, second section within $a \leq x < 2a$, third section within $2a \leq x < 3a$, fourth section within $0 \leq x < a$, fifth section within $a \leq x < 2a$.

BENDING MOMENTS, CUTTING FORCES, NORMAL FORCES

7. At this point, we can move on to determining internal forces. To do this we have to go through each of the sections. We will start from the left side. In this example, we will determine equations for all internal forces interval by interval. In addition, information about the bending moment will appear at each of the intervals.

   I section within $0 \leq x < a$.

   The bending moment will be only from the R_{Ay} force and will look like in the picture (dashed purple line). Cutting force only from R_{Ay}. Normal force R_{Ax}.

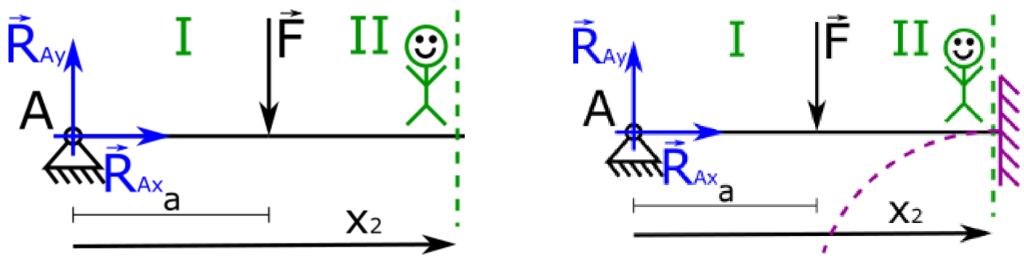


   Equations for all types of internal forces

$$M(x_1) = R_{Ay} * x_1$$
$$T(x_1) = R_{Ay}$$
$$N(x_1) = -R_{Ax}$$

8. II section within $a \leq x < 2a$.

   The bending moment will be from the R_{Ay} force and F force. The bending from R_{Ay} will be the same as it was in previous section. Bending moment from F force will look like in the picture (dashed purple line). Cutting forces from R_{Ay} and F. Normal force R_{Ax}.

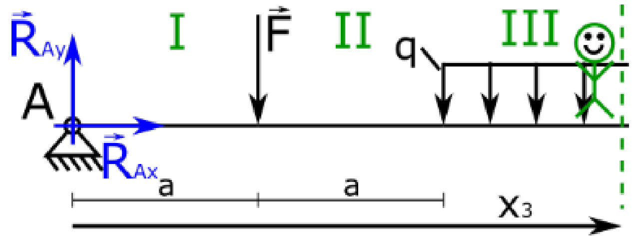


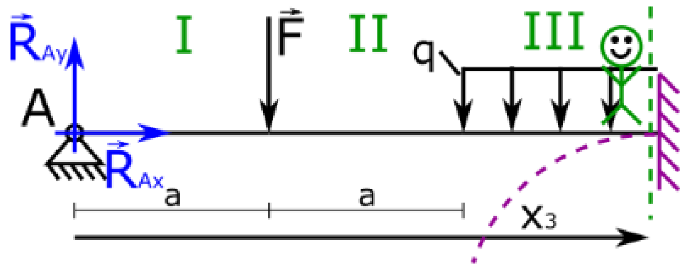
   Equations for all types of internal forces

$$M(x_2) = R_{Ay} * x_2 - F * (x_2 - a)$$
$$T(x_2) = R_{Ay} - F$$
$$N(x_2) = -R_{Ax}$$

9. III section within $2a \leq x < 3a$.

The bending moment will be from forces: R_{Ay} and F, and from distributed force q. The bending moment from R_{Ay} and F will be the same as it was in previous section. Bending moment from distributed force q will look like in the picture (dashed purple line). Cutting forces from R_{Ay}, F and distributed force q. Normal force R_{Ax}.



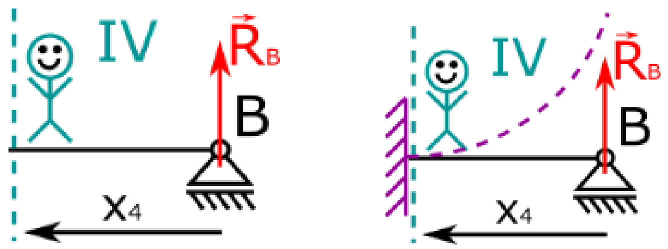


Equations for all types of internal forces

$$M(x_3) = R_{Ay} * x_3 - F * (x_3 - a) - q * (x_3 - 2a) * \frac{(x_3 - 2a)}{2}$$
$$T(x_3) = R_{Ay} - F - q * (x_3 - 2a)$$
$$N(x_3) = -R_{Ax}$$

10. IV section within $0 \leq x < a$ from right side **it is important to mark that we are calculating sections from the right side.**

The bending moment will be only from the R_B force and will look like in the picture (dashed purple line). Cutting force only from R_B. There will be no normal force.

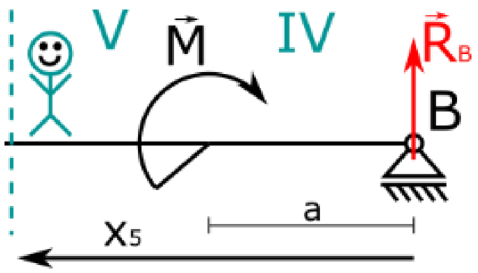


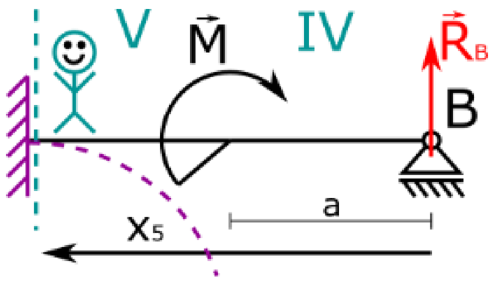
Equations for all types of internal forces

$$M(x_4) = R_B * x_4$$
$$T(x_4) = -R_B$$
$$N(x_4) = 0$$

11. V section within $a \leq x < 2a$ from right side **it is important to mark that we are calculating sections from the right side.**

    The bending moment will be from the R_B force and moment M. The bending from R_B will be the same as it was in previous section. Bending moment from moment M will look like in the picture (dashed purple line). Cutting forces from R_B. There will be no normal force.

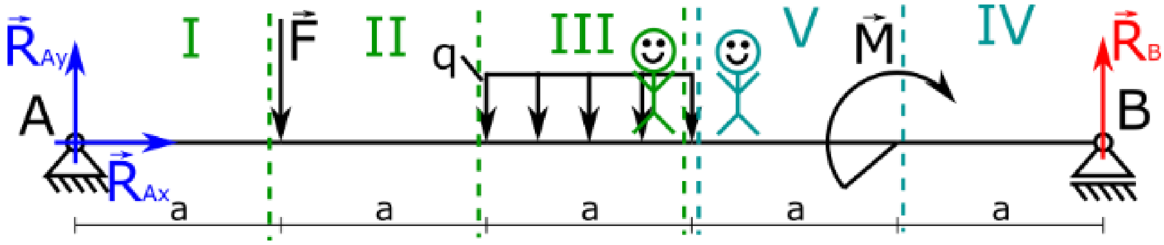




Equations for all types of internal forces

$$M(x_5) = R_B * x_5 - M$$
$$T(x_2) = -R_B$$
$$N(x_2) = 0$$

12. Let's write equations for all sections in one place to clarify everything.



I Section $0 \leq x_1 < a$

$$M(x_1) = R_{Ay} * x_1$$
$$T(x_1) = R_{Ay}$$
$$N(x_1) = -R_{Ax}$$

II Section $a \leq x_1 < 2a$

$$M(x_2) = R_{Ay} * x_2 - F * (x_2 - a)$$
$$T(x_2) = R_{Ay} - F$$
$$N(x_2) = -R_{Ax}$$

III Section $2a \leq x_1 < 3a$

$$M(x_3) = R_{Ay} * x_3 - F * (x_3 - a) - q * (x_3 - 2a) * \frac{(x_3 - 2a)}{2}$$
$$T(x_3) = R_{Ay} - F - q * (x_3 - 2a)$$
$$N(x_3) = -R_{Ax}$$

IV Section $0 \leq x_1 < a$ right side

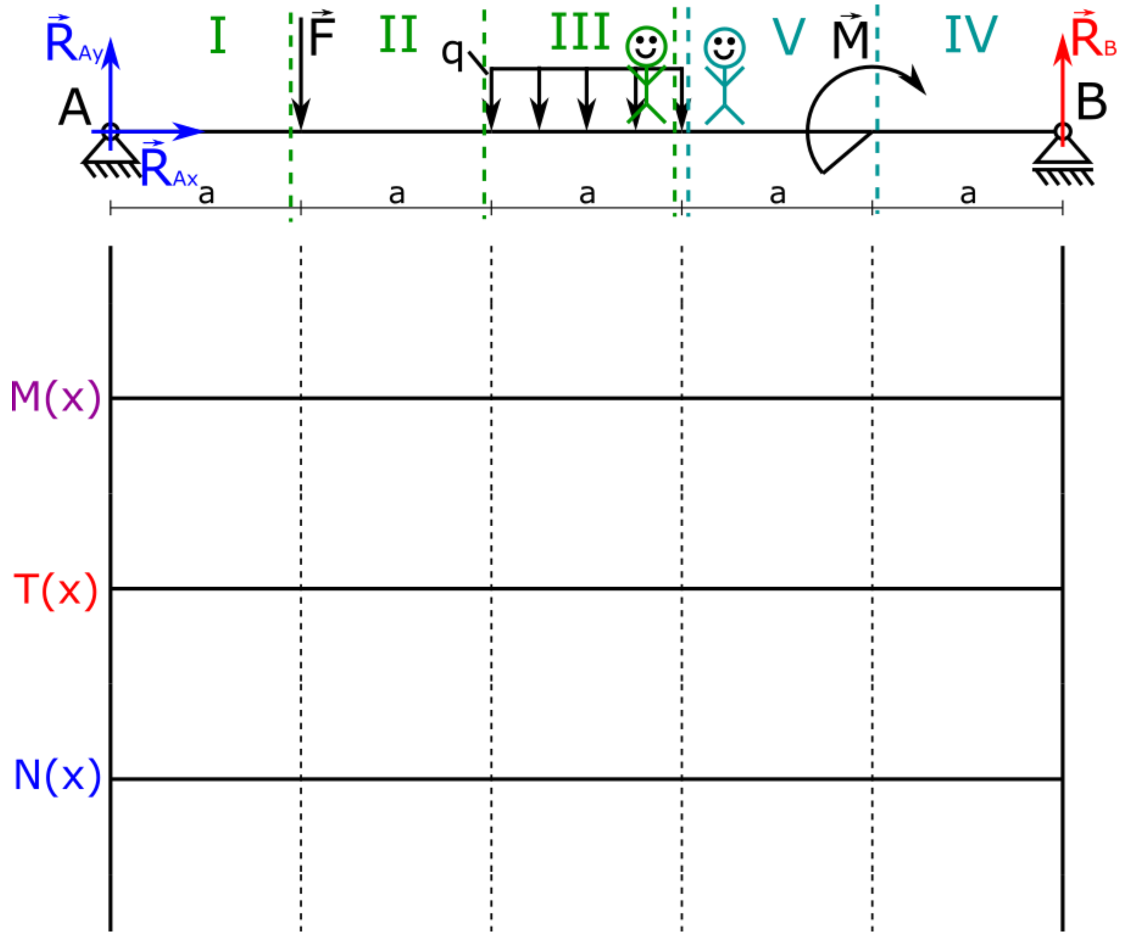
$$M(x_4) = R_B * x_4$$
$$T(x_4) = -R_B$$
$$N(x_4) = 0$$

V Section $a \leq x_1 < 2a$ right side

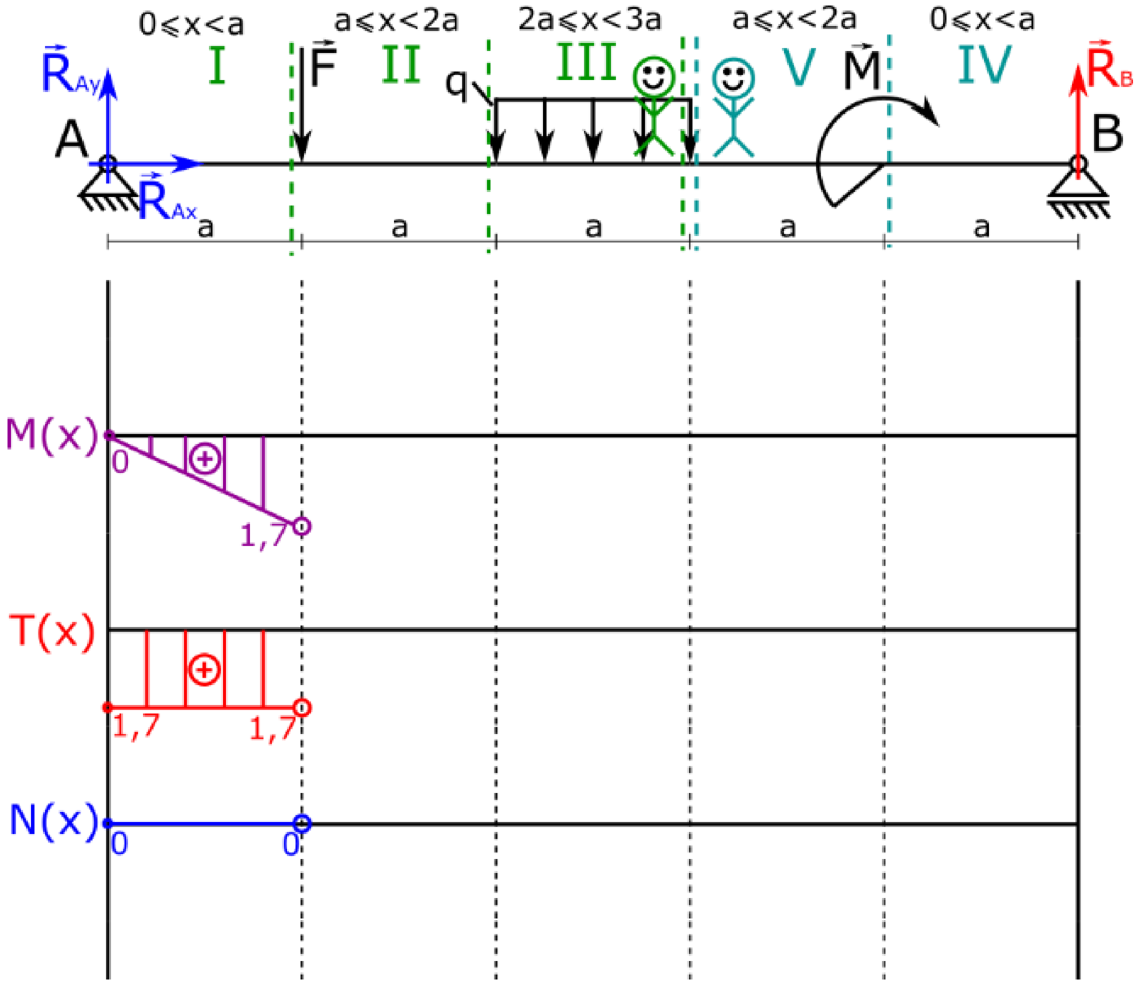
$$M(x_5) = R_B * x_5 - M$$
$$T(x_2) = -R_B$$
$$N(x_2) = 0$$

CHARTS

13. The last part related to solving beams – charts. To easily draw charts, it is best to draw them under the beam, which we solve, as shown in the figure.



14. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the first section.

$$M(x_1) = R_{Ay} * x_1$$
$$T(x_1) = R_{Ay}$$
$$N(x_1) = -R_{Ax}$$

We know the limits of the first section.

$$0 \le x < a$$

We substitute the boundary values into our equations (for x_1).

$$M(0) = R_{Ay} * 0 = 0$$

$$M(a) = R_{Ay} * a = 1{,}7 * 1 = 1{,}7kNm$$

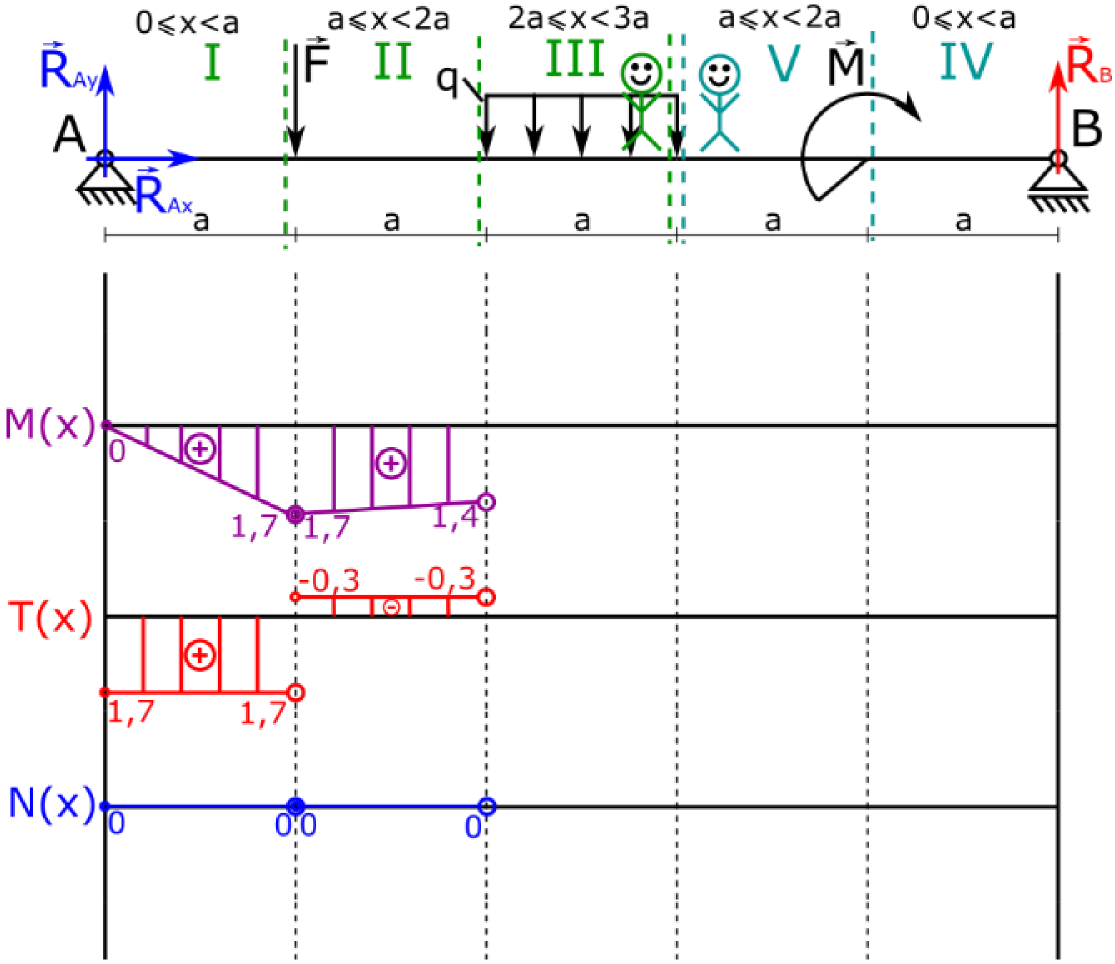
$$T(0) = R_{Ay} = 1{,}7kN$$

$$T(a) = R_{Ay} = 1{,}7kN$$

$$N(0) = -R_{Ax} = 0$$

$$N(a) = -R_{Ax} = 0$$

15. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the second section.

$$M(x_2) = R_{Ay} * x_2 - F * (x_2 - a)$$
$$T(x_2) = R_{Ay} - F$$
$$N(x_2) = -R_{Ax}$$

We know the limits of the second section.

$$a \leq x < 2a$$

We substitute the boundary values into our equations (for x_1).

$$M(a) = R_{Ay} * a - F * (a - a) = 1{,}7kNm$$

$$M(2a) = R_{Ay} * 2a - F * (2a - a) = 1{,}7 * 2 - 2 * 1 = 1{,}4kNm$$

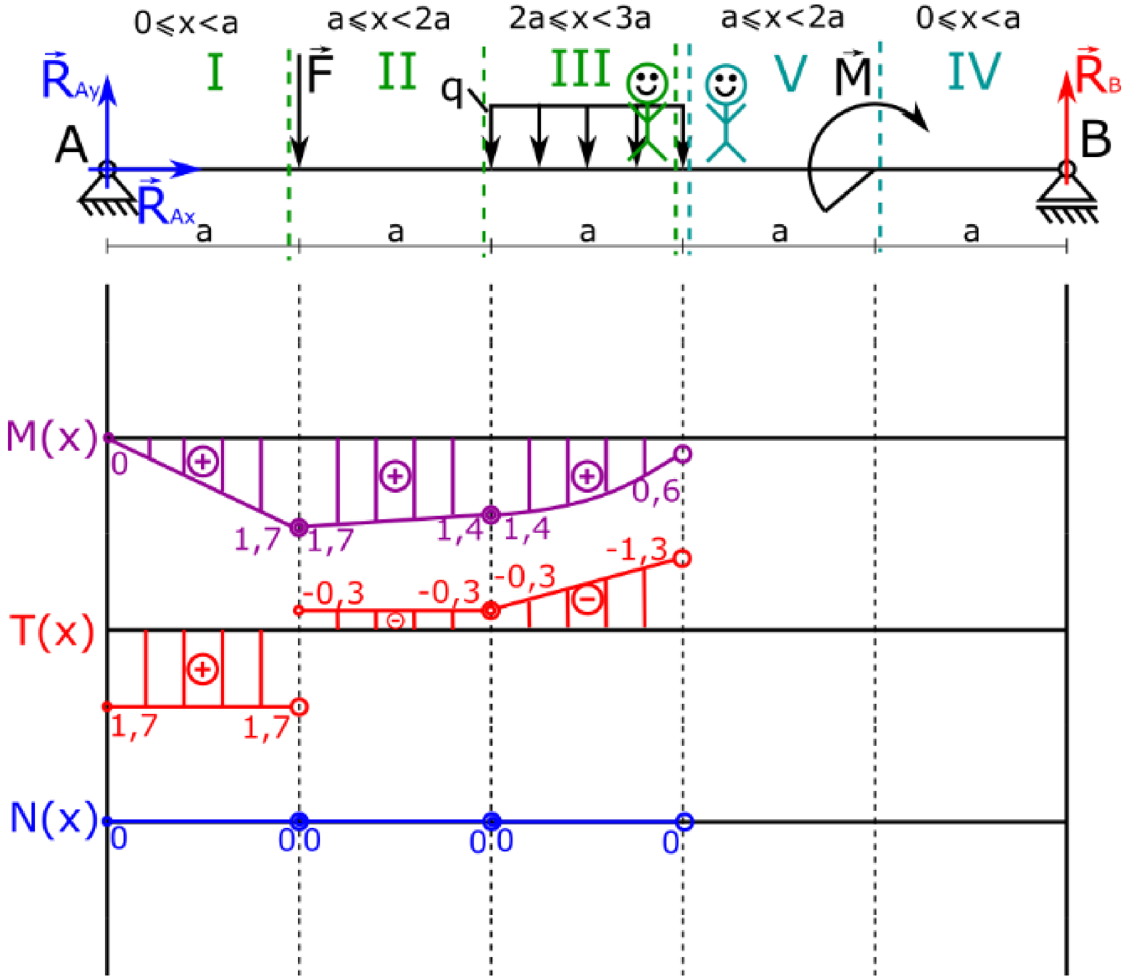
$$T(a) = R_{Ay} - F = 1{,}7 - 2 = -0{,}3kN$$

$$T(2a) = R_{Ay} - F = 1{,}7 - 2 = -0{,}3kN$$

$$N(a) = -R_{Ax} = 0$$

$$N(2a) = -R_{Ax} = 0$$

16. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the third section.

$$M(x_3) = R_{Ay} * x_3 - F * (x_3 - a) - q * (x_3 - 2a) * \frac{(x_3 - 2a)}{2}$$

$$T(x_3) = R_{Ay} - F - q * (x_3 - 2a)$$

$$N(x_3) = -R_{Ax}$$

We know the limits of the third section.

$$2a \leq x < 3a$$

We substitute the boundary values into our equations (for x_1).

$$M(2a) = R_{Ay} * 2a - F * (2a - a) - q * (2a - 2a) * \frac{(2a - 2a)}{2} = 1{,}4kNm$$

$$M(3a) = R_{Ay} * 3a - F * (3a - a) - q * (3a - 2a) * \frac{(3a - 2a)}{2} = 0{,}6kNm$$

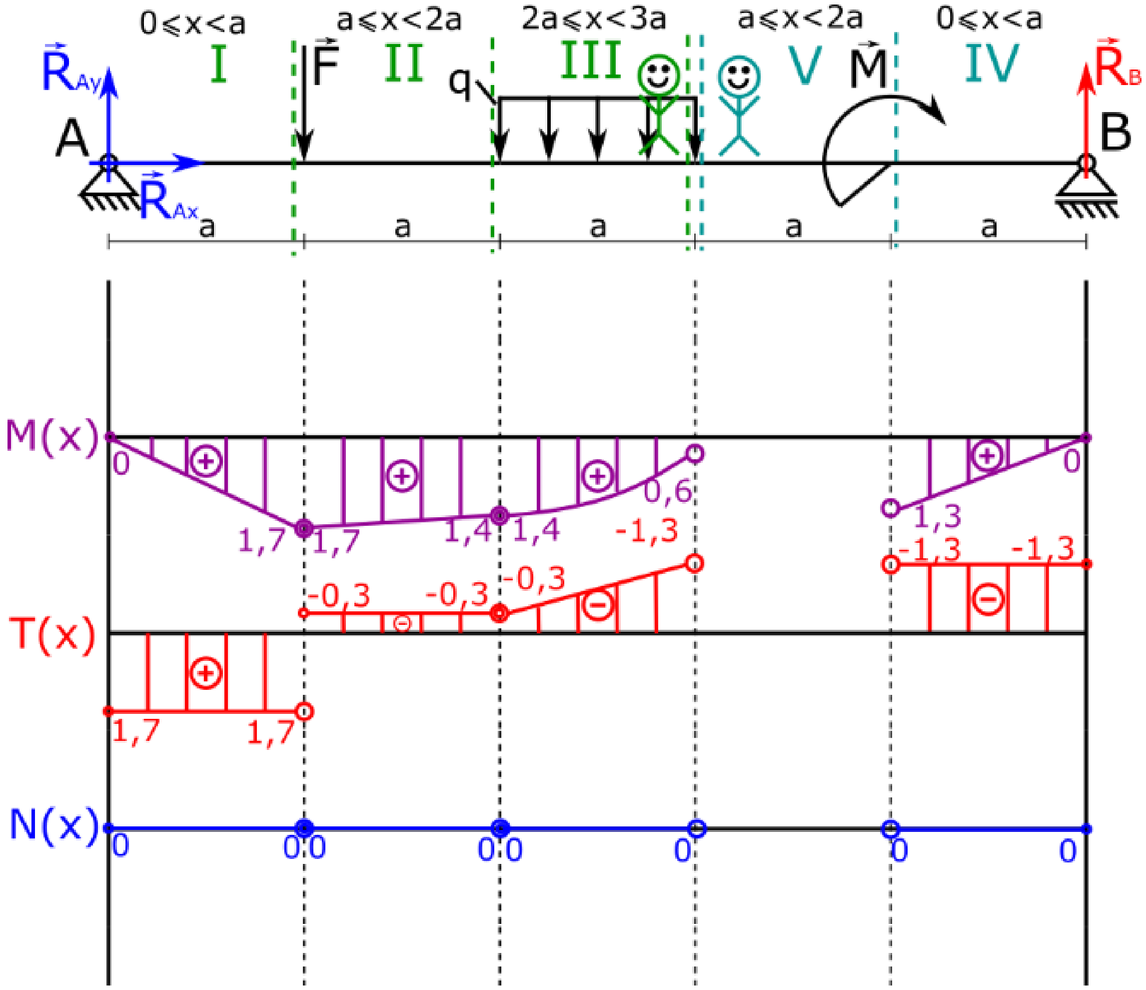
$$T(2a) = R_{Ay} - F - q * (2a - 2a) = 1{,}7 - 2 = -0{,}3kN$$

$$T(3a) = R_{Ay} - F - q * (3a - 2a) = 1{,}7 - 2 - 1 * (3 - 2) = -1{,}3kN$$

$$N(2a) = -R_{Ax} = 0$$

$$N(3a) = -R_{Ax} = 0$$

17. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the fourth section.

$$M(x_4) = R_B * x_4$$
$$T(x_4) = -R_B$$
$$N(x_4) = 0$$

We know the limits of the fourth section.

$$0 \le x < a$$

We substitute the boundary values into our equations (for x_1).

$$M(0) = R_B * 0 = 0$$

$$M(a) = R_B * a = 1{,}3 * 1 = 1{,}3kNm$$

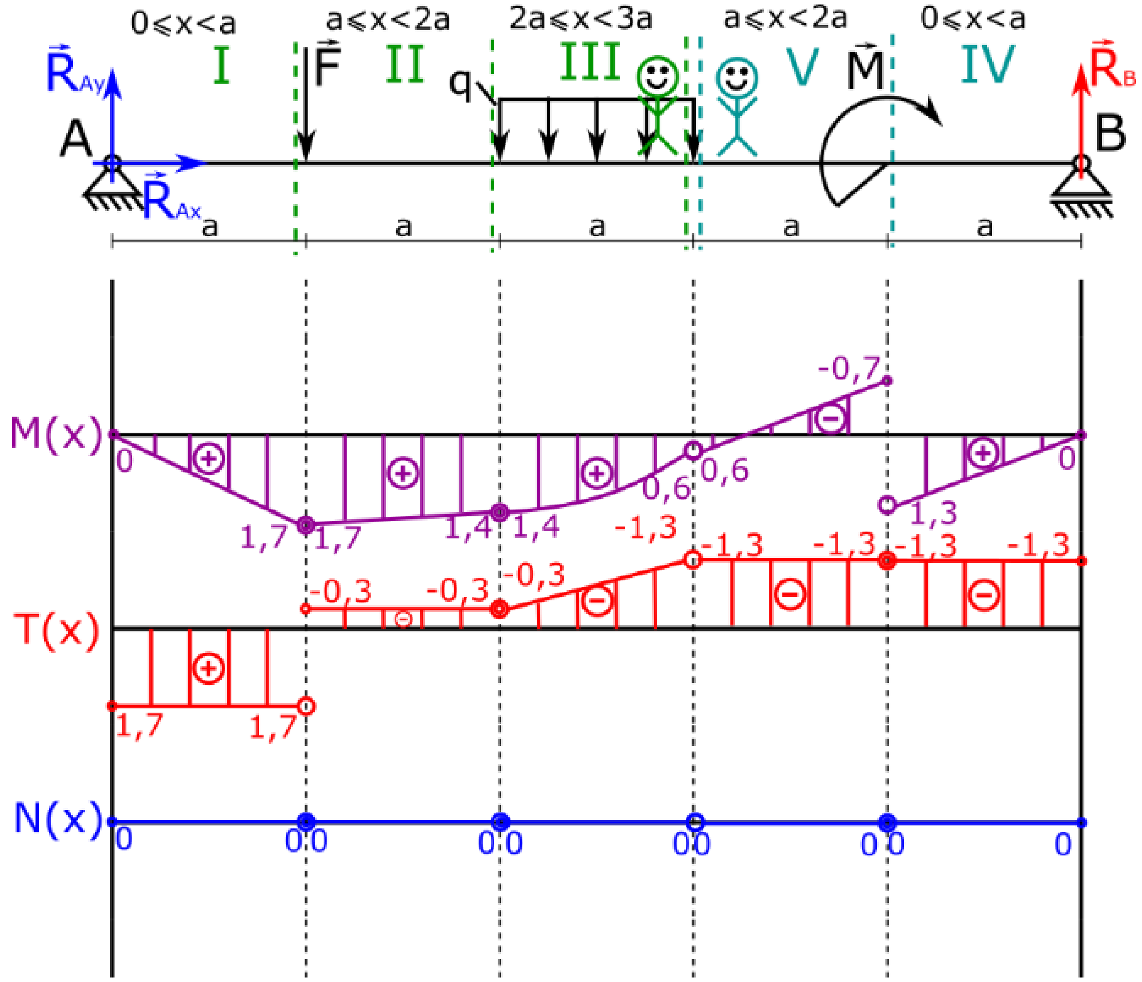
$$T(0) = -R_B = -1{,}3kN$$

$$T(a) = -R_B = -1{,}3kN$$

$$N(0) = 0$$

$$N(a) = 0$$

18. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the fifth section.

$$M(x_5) = R_B * x_5 - M$$

$$T(x_2) = -R_B$$

$$N(x_2) = 0$$

We know the limits of the fifth section.

$$a \leq x < 2a$$

We substitute the boundary values into our equations (for x_1).

$$M(a) = R_B * a - M = 1{,}3 - 2 = -0{,}7kNm$$

$$M(2a) = R_B * 2a - M = 1{,}3 * 2 - 2 = 2{,}6 - 2 = 0{,}6kNm$$

$$T(a) = -R_B = -1{,}3kN$$

$$T(2a) = -R_B = -1{,}3kN$$

$$N(a) = -R_{Ax} = 0$$

$$N(2a) = -R_{Ax} = 0$$