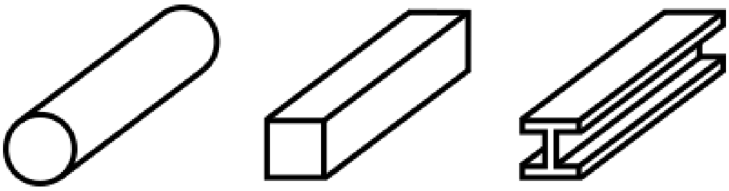
Beams - basics

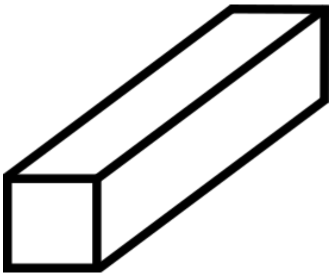
First of all it is necessary to define what we will describe as a beam.

For our purpose we will describe beam as: a rod of any cross-section, supported at one or several points, loaded with external forces that cause it to bend.

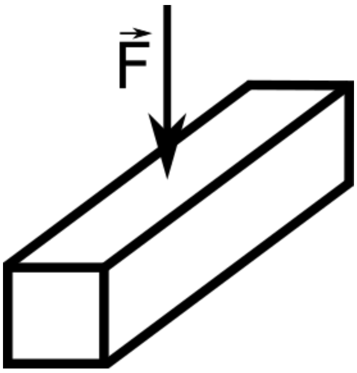


Different types of cross-sections of beams

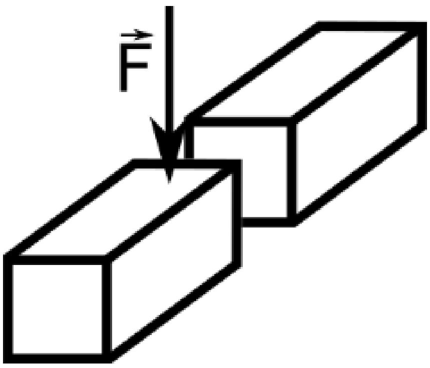
Let's choose a square beam for further considerations.



Let's assume that a certain load has been applied to this beam

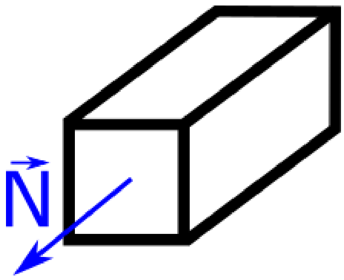


In order to check how the applied load affects the beam deformation, we will have to cut the beam in our thoughts. Such a cut of the beam causes that each of the two parts hangs in the air and in fact would collapse. To maintain system stability, internal forces must be introduced to keep the beam in balance.

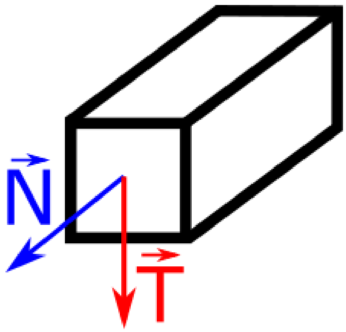


By static beam solution, we mean determining the reactions occurring at the support points and determining the bending moments, cutting forces and normal forces in the beam. These forces are the forces that balance our beam after cutting it.

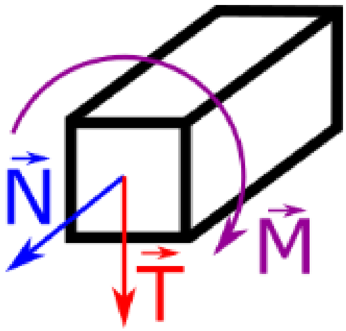
1. Let's start with normal forces. Normal forces (N) - horizontal force (acting along the length), equal to the algebraic sum of all external forces acting on one side of the section under consideration.



2. Cutting forces (T) - vertical force (acting transversely), equal to the algebraic sum of all external forces acting on one side of the section under consideration.



3. Bending moments (M) - a vector lying in the plane of the section, with a numerical value equal to the algebraic sum of moments of all external forces acting on one side of the section under consideration, due to the center of this section.

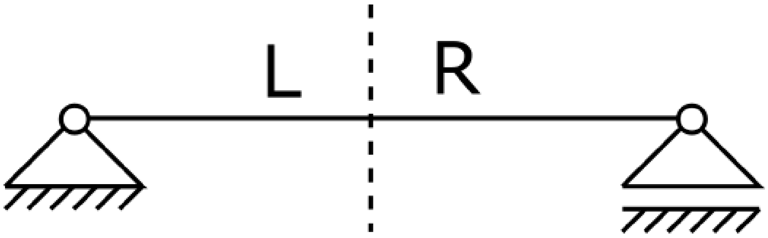


The abovementioned internal forces will be the forces we will count when solving the beams.

How will we draw beams? You could draw beams as shown below.

However, additional simplification is usually performed and the beams look schematically as follows. As someone previously noted, it can be quite misleading compared to trusses. However, the method of loading immediately dispels doubts whether we are dealing with a beam or a truss.

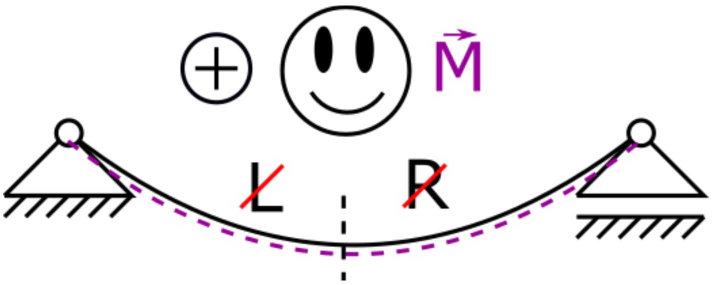
Another important issue related to beams is, as I mentioned before, the need to make cuts. When we make the cut, our beam automatically divides into two parts. Left and right. And so we will look at the beams. Depending on which side of the beam we are on, we colloquially say that we enter the beam from its left or right side and move to the place of cutting.



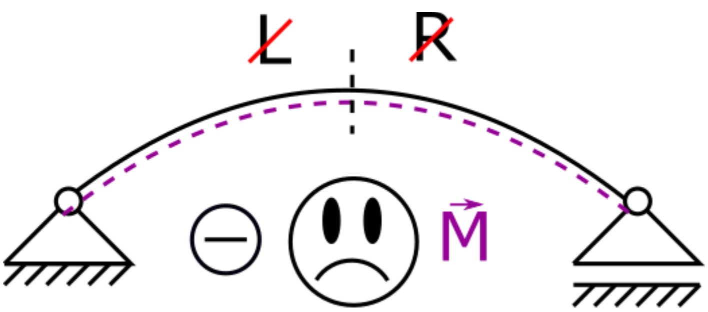
At this point, we will introduce a very important thing, namely the notation according to which we will take positive and negative values of our internal forces: bending moments, cutting forces and normal forces.

Let's start with the bending moments.

Bending moments will bend the beam. The beam can bend down so that the lower fibers (dashed line) will be stretching. Then we will assume that such bending moments are positive. It can be said that the beam is smiling at this moment, i.e. we have positive value.

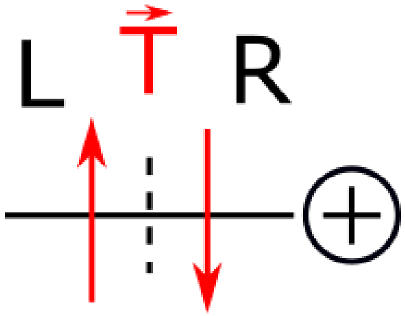


Of course, the beam can also bend upwards, so that the lower fibers (dashed line) are compressed. Then we will assume that such bending moments are negative. It can be said that the beam is sad at the moment, i.e. we have negative values.

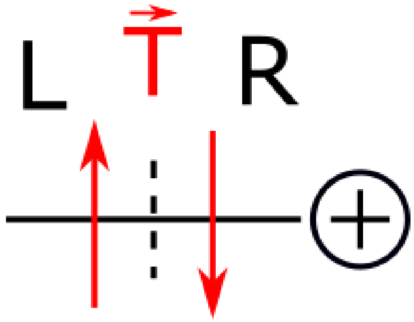


In the case of bending moments, the sign (positive or negative) will depend only on the way the beam is bent, regardless of whether we are on its left or right side.

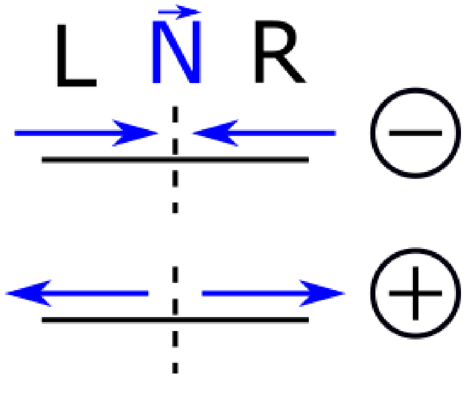
Then let's get to the cutting forces. In the case of cutting forces, the mark will depend on how these forces act on one side of the cross-section. If on the left side of the cross-section the cutting forces go up and on the right down, then the cutting forces will be taken with a positive sign.



Of course, if on the left side of the cross-section the forces are going down and on the right up, then we have the opposite situation and the cutting forces will be with a negative sign.



Finally, normal forces remained. For normal forces, the sign depends on whether the forces are directed towards the cut or to the cut. When the forces come out of the beam (the sense is from the cutting point), then we assume that they have positive values. In the opposite case, i.e. when their sense is into the cutting direction (they enter the beam), we assume that these forces have a negative sign.

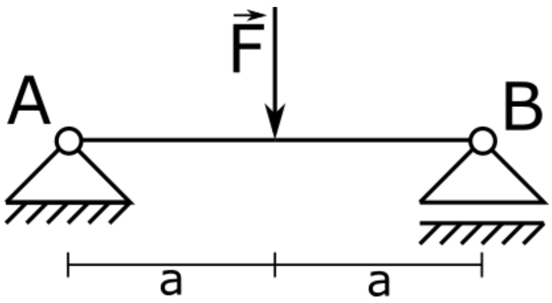


The above notation is very important because it will be used when solving all beams and frames.

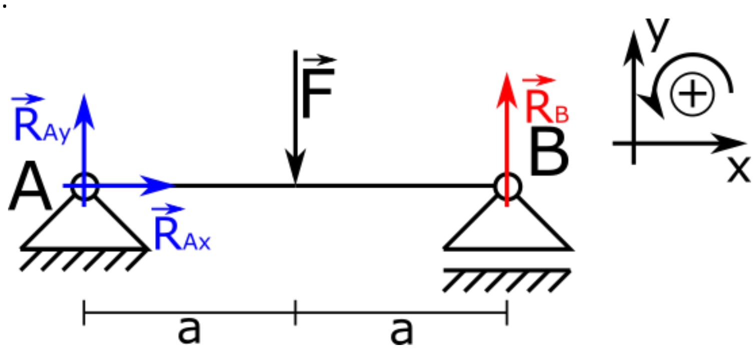
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam. An important information, the beam solution also includes drawing internal force diagrams.

Ex.1

For the beam shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a = 1 [m], F = 10 [kN].



1. The first thing to do when solving beams is to determine the number of supporting unknowns, as was the case with trusses. It is clear that in this case we have three unknown supports. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, anticlockwise turning moments will be positive.



2. Now we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

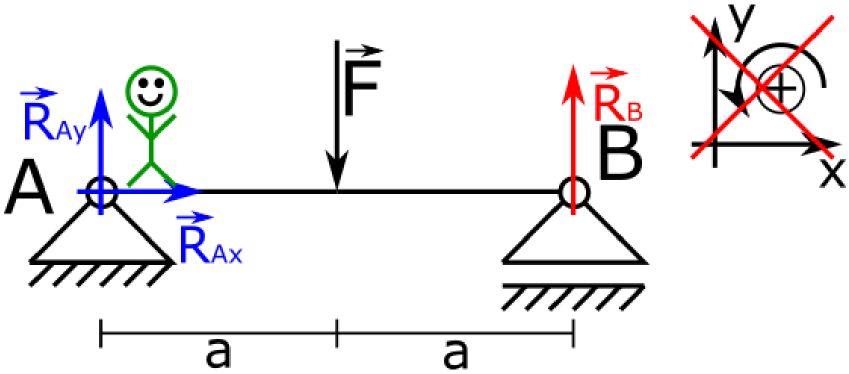
$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

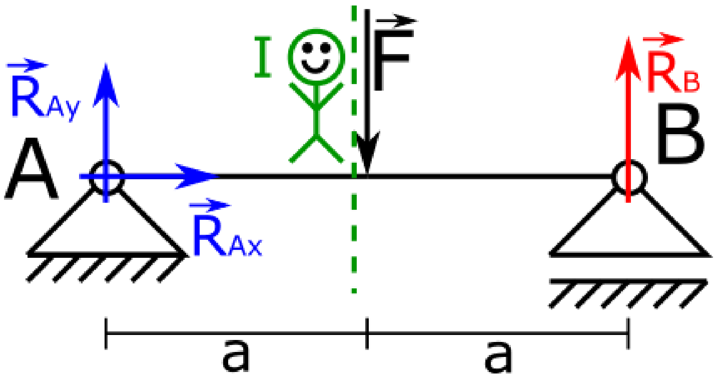
$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - F + R_B \rightarrow R_{Ay} = F - R_B = 10 - 5 = 5kN$$

$$\sum_{i=1}^{n} M_A = 0 = -F * a + R_B * 2a \rightarrow R_B = \frac{F}{2} = 5kN$$

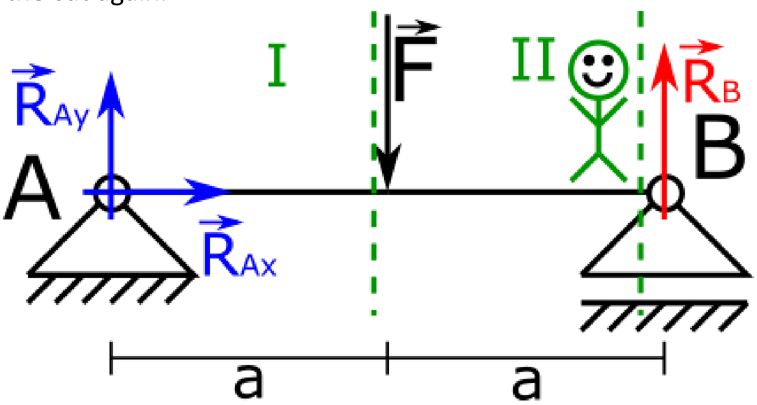
3. This is where the similarity to solving trusses ends.
4. After determining the reaction in the supports, in the next step we need to determine how many cuts the beam needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces that was introduced at the beginning. We enter the beam from its left (you can also enter the beam from the right).



5. Cuts will be made on the beam in places where something happens on the beam (something appears or disappears). The important information is that we always cut before something on the beam has happened.
   We see that when we enter the beam from its left, forces R_{Ax} and R_{Ay} appear. However, we cannot make the cut before these forces, because then we are not yet on the beam.
   We go further along the beam and come across force F. Something happened, force appeared. So we know that in this place just before the appearance of force F should be cut.



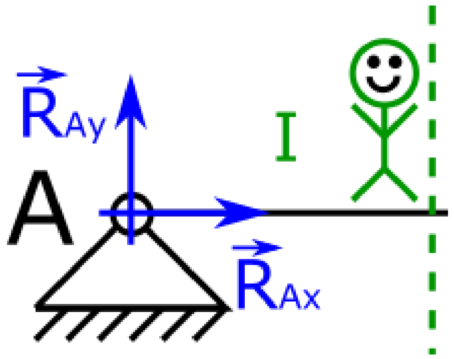
6. After making this cut, we bypass the F force and continue until we come to the R_B force. Again, something is happening, strength comes to our way, so at this point just before the R_B force we make the cut again.



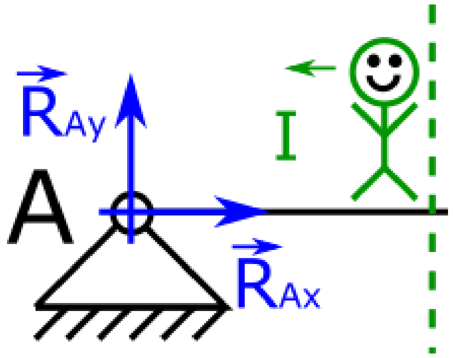
7. In this way, we divided the beam into two sections I and II. The first section within $0 \leq x < a$ second section within $a \leq x < 2a$.

BENDING MOMENTS

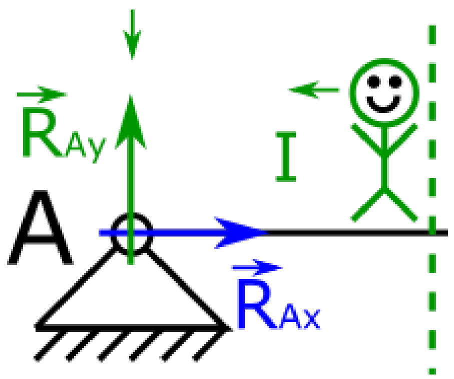
8. At this point, we can move on to determining internal forces. To do this we have to go through each of the sections. We assume that we enter the beam on the left, as before and reach the end of the first section. We will begin to determine internal forces from bending moments, then cutting forces and finally normal forces.



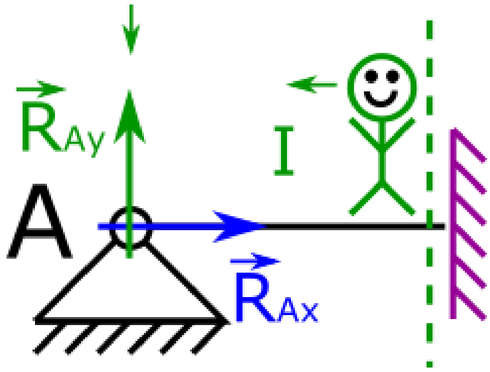
9. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



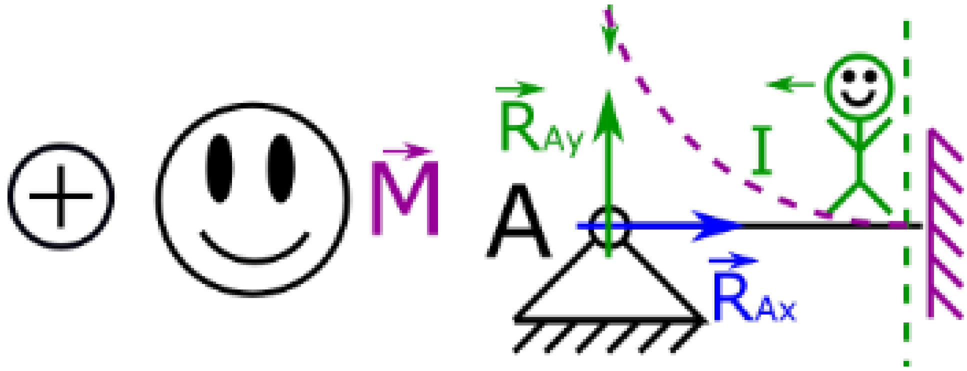
10. We see that at the beginning of the beam there is one force that causes the beam to bend, R_{Ay} force.



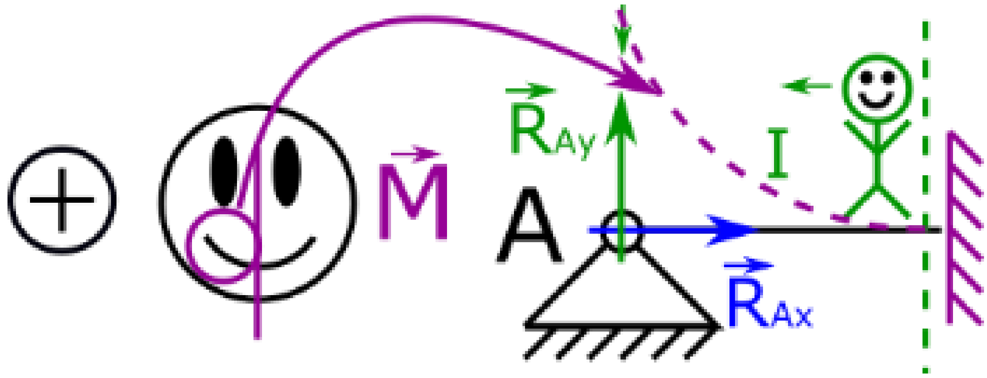
11. Now, to determine how the force bends the beam, we assume that at the place where we stand (where the cut), we catch the beam rigidly.



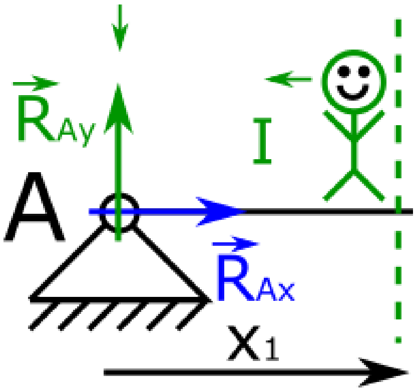
12. At this point, when one end of the beam is fixed, we imagine how the force R_{Ay} tries to bend the beam. You can see clearly in this situation, under the influence of this force, the beam would bend, as shown by the purple dashed line.



13. According to the notation introduced earlier, if the beam under the influence of force smiles, then we assume that the bending moment from such force is a positive sign. Of course, when cutting, it is clearly seen that we get only half of the smile.



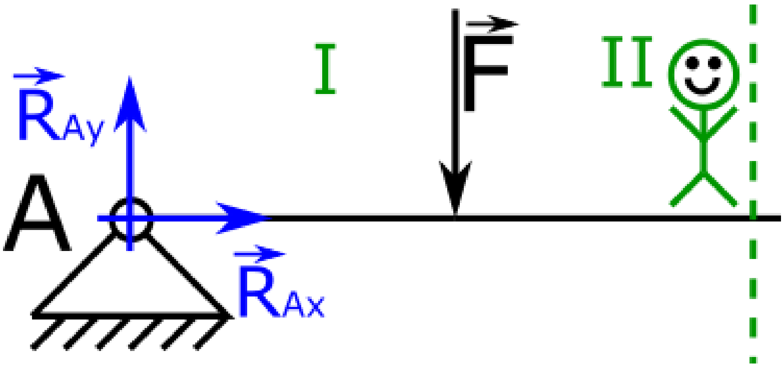
14. We already know what sign the bending moment from R_{Ay} will have, now the arm of this moment should be determined. Because we are standing just before the appearance of the force F, it only means that we are at a distance of some x from the beginning of the beam. Let's call this distance x_1, since we're in the first section.



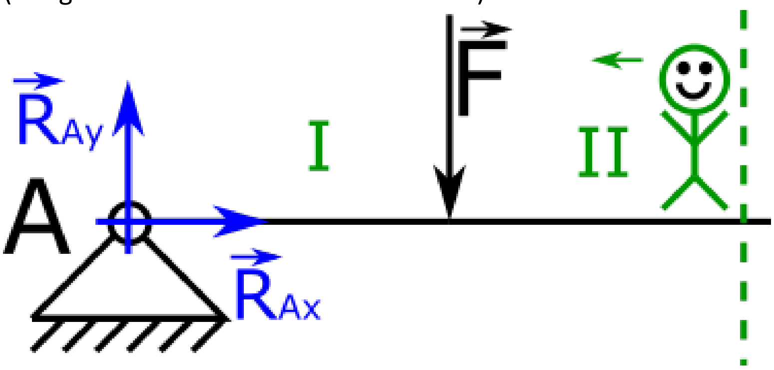
15. After these considerations, we can write the first equation for bending moments in the first section.

$$M(x_1) = R_{Ay} * x_1$$

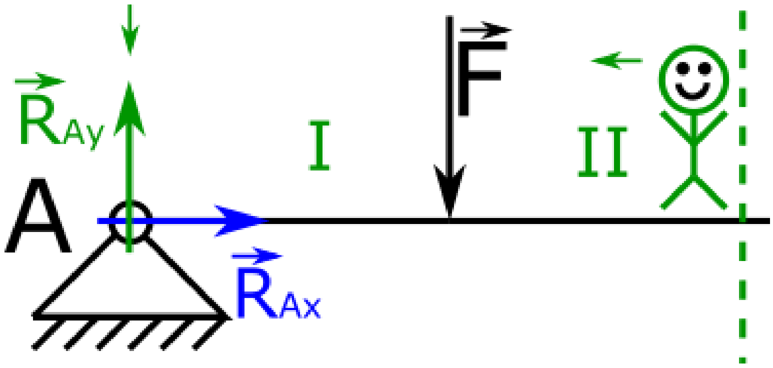
16. Now we move to the second section. The important thing is that the first section remains. You can imagine that we just walk along the beam and we can't cut what is behind us, and the beam just lengthens.



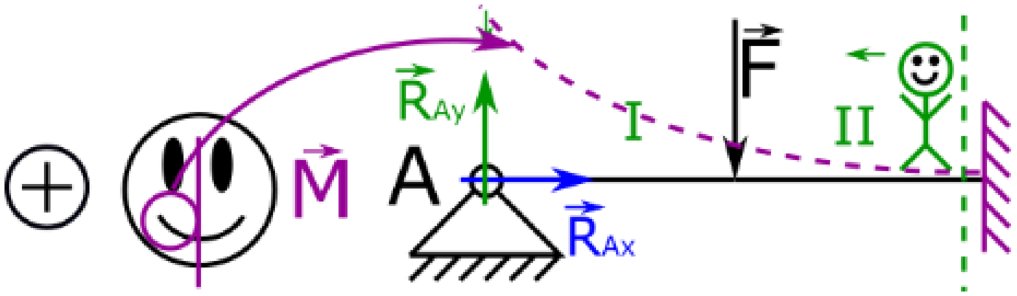
17. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



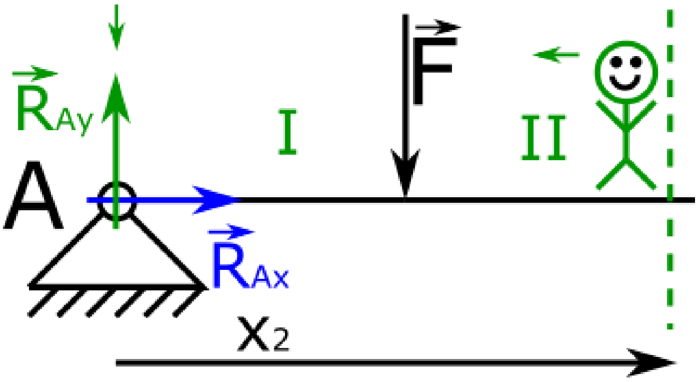
18. We see that at the beginning of the beam there is one force that causes the beam to bend, R_{Ay} force.



19. Based on the previous section, we can see that this force still bends the beam upwards, hence the moment bending from this force will still be with a positive sign.



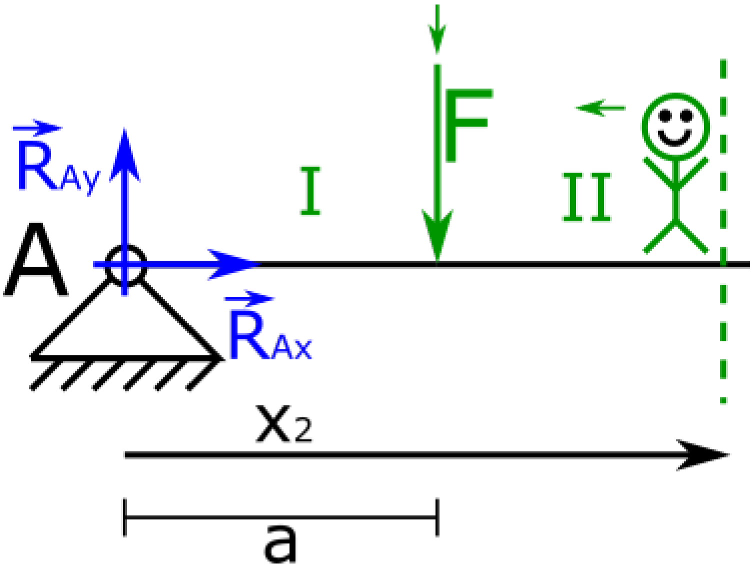
20. Now it is necessary to determine again what the arm of the created moment is. It is clear that we are standing further and we will call this distance as x_2.



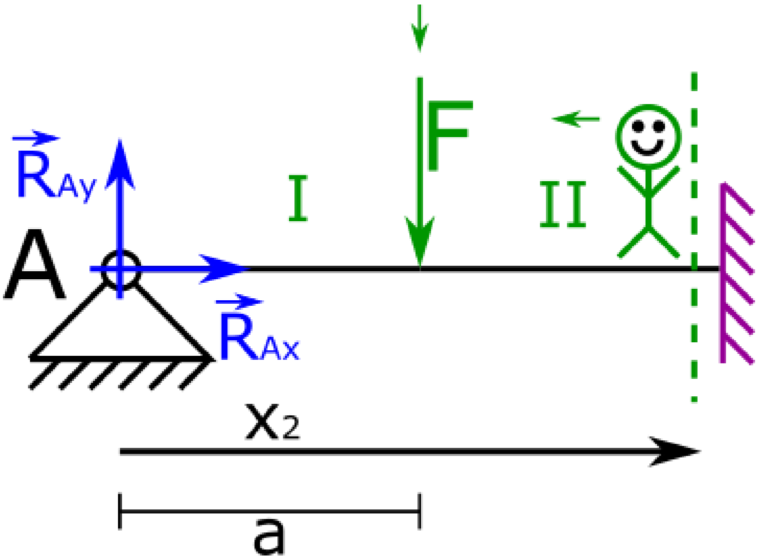
21. We can write first part of equation for bending moments in the second section. We will write the next part after further considerations.

$$M(x_2) = R_{Ay} * x_2 \dots$$

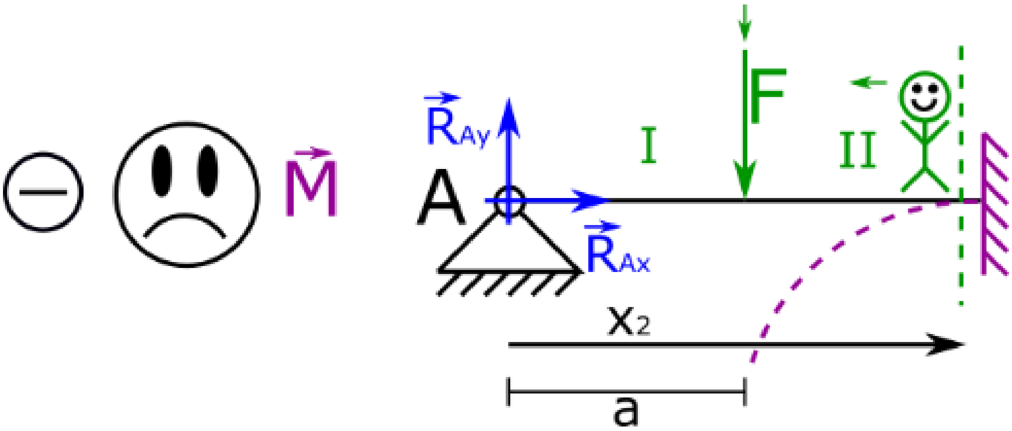
22. We see that next force on the beam that causes the beam to bend is F force.



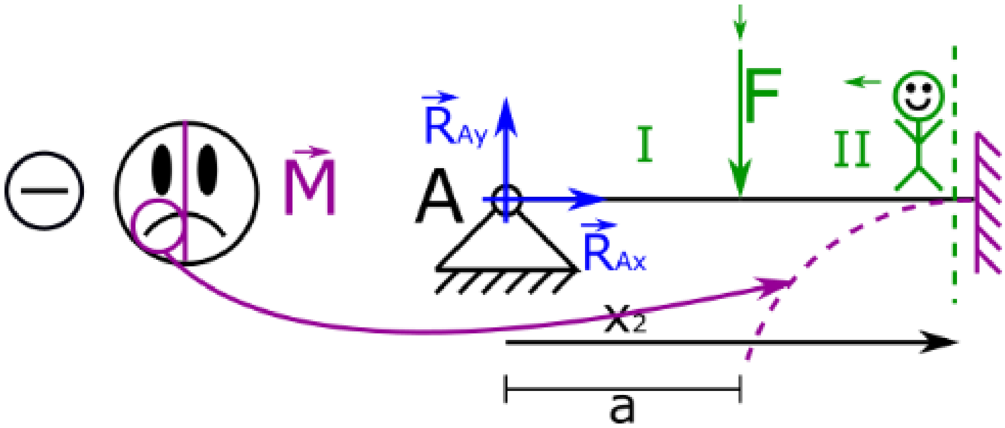
23. Now, to determine how the force bends the beam, we assume that at the place where we stand (where the cut), we catch the beam rigidly.



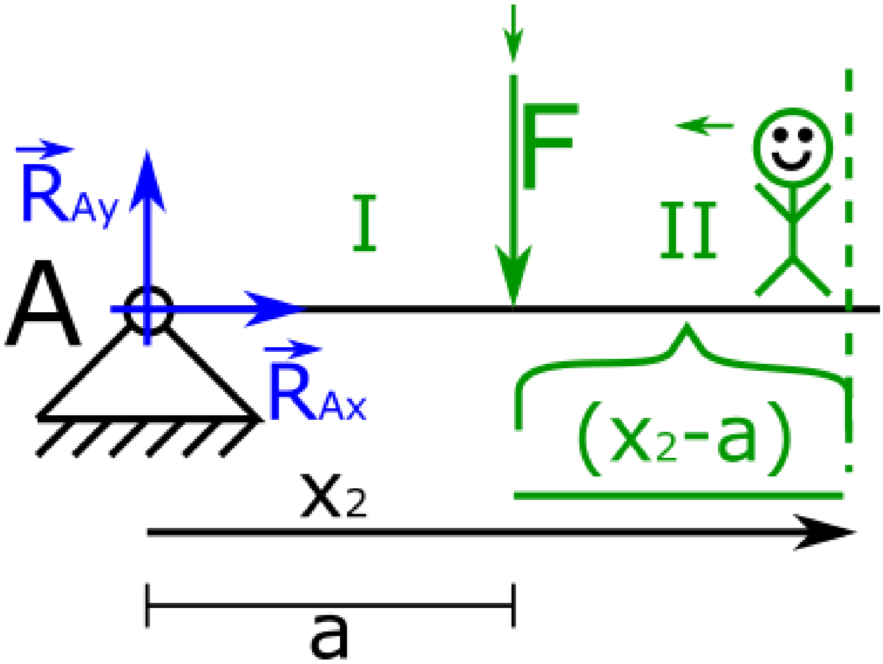
24. At this point, when one end of the beam is fixed, we imagine how the force F tries to bend the beam. You can see clearly in this situation, under the influence of this force, the beam would bend, as shown by the purple dashed line.



25. According to the notation introduced earlier, if the beam under the influence of force is sad, then we assume that the bending moment from such force is a negative sign. Of course, when cutting, it is clearly seen that we get only half of sad face.



26. We already know what sign the bending moment from F will have, now the arm of this moment should be determined. Because we are standing just before the appearance of the force R_B, it only means that we are at a distance of difference between x_2 and a from the first section of the beam. The arm of this bending moment will be equal (x_2-a).

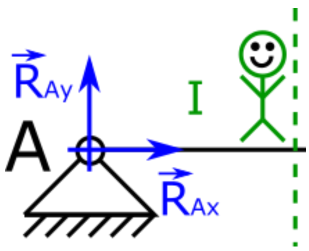


27. We can write final of equation for bending moments in the second section. We need to take first part and add second part.

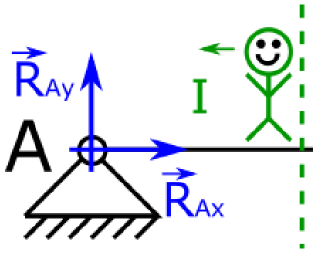
$$M(x_2) = R_{Ay} * x_2 - F * (x_2 - a)$$

CUTTING FORCES

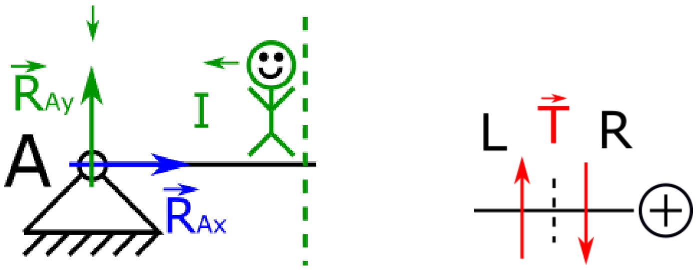
28. Now we can move to the next internal forces, i.e. the cutting forces. We will consider these forces similarly to the previous section after the section.



29. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



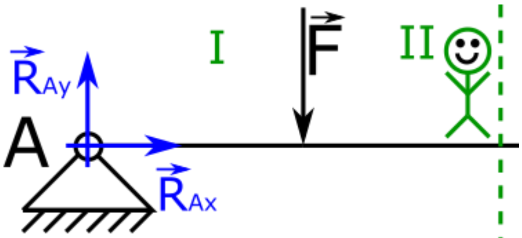
30. We see that at the beginning of the beam there is one force that seems to cut the beam, R_{Ay} force. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ay}'s is up, which means that we will take this force with a positive sign.



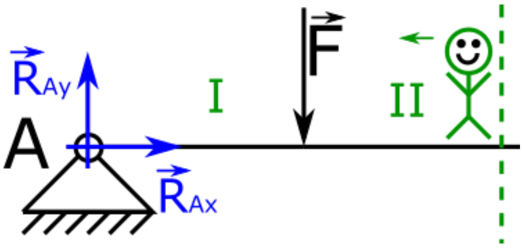
31. After these considerations, we can write the first equation for cutting forces in the first section.

$$T(x_1) = R_{Ay}$$

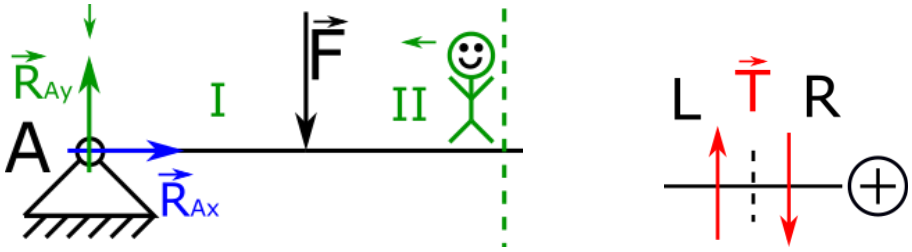
32. Now we move to the second section. The important thing is that the first section remains. You can imagine that we just walk along the beam and we can't cut what is behind us, and the beam just lengthens.



33. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



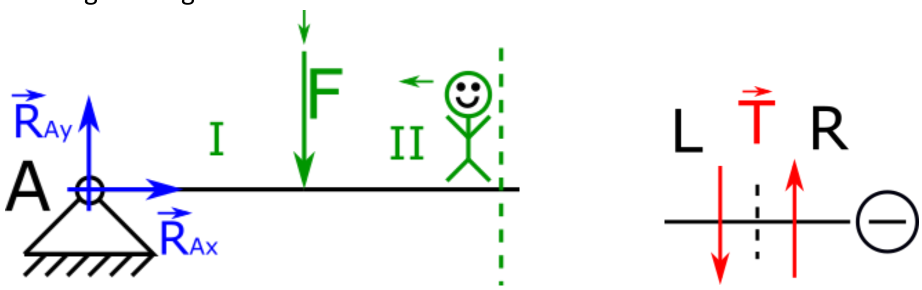
34. We see that at the beginning of the beam there is one force that seems to cut the beam, R_{Ay} force. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ay}'s is up, which means that we will take this force with a positive sign. So it is the same, as it was for the first section.



35. We can write first part of equation for cutting forces in the second section. We will write the next part after further considerations.

$$T(x_2) = R_{Ay} \dots$$

36. We see that next force on the beam that seems to cut the beam is F force. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of F's is down, which means that we will take this force with a negative sign.

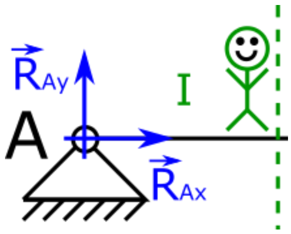


37. We can write final of equation for cutting forces in the second section. We need to take first part and add second part.

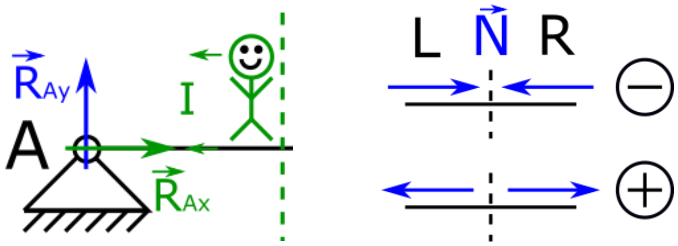
$$T(x_2) = R_{Ay} - F$$

NORMAL FORCES

38. Now we can move to the final internal forces, i.e. the normal forces. We will consider these forces similarly to the previous section after the section.



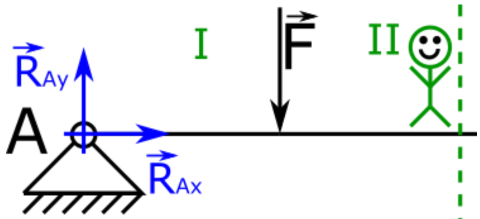
39. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look). We see that at the beginning of the beam there is one force acting along the beam, R_{Ax} force. Now let's get back to our assumptions, from the beginning, for normal forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ax}'s is into the cut, which means that we will take this force with a negative sign.



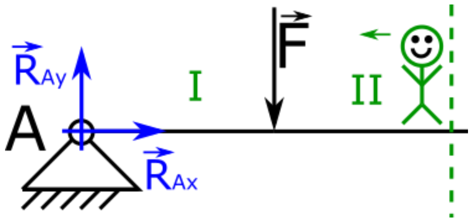
40. After these considerations, we can write the first equation for cutting forces in the first section.

$$N(x_1) = -R_{Ax}$$

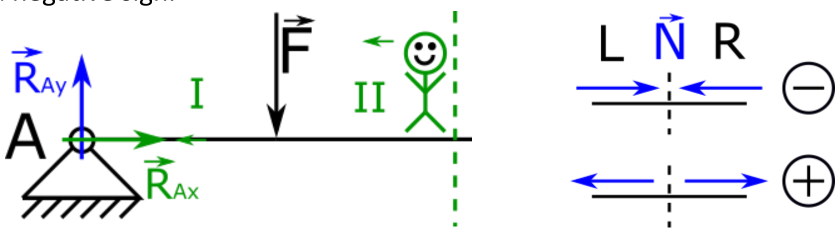
41. Now we move to the second section. The important thing is that the first section remains. You can imagine that we just walk along the beam and we can't cut what is behind us, and the beam just lengthens.



42. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



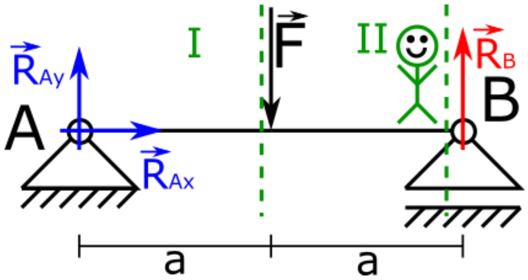
43. We see that at the beginning of the beam there is one force acting along the beam, R_{Ax} force, what is more this is the only force which is acting along the beam. Now let's get back to our assumptions, from the beginning, for normal forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ax}'s is into the cut, which means that we will take this force with a negative sign.



44. After these considerations, we can write the first equation for cutting forces in the first section.

$$N(x_2) = -R_{Ax}$$

45. Let's write equations for both of sections in one place to clarify everything.



I Section $0 \leq x_1 < a$

$$M(x_1) = R_{Ay} * x_1$$

$$T(x_1) = R_{Ay}$$

$$N(x_1) = -R_{Ax}$$

II Section $a \leq x_2 < 2a$

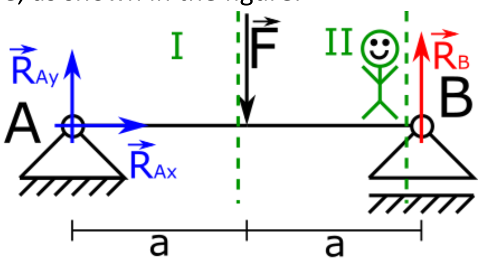
$$M(x_2) = R_{Ay} * x_1 - F * (x_2 - a)$$

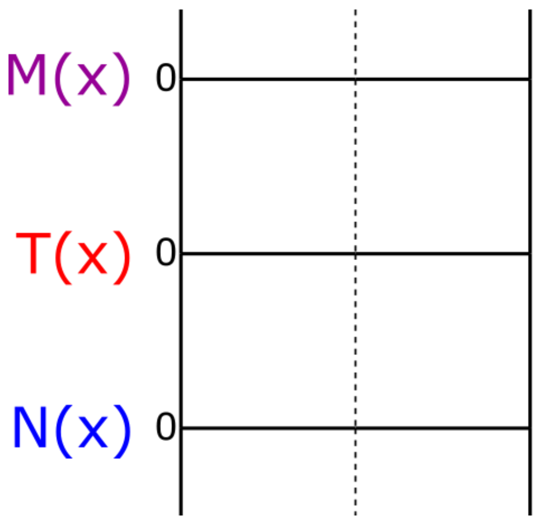
$$T(x_2) = R_{Ay} - F$$

$$N(x_2) = -R_{Ax}$$

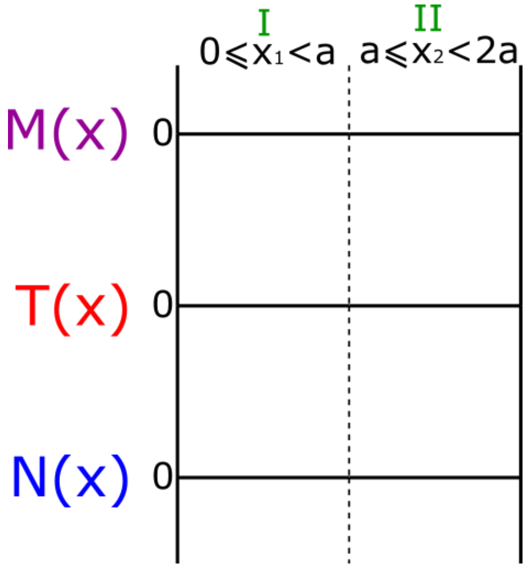
CHARTS

46. The last part related to solving beams – charts. To easily draw charts, it is best to draw them under the beam, which we solve, as shown in the figure.





47. In this example, the charts will be drawn step by step so that you can understand their creation. The first graphs will be made for the first section starting from the bending moments graph.



48. We will need the equation of bending moments for the first section.

$$M(x_1) = R_{Ay} * x_1$$

We know the limits of the first section.

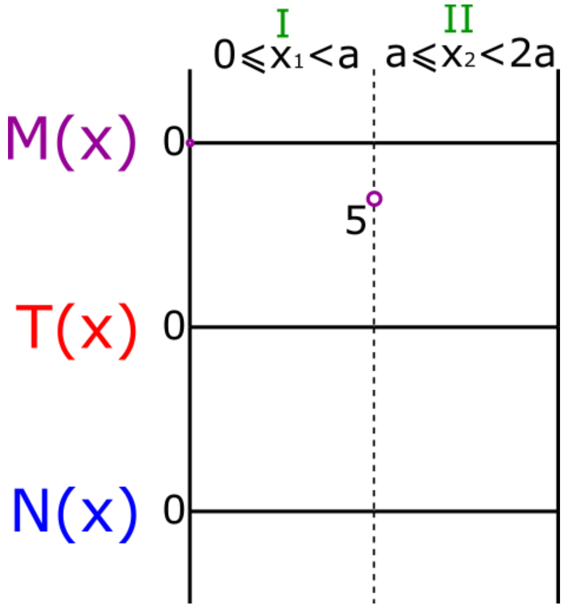
$$0 \leq x < a$$

We substitute the boundary values into our equation (for x_1).

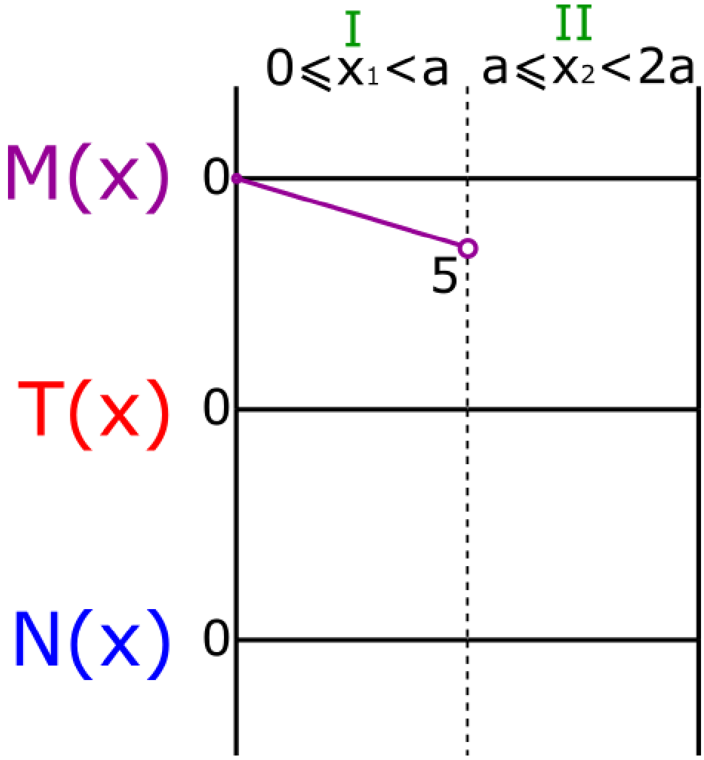
$$M(0) = R_{Ay} * 0 = 0$$

$$M(a) = R_{Ay} * a = 5 * 1 = 5kNm$$

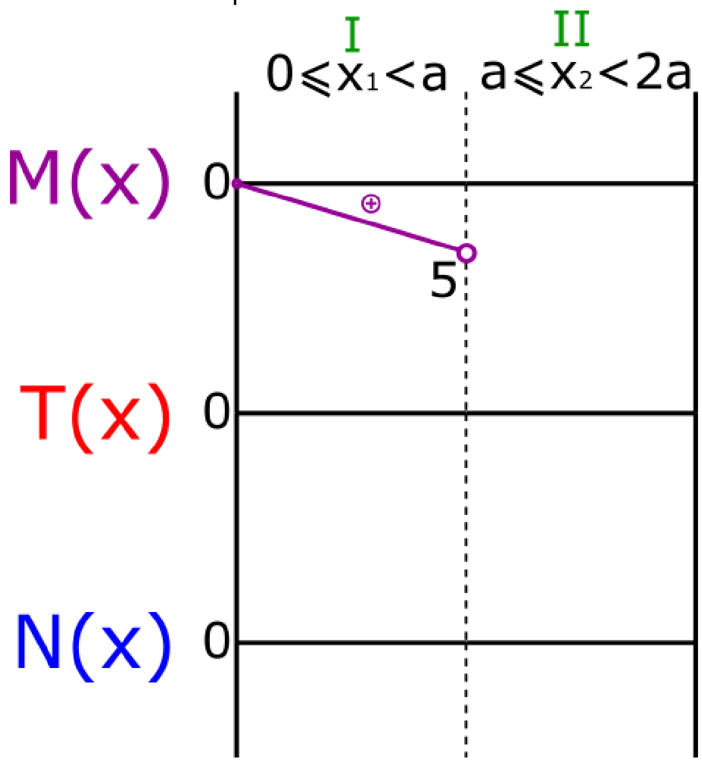
49. When we know the values of the bending moment at the ends of the section we put these values on the graph. **Important information is that we will write positive values at the bottom of the chart and negative values at the top.**



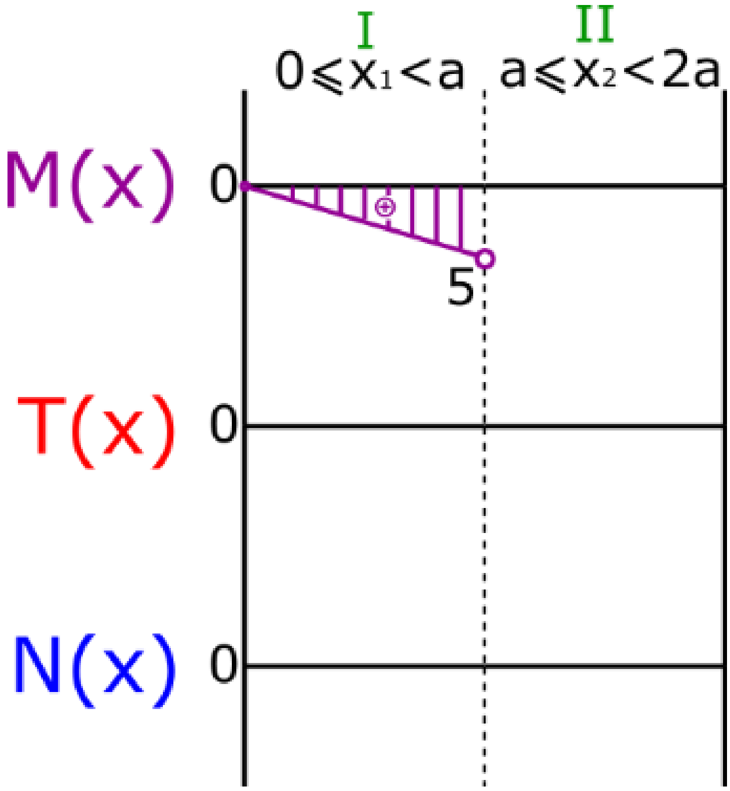
50. Then we consider what kind of function describes the bending moment equation we wrote. It can be clearly seen that this is also a linear equation, what's more it is a growing function. Based on this information, we can connect the points previously marked as shown below.



51. We further mark that we are on the positive side.



52. Finally, we dash this chart. **Important information that the lines must be vertical, any other hatching, painting the chart will be incorrect.**



53. We will need the equation of cutting forces for the first section.

$$T(x_1) = R_{Ay}$$

We know the limits of the first section.

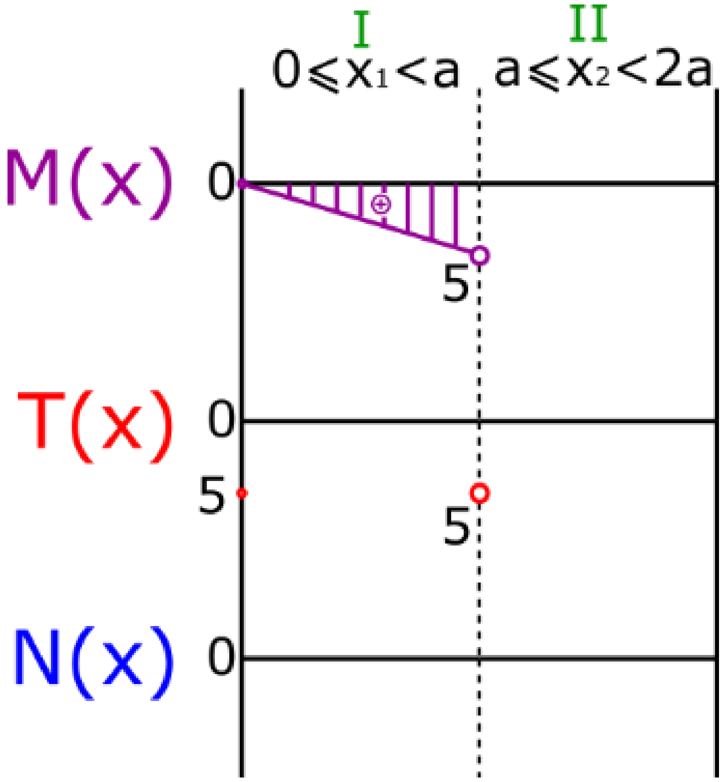
$$0 \leq x < a$$

We substitute the boundary values into our equation (for x_1).

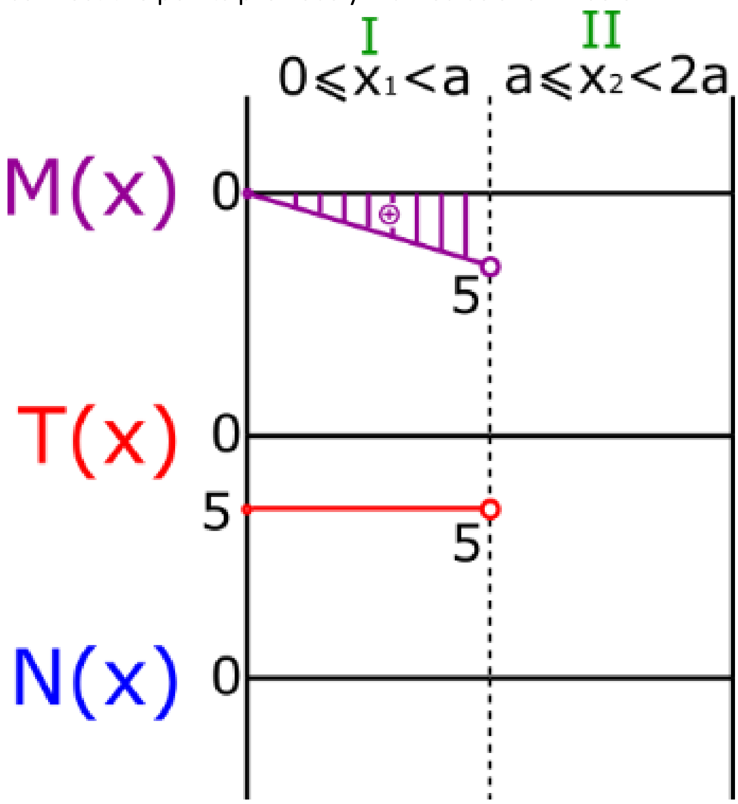
$$T(0) = R_{Ay} = 5kN$$

$$T(a) = R_{Ay} = 5kN$$

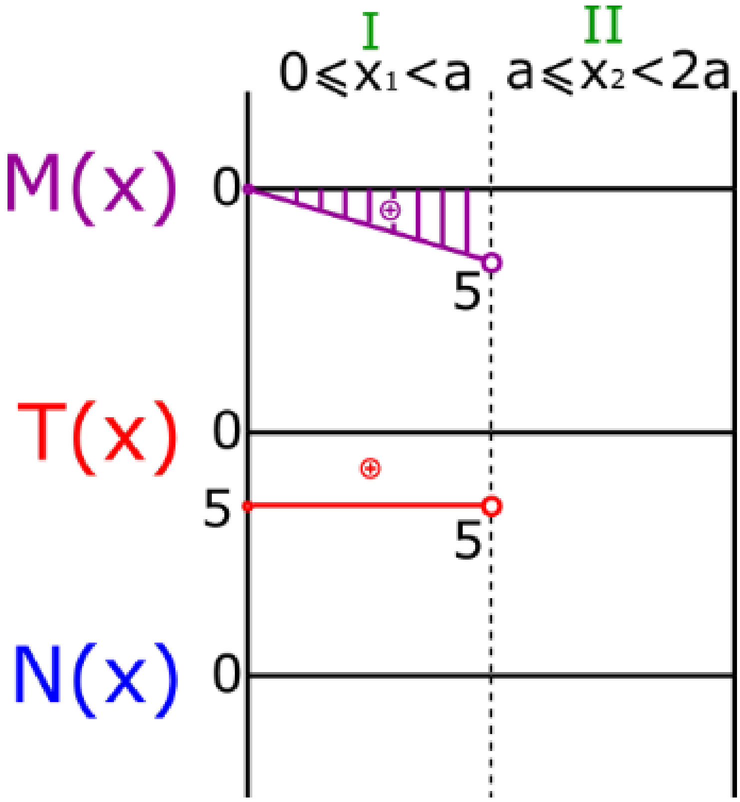
54. When we know the values of the cutting forces at the ends of the section we put these values on the graph.



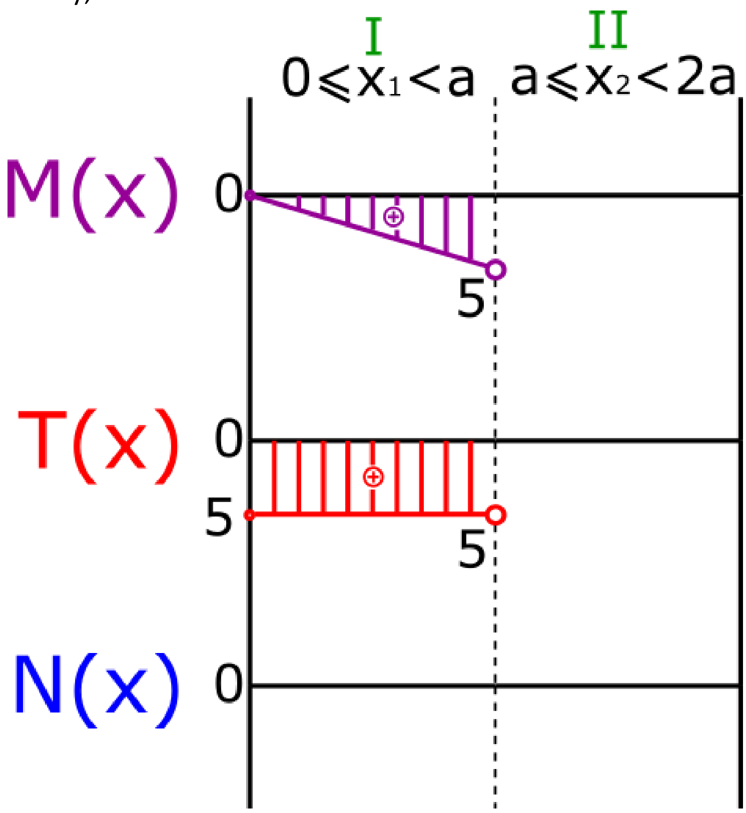
55. Then we consider what kind of function describes the cutting forces equation we wrote. It can be clearly seen that this is a constant equation. Based on this information, we can connect the points previously marked as shown below.



56. We further mark that we are on the positive side.



57. Finally, we dash this chart.



58. We will need the equation of normal forces for the first section.

$$N(x_1) = -R_{Ax}$$

We know the limits of the first section.

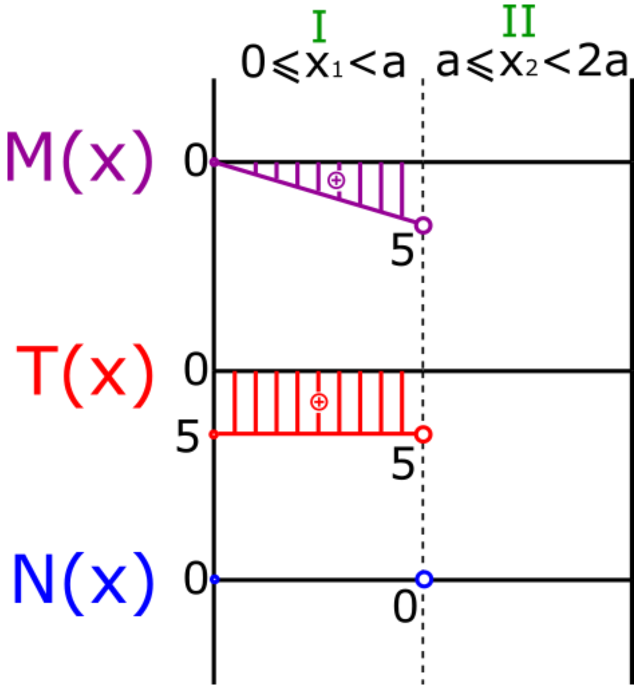
$$0 \le x < a$$

We substitute the boundary values into our equation (for x_1).

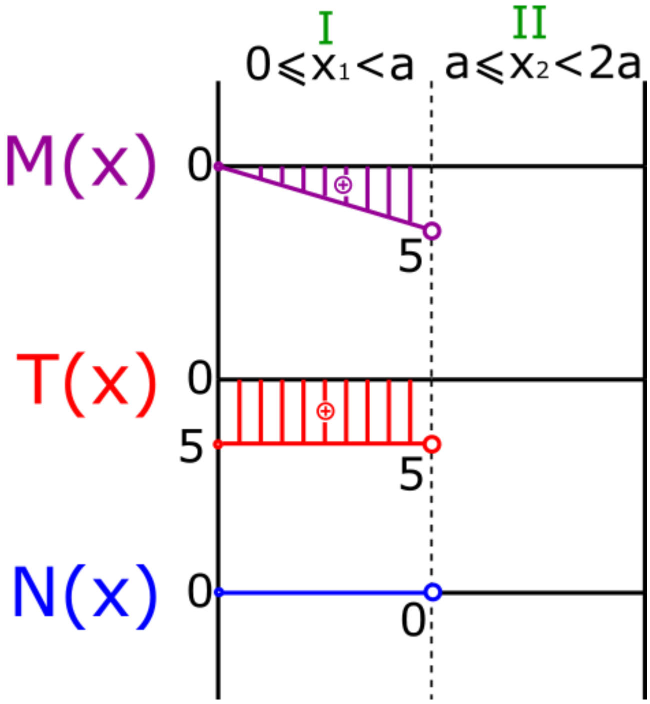
$$N(0) = -R_{Ax} = 0kN$$

$$N(a) = -R_{Ax} = 0kN$$

59. When we know the values of the normal forces at the ends of the section we put these values on the graph.



60. Then we consider what kind of function describes the normal forces equation we wrote. It can be clearly seen that this is a constant equation. Based on this information, we can connect the points previously marked as shown below.



61. We repeat the steps for the second section.
    We will need the equation of bending moments for the second section.

$$M(x_2) = R_{Ay} * x_2 - F * (x_2 - a)$$

We know the limits of the second section.

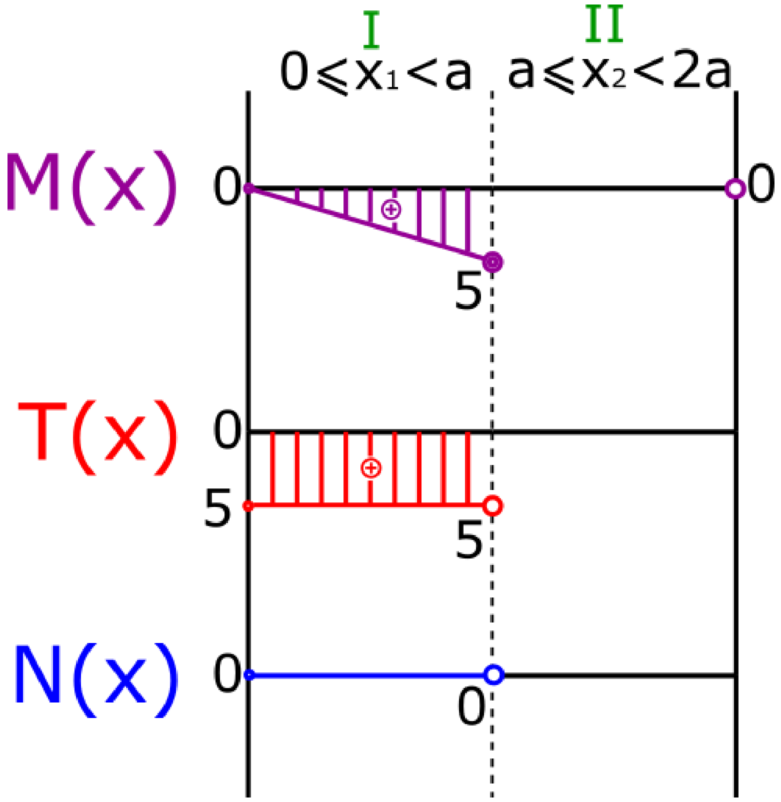
$$a \leq x < 2a$$

We substitute the boundary values into our equation (for x_2).

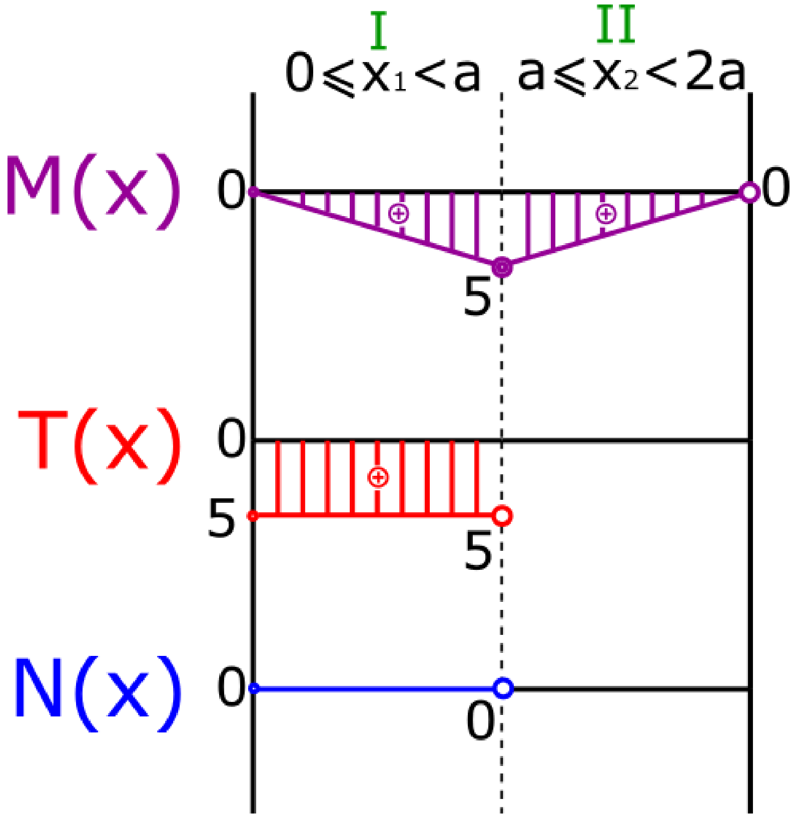
$$M(a) = R_{Ay} * a - F * (a - a) = 5kNm$$

$$M(2a) = R_{Ay} * 2a - F * (2a - a) = 5 * 2 - 10 * 1 = 0kNm$$

62. When we know the values of the bending moment at the ends of the section we put these values on the graph.



63. Now we consider what kind of function describes the bending moment equation we wrote. It can be clearly seen that this is also a linear equation, what's more it is a descending function. Based on this information, we can connect the points previously marked, mark that we are on the positive side, and finally, we dash this chart.



64. We will need the equation of cutting forces for the second section.

$$T(x_2) = R_{Ay} - F$$

We know the limits of the second section.

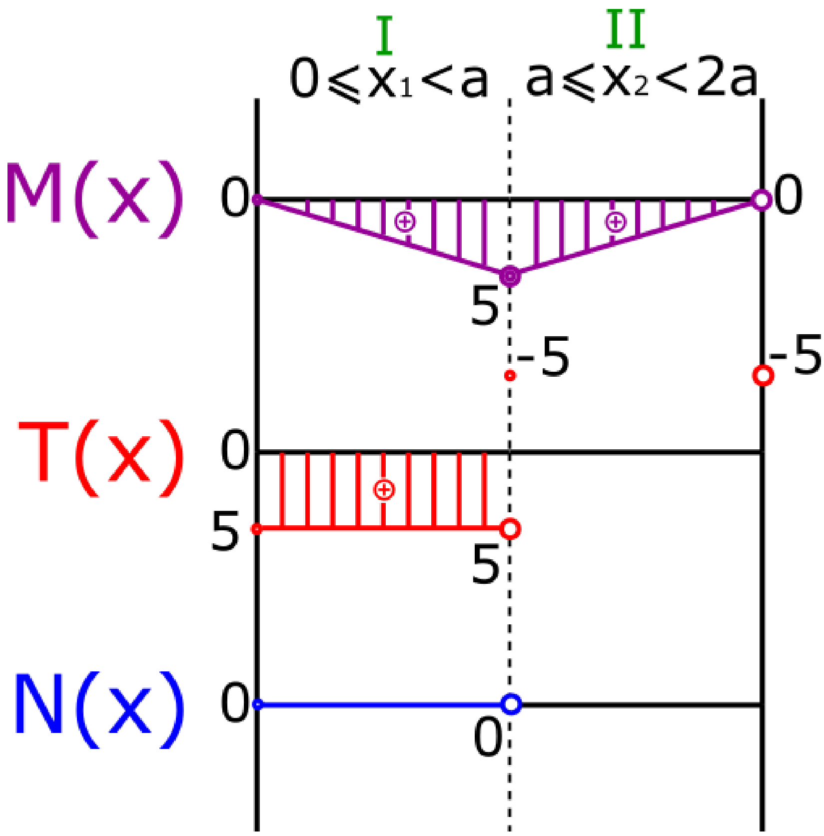
$$a \leq x < 2a$$

We substitute the boundary values into our equation (for x_1).

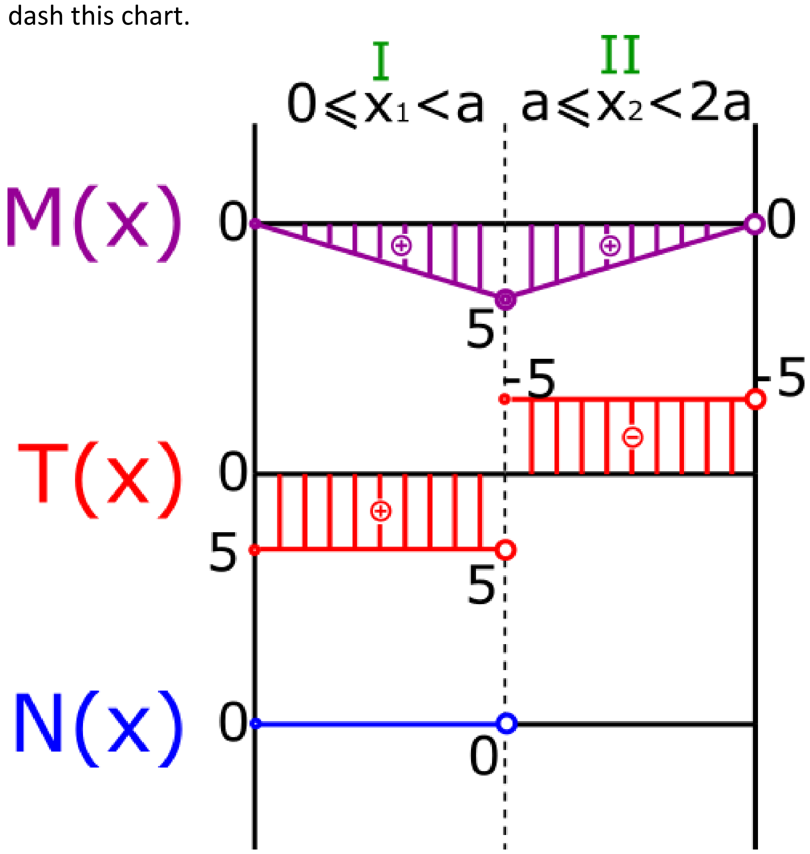
$$T(a) = R_{Ay} - F = -5kN$$

$$T(2a) = R_{Ay} - F = -5kN$$

65. When we know the values of the bending moment at the ends of the section we put these values on the graph.



66. Now we consider what kind of function describes the cutting forces equation we wrote. It can be clearly seen that this is also a constant equation. Based on this information, we can connect the points previously marked, mark that we are on the positive side, and finally, we dash this chart.



67. We will need the equation of normal forces for the second section.

$$N(x_2) = -R_{Ax}$$

We know the limits of the second section.

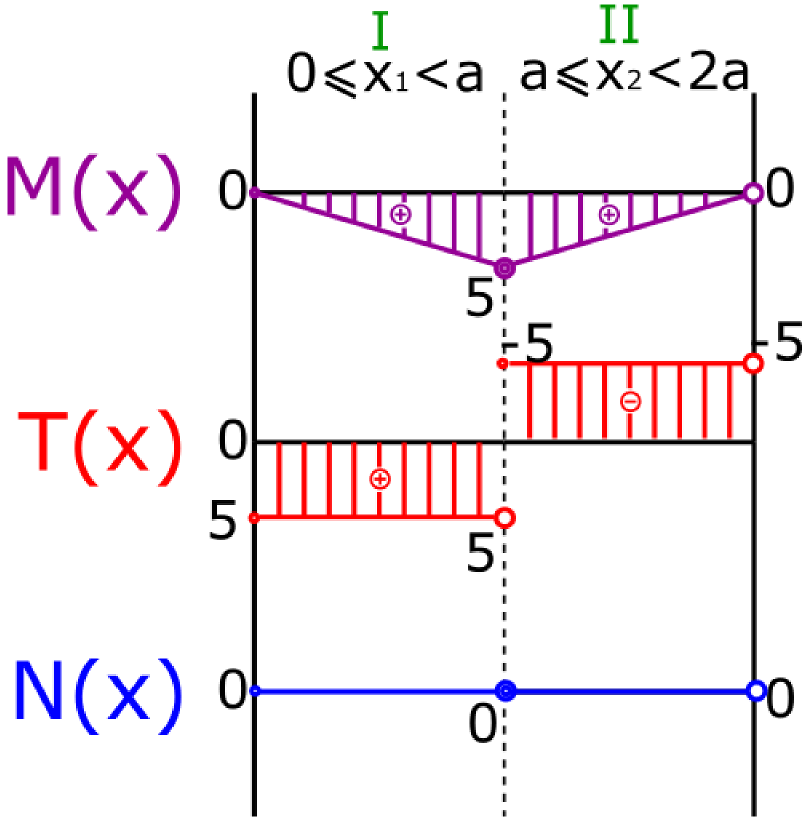
$$a \leq x < 2a$$

We substitute the boundary values into our equation (for x_2).

$$N(a) = -R_{Ax} = 0kN$$

$$N(2a) = -R_{Ax} = 0kN$$

68. When we know the values of the bending moment at the ends of the section we put these values on the graph. Now we consider what kind of function describes the cutting forces equation we wrote. It can be clearly seen that this is also a constant equation. Based on this information, we can connect the points previously marked, mark that we are on the positive side, and finally, we dash this chart.



69. Looking at the charts above, we can pull out some information that will also allow us to check whether we have solved our beam well.
    - First of all, when we look at the bending moments chart, we see that in the place where we have the joints, the moments values are equal to 0. What's more, the moments values, when the beam is loaded only by forces, are equal on the connection of the sections, as it is between the sections I and section II, the moment value at the end of section I is equal to the value at the beginning of section II. If there were a difference in these values, then it would mean that something is wrong in the calculations and the chart.

- Secondly, looking at the cutting forces diagram, it can be seen that at the connection of sections I and II there is a pitch in the value of the cutting force. This pitch is exactly equal to the value of the concentrated force F present at this place on the beam. If there was no pitch on the chart, or the pitch value was different than the value of the concentrated force, it would mean that something is wrong in the calculations and the chart.

The function of shear forces is derived from the function of bending moments

$$\frac{dM(x)}{dx} = T(x)$$

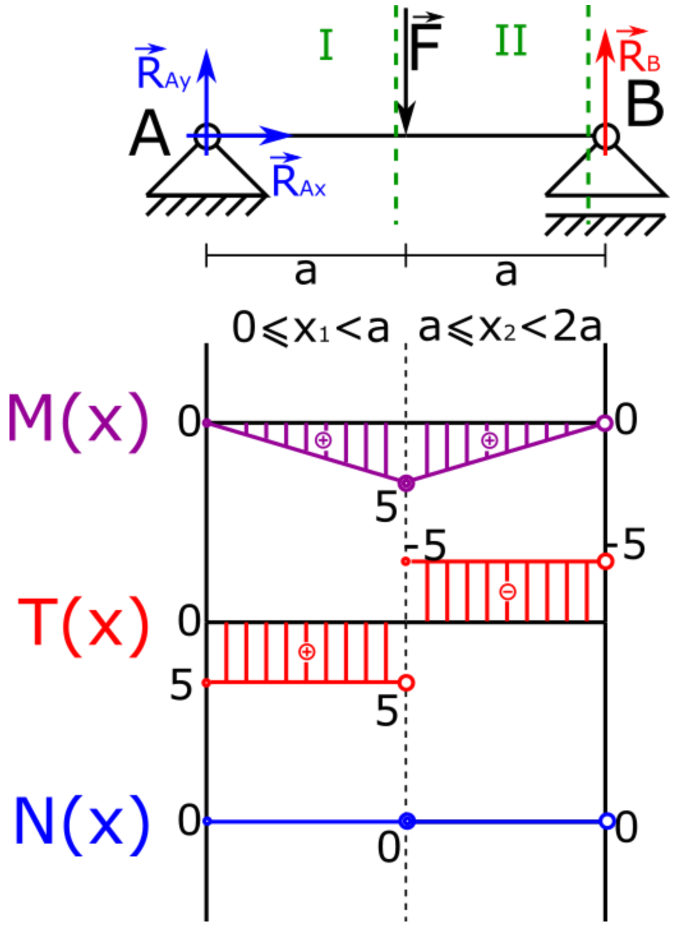
ATTENTION

By entering the beam on the right, the function of shear forces is a derivative of the function of bending moments with the opposite sign

$$\frac{dM(x)}{dx} = -T(x)$$

- Finally, normal forces. It is clear that if there are no forces acting along the beam, then the normal force must be zero.

70. Finally a beam with calculated reactions, internal force equations and graphs of these forces.



$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - F + R_B \rightarrow R_{Ay} = F - R_B = 10 - 5 = 5kN$$

$$\sum_{i=1}^{n} M_A = 0 = -F * a + R_B * 2a \rightarrow R_B = \frac{F}{2} = 5kN$$

I Section $0 \leq x_1 < a$

$$M(x_1) = R_{Ay} * x_1$$

$$T(x_1) = R_{Ay}$$

$$N(x_1) = -R_{Ax}$$

II Section $a \leq x_2 < 2a$

$$M(x_2) = R_{Ay} * x_1 - F * (x_2 - a)$$

$$T(x_2) = R_{Ay} - F$$

$$N(x_2) = -R_{Ax}$$