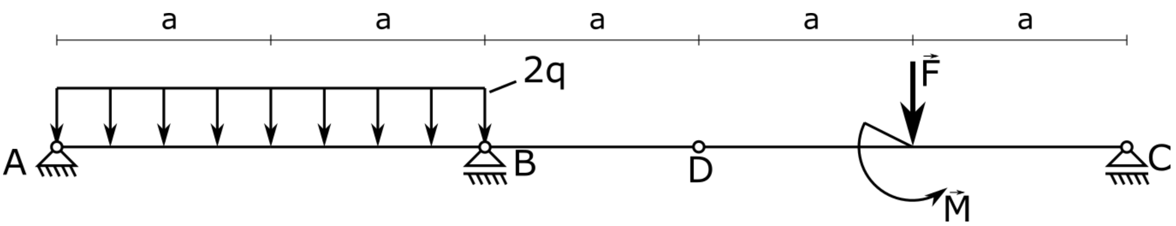
Beams – beams with joints

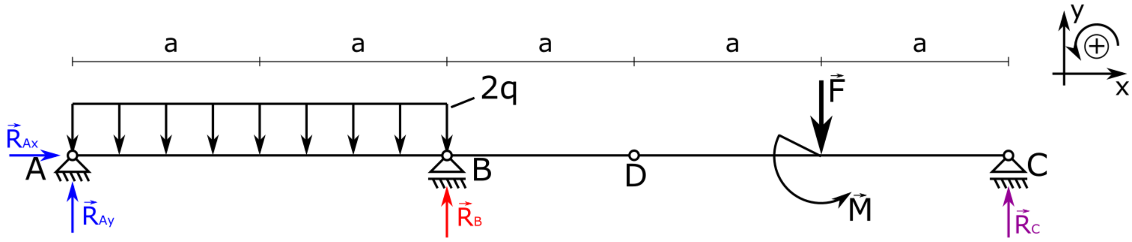
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam with all types of loads. An important information, the beam solution also includes drawing internal force diagrams.

Ex.7

For the beam shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a=1[m], F=1[kN], M=2[kNm], q=1[kN/m].



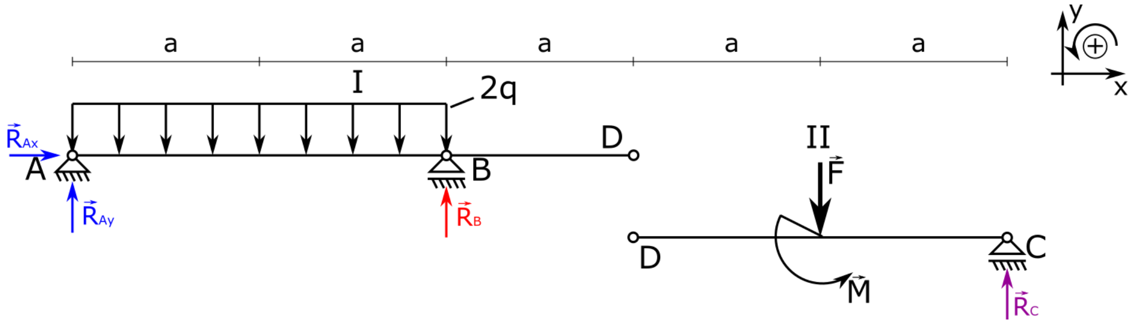
1. In this case, it is clear that the beam shown is different from the previous ones. In this case we have three supports, and on the beam at point D there is a hinge. For now, these are the only differences. As before, we must first identify support unknowns. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, clockwise turning moments will be positive.



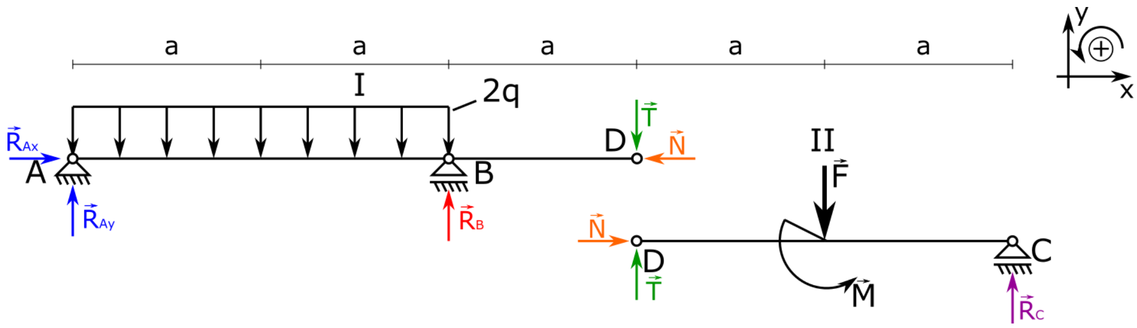
2. It is clear that in this case we have four supporting unknowns, and we can only write three equations of equilibrium, as below.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

3. Below I will show two ways to solve the above problem.
4. **The first method** consists in disconnecting our beam in the place where the joint is. As shown in the picture.

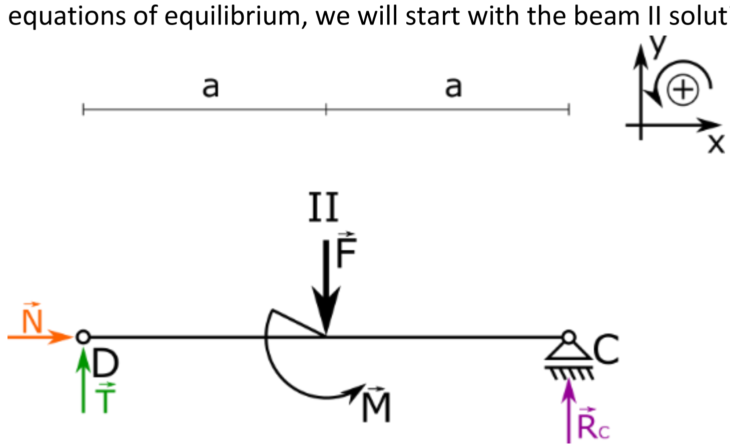


5. In this way we received two new beams. However, to balance the system in the place where we disconnected we need to introduce some (internal) forces. **We know that in the case of joints, there are no moments (no friction).** Therefore, we only introduce normal and cutting forces. As shown.



$$|\vec{T}| = |-\vec{T}|$$
$$|\vec{N}| = |-\vec{N}|$$

6. The introduced forces also have unknown values for us. It can be said that we have increased the number of unknowns to six, which means that we have only made things worse. However, it is important that we assume that we are dealing with two beams. Therefore, check for each of the beams how many are unknown. In the case of beam I we have five unknowns (3 supporting and two already introduced), in the case of beam II we have 3 unknowns (1 supporting and two already introduce). Because we are able to make three equations of equilibrium, we will start with the beam II solution.



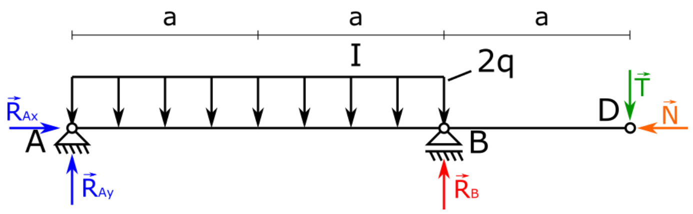
$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

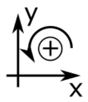
$$\sum_{i=1}^{n} F_{xi} = 0 = N \rightarrow N = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = T - F + R_C \rightarrow R_C = F - T = 1 - 1{,}5 = -0{,}5kN$$

$$\sum_{i=1}^{n} M_C = 0 = -T * 2a + F * a + M \rightarrow T = \frac{F * a + M}{2a} = \frac{1 * 1 + 2}{2} = 1{,}5kN$$

7. After calculating the unknowns for the second beam, we see that in the case of the first beam we have only three unknowns (3 supporting unknowns). That is why we can also write equilibrium equations here.





$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} - N \rightarrow R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - 2q * 2a + R_B - T \rightarrow R_{Ay} = 2q * 2a - R_B + T = 4 - 4{,}25 + 1{,}5 = 1{,}25kN$$

$$\sum_{i=1}^{n} M_A = 0 = -2q * 2a * a + R_B * 2a - T * 3a \rightarrow R_B = \frac{2q * 2a * a + T * 3a}{2a} = \frac{2 * 2 * 1 + 1{,}5 * 3}{2} = \frac{8{,}5}{2} = 4{,}25kN$$

Finally, we get the values of our unknown reactions as below

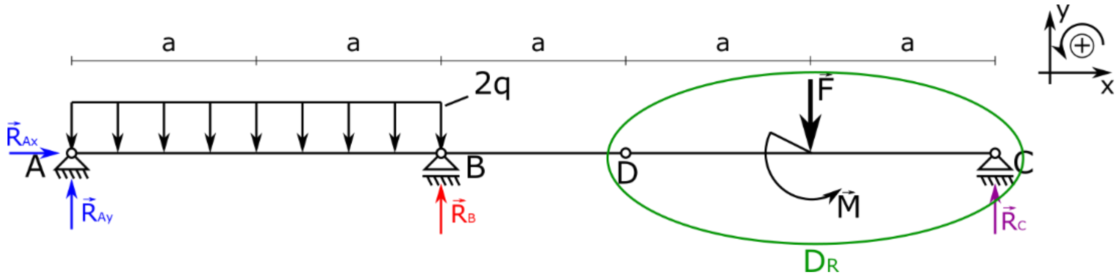
$$R_{Ax} = 0$$

$$R_{Ay} = 1{,}25kN$$

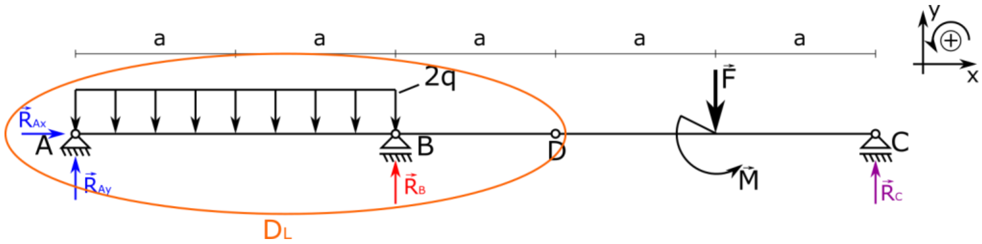
$$R_B = 4{,}25kN$$

$$R_C = -0{,}5kN$$

8. **For the second method**, things look a little different. We will continue to work with the whole beam while we use the fact that the joints are zero (no moments). Therefore, we will write the equation for the sum of the moments **on one side of our joint on its right**



or to his left.



In this way, they received the missing equation to calculate support unknowns.

9. It is best to take a side for the equation where there will be less to count. In this case, we will count the sum of the moments on the right side of our joint.

So we write three equations of equilibrium for our entire beam and additionally the equation for the joint.

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - 2q * 2a + R_B - F + R_C \rightarrow R_{Ay} = 2q * 2a - R_B + F - R_C$$
$$= 4 - 4{,}25 + 1 + 0{,}5 = 1{,}25kN$$

$$\sum_{i=1}^{n} M_A = 0 = -2q * 2a * a + R_B * 2a - F * 4a + M + R_C * 5a \rightarrow R_B$$
$$= \frac{2q * 2a * a + F * 4a - M - R_C * 5a}{2a} = \frac{2 * 2 * 1 + 1 * 4 - 2 + 0.5 * 5}{2} = \frac{8{,}5}{2}$$
$$= 4{,}25kN$$

$$\sum_{i=1}^{n} M_D^R = 0 = -F * a + M + R_C * 2a \rightarrow R_C = \frac{F * a - M}{2a} = \frac{1 * 1 - 2}{2} = -0.5kN$$

Finally, we get the values of our unknown reactions as below

$$R_{Ax} = 0$$

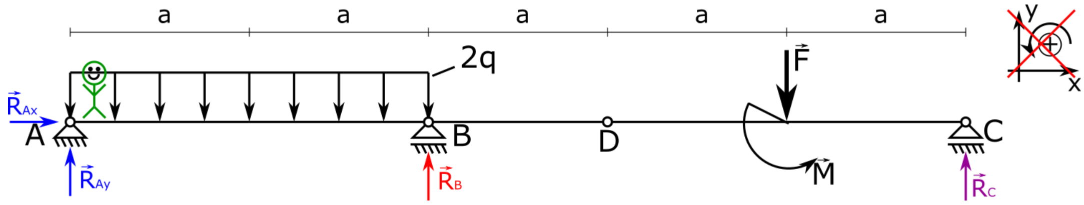
$$R_{Ay} = 1{,}25kN$$

$$R_B = 4{,}25kN$$

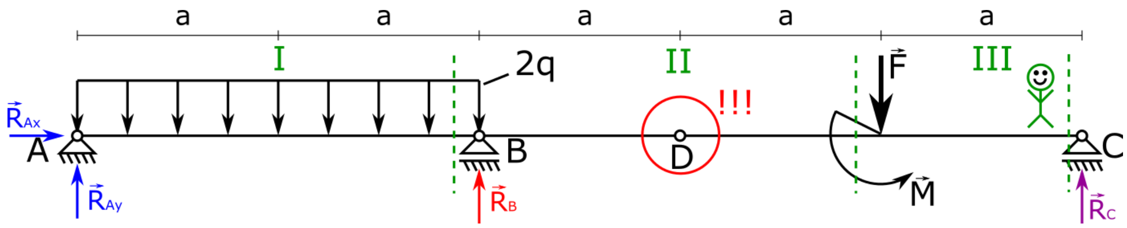
$$R_C = -0{,}5kN$$

It is clear that regardless of the way we get exactly the same results.

10. After determining the reaction in the supports, in the next step we need to determine how many cuts the beam needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces that was introduced at the beginning. We enter the beam from its left (you can also enter the beam from the right).



11. Cuts will be made on the beam in places where something happens on the beam (something appears or disappears). The important information is that we always cut before something on the beam has happened.
We see that when we enter the beam from its left, forces R_{Ax}, R_{Ay} and distributed force appears. However, we cannot make the cut before these forces, because then we are not yet on the beam. We go further along the beam and come across force R_B and the end of distributed force. Something happened. So we know that in this place just before the appearance of force R_B should be first cut. Going further we reach the place where joint appears but **joint do not introduce the cut.** . Going further we reach the place where force F and moment M appears. So in this place just before this force and moment appeared we also need to make a cut. Continuing the journey we reach the place where the force R_C appears. i.e. something has happened. This means that in this place just before the force R_C we have to make another cut. In this place we end our journey.



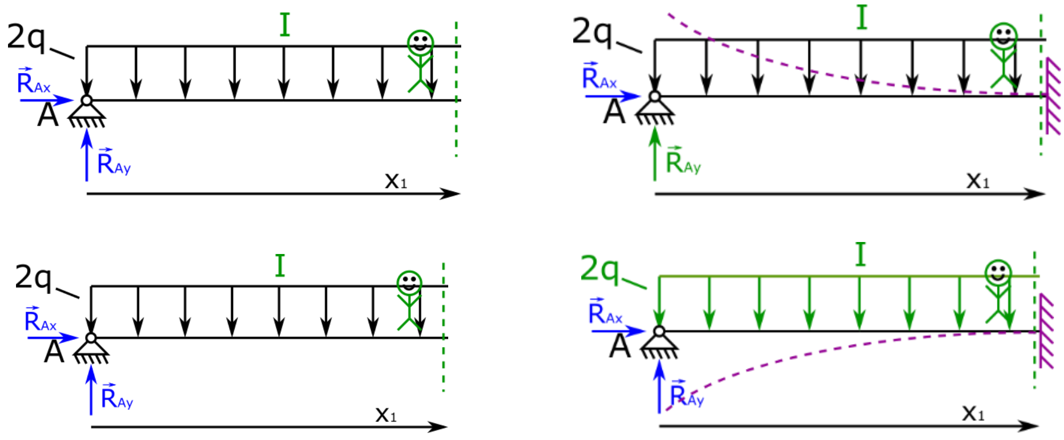
12. In this way, we divided the beam into three sections . The first section within $0 \leq x < 2a$, second section within $2a \leq x < 4a$, third section within $4a \leq x < 5a$.

BENDING MOMENTS, CUTTING FORCES, NORMAL FORCES

13. At this point, we can move on to determining internal forces. To do this we have to go through each of the sections. In this example, we will determine equations for all internal forces section by section. In addition, information about the bending moment will appear at each of the section.

I section within $0 \leq x < 2a$.

The bending moment will be from the R_{Ay} force and from distributed force and will look like in the pictures (dashed purple line). Cutting force from R_{Ay} and distributed force. Normal force R_{Ax}.

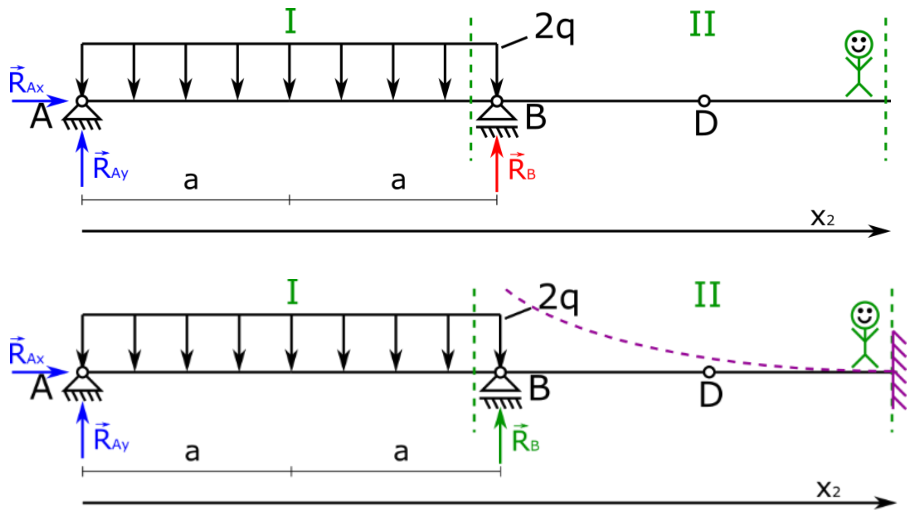


Equations for all types of internal forces

$$M(x_1) = R_{Ay} * x_1 - \frac{2q * x_1^2}{2}$$
$$T(x_1) = R_{Ay} - 2q * x_1$$
$$N(x_1) = -R_{Ax}$$

14. II section within $2a \le x < 4a$.

The bending moment will be from the R_{Ay} force, distributed force, R_B force. The bending from R_{Ay} and distributed force will be the same as it was in previous section. Bending moment from R_B force will look like in the picture (dashed purple line). Cutting forces from R_{Ay} distributed force and R_B. Normal force R_{Ax}.

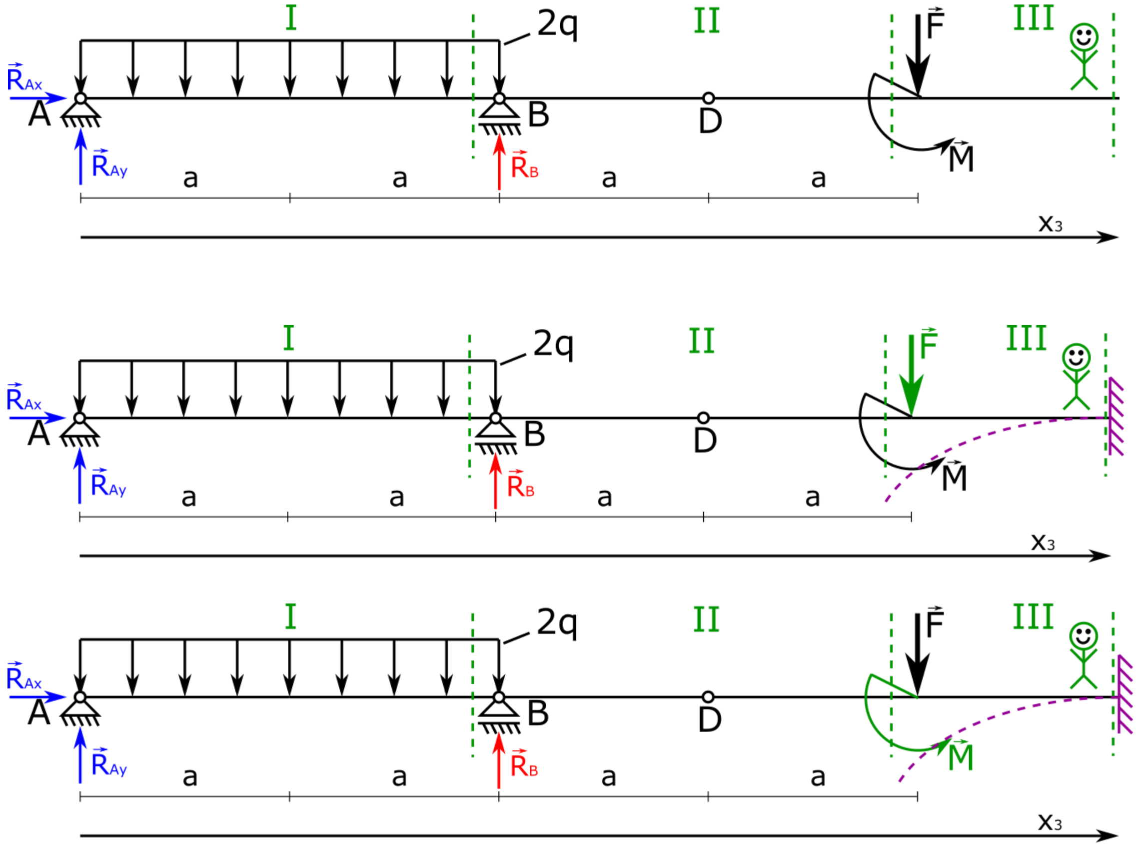


Equations for all types of internal forces

$$M(x_2) = R_{Ay} * x_2 - 2q * 2a * (x_2 - a) + R_B * (x_2 - 2a)$$
$$T(x_2) = R_{Ay} - 2q * 2a + R_B$$
$$N(x_2) = -R_{Ax}$$

15. III section within $4a \leq x < 5a$.

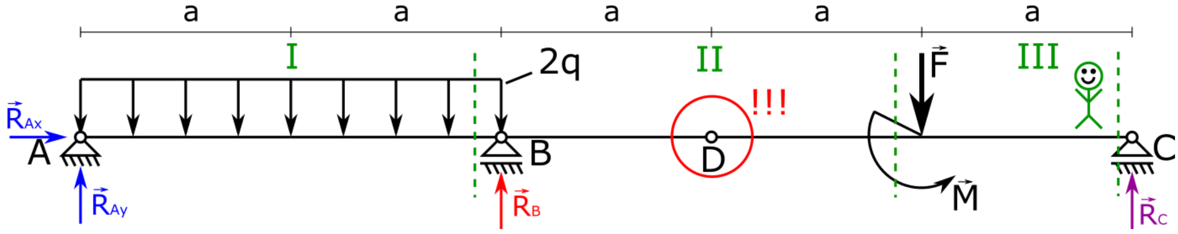
The bending moment will be from the R_{Ay} force, distributed force, R_B force and force F and moment M. The bending from R_{Ay}, distributed force and force R_B will be the same as it was in previous section. Bending moment from F force and moment M will look like in the picture (dashed purple line). Cutting forces from R_{Ay} distributed force, R_B and F. Normal force R_{Ax}.



Equations for all types of internal forces

$$M(x_3) = R_{Ay} * x_3 - 2q * 2a * (x_3 - a) + R_B * (x_3 - 2a) - F * (x_3 - 4a) - M$$
$$T(x_3) = R_{Ay} - 2q * 2a + R_B - F$$
$$N(x_3) = -R_{Ax}$$

16. Let's write equations for all sections in one place to clarify everything.



I Section $0 \leq x_1 < 2a$

$$M(x_1) = R_{Ay} * x_1 - \frac{2q * x_1^2}{2}$$
$$T(x_1) = R_{Ay} - 2q * x_1$$
$$N(x_1) = -R_{Ax}$$

II Section $2a \leq x_1 < 4a$

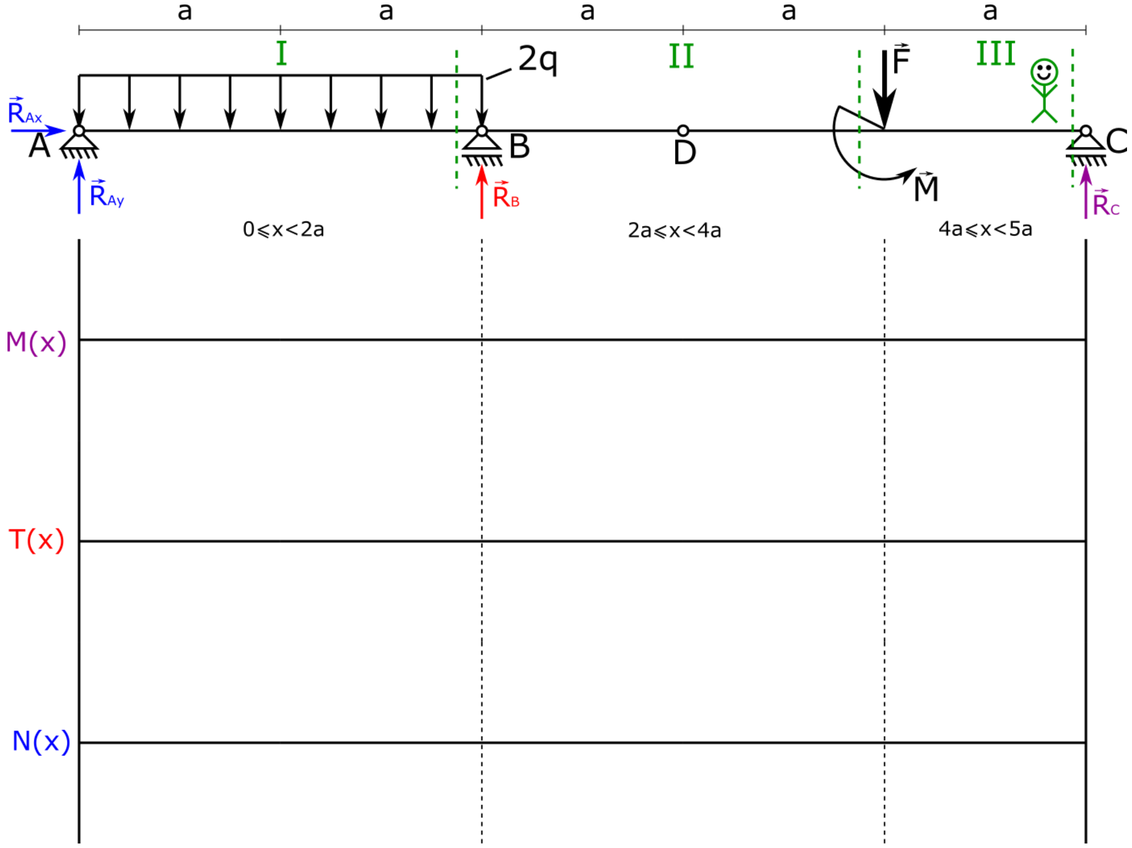
$$M(x_2) = R_{Ay} * x_2 - 2q * 2a * (x_2 - a) + R_B * (x_2 - 2a)$$
$$T(x_2) = R_{Ay} - 2q * 2a + R_B$$
$$N(x_2) = -R_{Ax}$$

III Section $4a \leq x_1 < 5a$

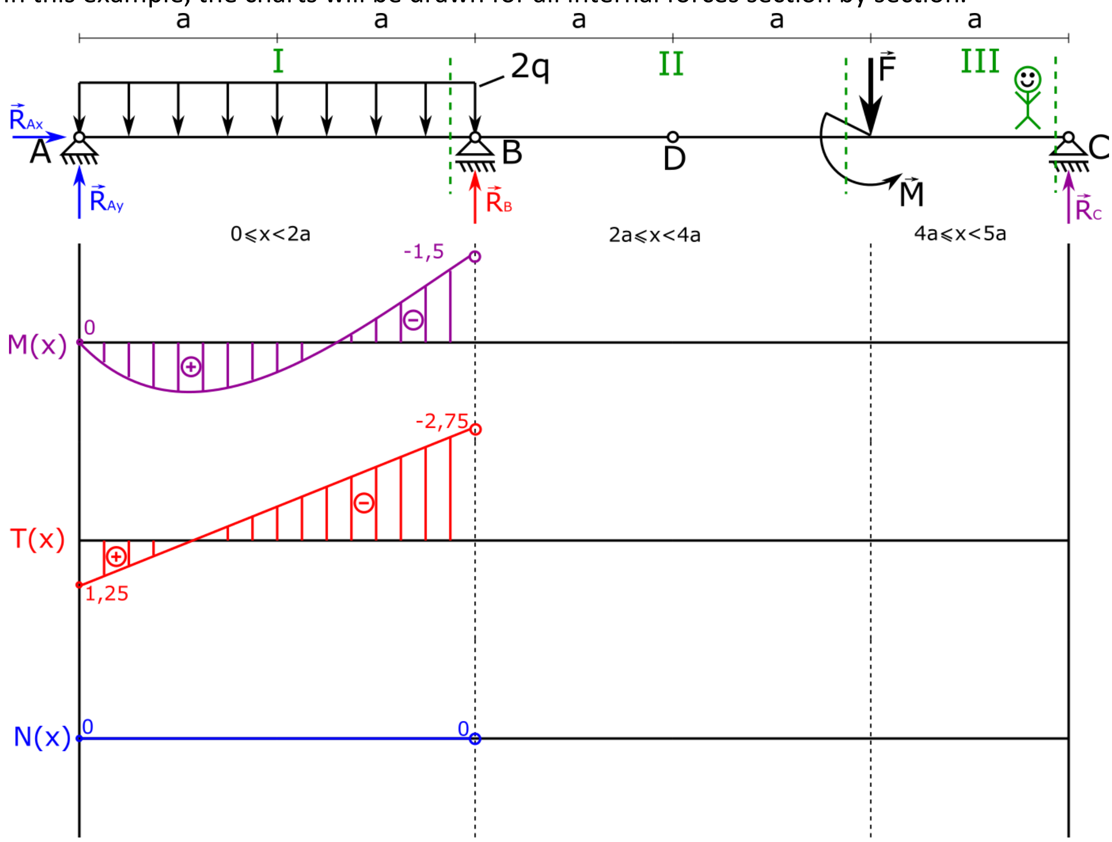
$$M(x_3) = R_{Ay} * x_3 - 2q * 2a * (x_3 - a) + R_B * (x_3 - 2a) - F * (x_3 - 4a) - M$$
$$T(x_3) = R_{Ay} - 2q * 2a + R_B - F$$
$$N(x_3) = -R_{Ax}$$

CHARTS

17. The last part related to solving beams – charts. To easily draw charts, it is best to draw them under the beam, which we solve, as shown in the figure.



18. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the first section.

$$M(x_1) = R_{Ay} * x_1 - \frac{2q * x_1^2}{2}$$
$$T(x_1) = R_{Ay} - 2q * x_1$$
$$N(x_1) = -R_{Ax}$$

We know the limits of the first section.

$$0 \leq x < 2a$$

We substitute the boundary values into our equations (for x_1).

$$M(0) = R_{Ay} * 0 - \frac{2q * 0}{2} = 0$$

$$M(2a) = R_{Ay} * 2a - \frac{2q * (2a)^2}{2} = 1{,}25 * 2 - \frac{2 * 4}{2} = 2{,}5 - 4 = -1{,}5kNm$$

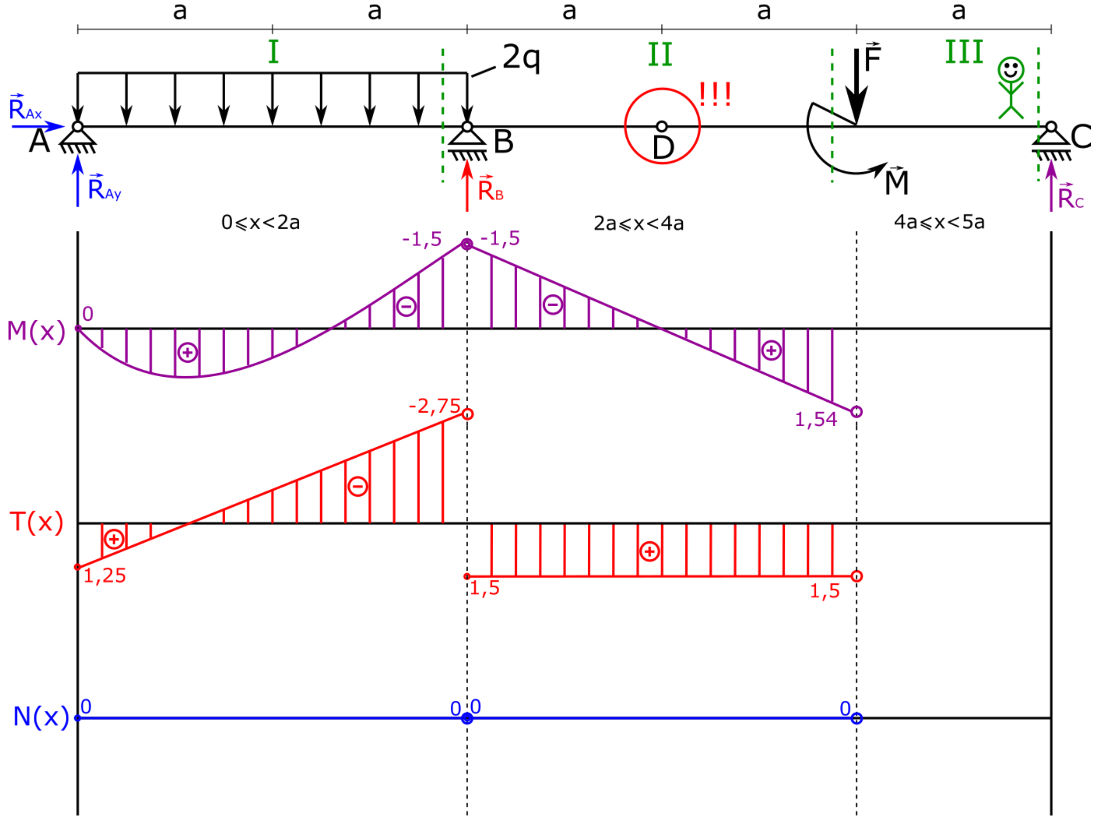
$$T(0) = R_{Ay} - 2q * 0 = 1{,}25kN$$

$$T(2a) = R_{Ay} - 2q * 2a = 1{,}25 - 2 * 2 = -2{,}75kN$$

$$N(0) = -R_{Ax} = 0$$

$$N(2a) = -R_{Ax} = 0$$

19. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the second section.

$$M(x_2) = R_{Ay} * x_2 - 2q * 2a * (x_2 - a) + R_B * (x_2 - 2a)$$
$$T(x_2) = R_{Ay} - 2q * 2a + R_B$$
$$N(x_2) = -R_{Ax}$$

We know the limits of the second section.

$$2a \leq x < 4a$$

We substitute the boundary values into our equations (for x_1).

$$M(2a) = R_{Ay} * 2a - 2q * 2a * (2a - a) + R_B * (2a - 2a) = -1{,}5kNm$$

$$M(4a) = R_{Ay} * 4a - 2q * 2a * (4a - a) + R_B * (4a - 2a) = 1{,}5kNm$$

$$T(2a) = R_{Ay} - 2q * 2a + R_B = 1{,}5kN$$

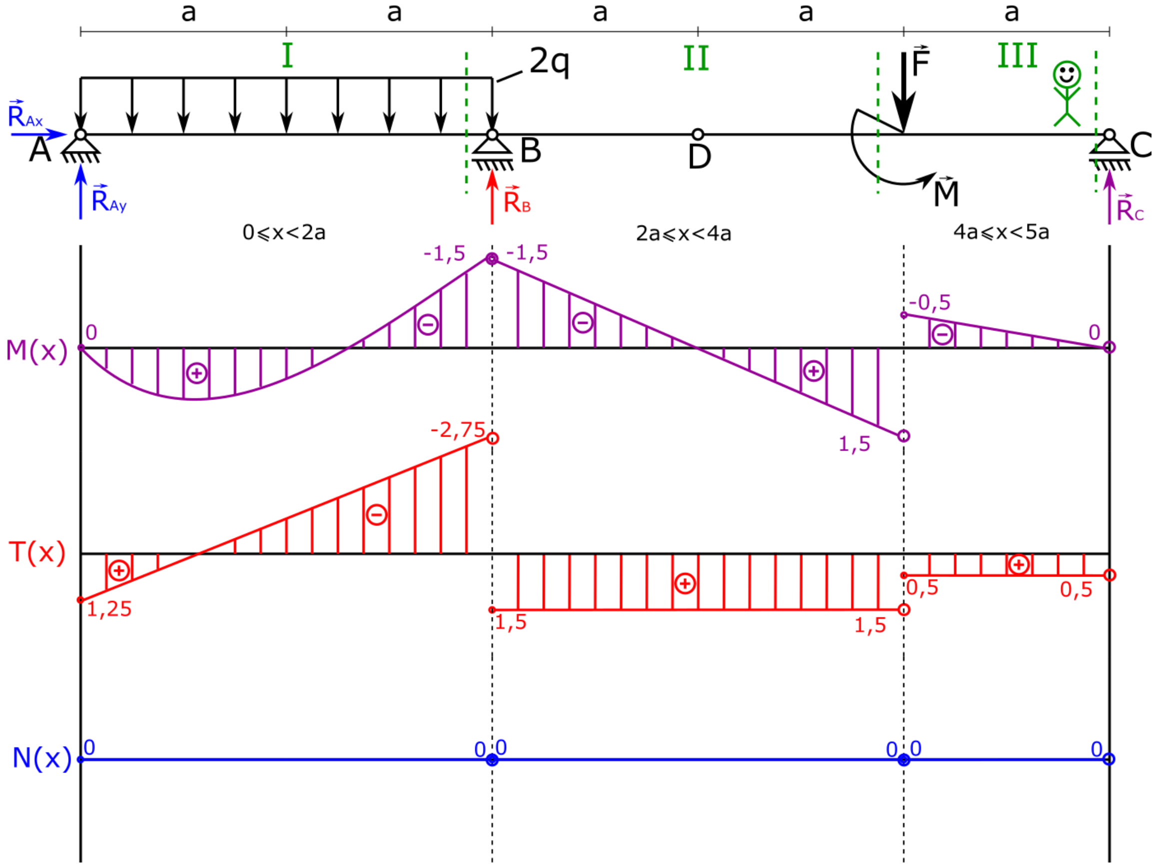
$$T(4a) = R_{Ay} - 2q * 2a + R_B = 1{,}5kN$$

$$N(2a) = -R_{Ax} = 0$$

$$N(4a) = -R_{Ax} = 0$$

Important information. The graph of bending moments where the joint is on the beam must pass through 0.

20. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the third section.

$$M(x_3) = R_{Ay} * x_3 - 2q * 2a * (x_3 - a) + R_B * (x_3 - 2a) - F * (x_3 - 4a) - M$$
$$T(x_3) = R_{Ay} - 2q * 2a + R_B - F$$
$$N(x_3) = -R_{Ax}$$

We know the limits of the third section.

$$4a \leq x < 5a$$

We substitute the boundary values into our equations (for x_1).

$$M(4a) = R_{Ay} * 4a - 2q * 2a * (4a - a) + R_B * (4a - 2a) - F * (4a - 4a) - M = -0{,}5kNm$$

$$M(5a) = R_{Ay} * 5a - 2q * 2a * (5a - a) + R_B * (5a - 2a) - F * (5a - 4a) - M = 0kNm$$

$$T(4a) = R_{Ay} - 2q * 2a + R_B - F = 0{,}5kN$$

$$T(5a) = R_{Ay} - 2q * 2a + R_B - F = 0{,}5kN$$

$$N(4a) = -R_{Ax} = 0$$

$$N(5a) = -R_{Ax} = 0$$