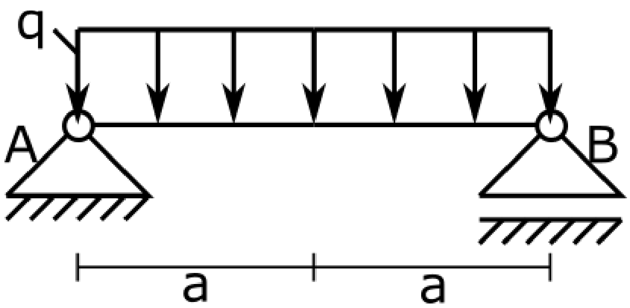
Beams – distributed force

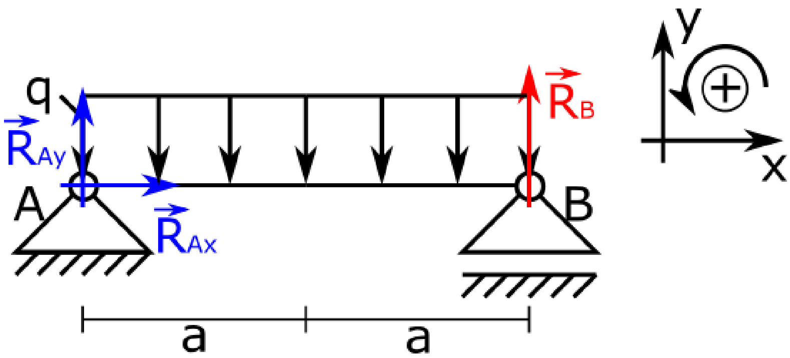
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam. An important information, the beam solution also includes drawing internal force diagrams.

Ex.2

For the beam shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a = 2 [m], q = 5 [kN/m].



1. The first thing to do when solving beams is to determine the number of supporting unknowns, as was the case with trusses. It is clear that in this case we have three unknown supports. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, anticlockwise turning moments will be positive.



2. Now we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

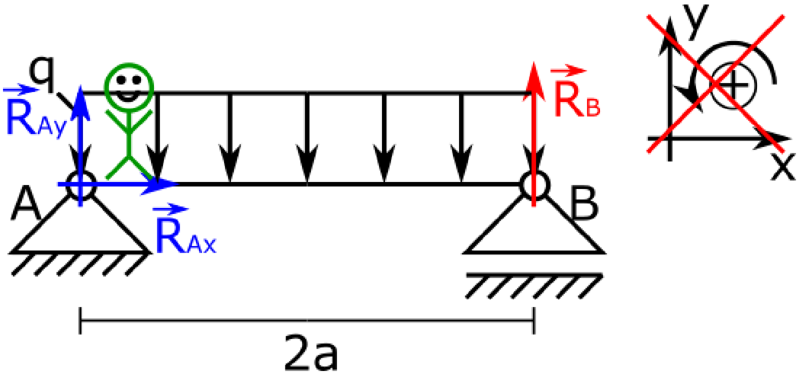
$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

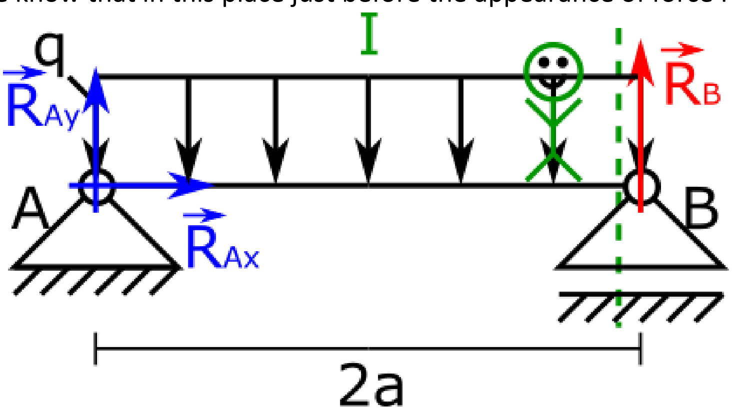
$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - q * 2a + R_B \rightarrow R_{Ay} = q * 2a - R_B = 20 - 10 = 10kN$$

$$\sum_{i=1}^{n} M_A = 0 = -q * 2a * a + R_B * 2a \rightarrow R_B = \frac{q * 2a * a}{2a} = 10kN$$

3. This is where the similarity to solving trusses ends.
4. After determining the reaction in the supports, in the next step we need to determine how many cuts the beam needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces that was introduced at the beginning. We enter the beam from its left (you can also enter the beam from the right).



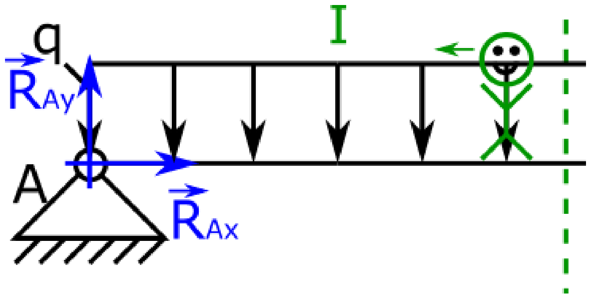
5. Cuts will be made on the beam in places where something happens on the beam (something appears or disappears). The important information is that we always cut before something on the beam has happened.
We see that when we enter the beam from its left, forces R_{Ax}, R_{Ay} and distributed force q appear. However, we cannot make the cut before these forces, because then we are not yet on the beam.
We go further along the beam and come across force R_B. Something happened, force appeared. So we know that in this place just before the appearance of force R_B should be cut.



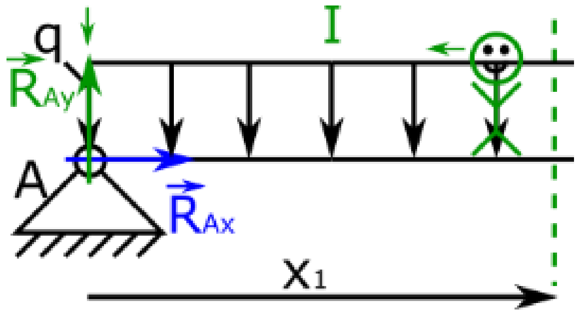
6. In this way, we divided the beam into one section I. The section within $0 \leq x < 2a$.

BENDING MOMENTS

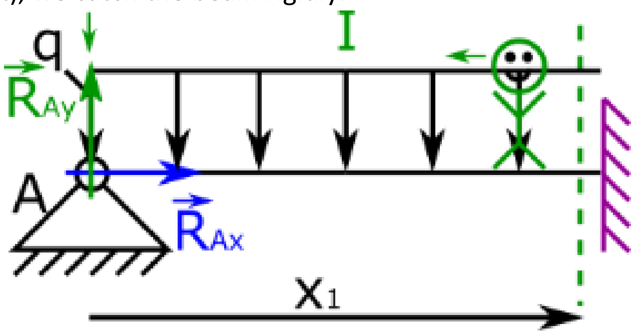
7. At this point, we can move on to determining internal forces. To do this we have to go through the section. We assume that we enter the beam on the left, as before and reach the end of the section. We will begin to determine internal forces from bending moments, then cutting forces and finally normal forces. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



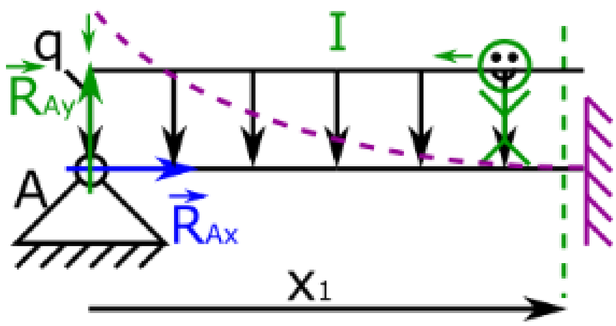
8. We see that at the beginning of the beam there is one force that causes the beam to bend, R_{Ay} force.



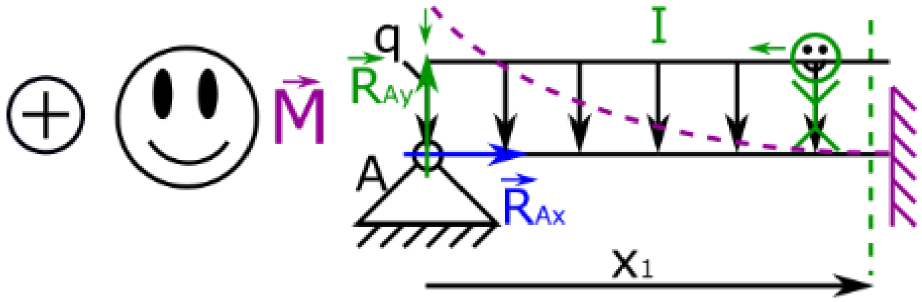
9. Now, to determine how the force bends the beam, we assume that at the place where we stand (where the cut), we catch the beam rigidly.



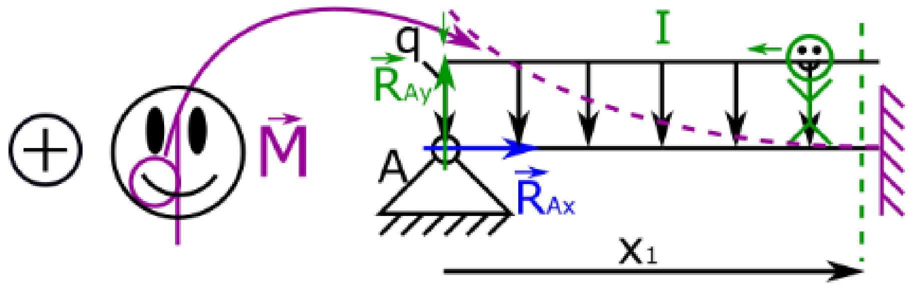
10. At this point, when one end of the beam is fixed, we imagine how the force R_{Ay} tries to bend the beam. You can see clearly in this situation, under the influence of this force, the beam would bend, as shown by the purple dashed line.



11. According to the notation introduced earlier, if the beam under the influence of force smiles, then we assume that the bending moment from such force is a positive sign. Of course, when cutting, it is clearly seen that we get only half of the smile.



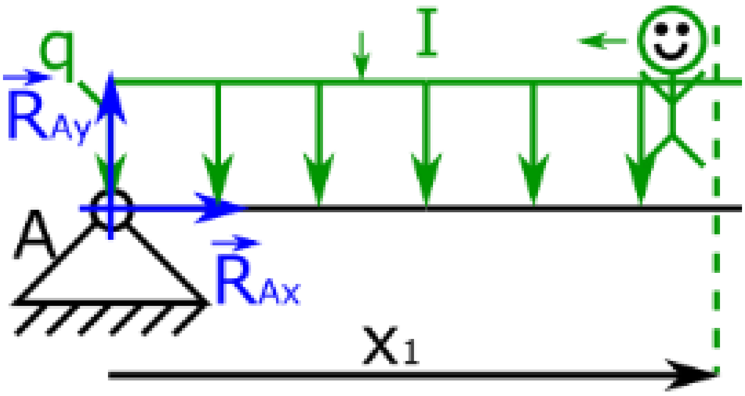
12. We already know what sign the bending moment from R_{Ay} will have, now the arm of this moment should be determined. Because we are standing just before the appearance of the force R_B, it only means that we are at a distance of some x from the beginning of the beam. Let's call this distance x_1.



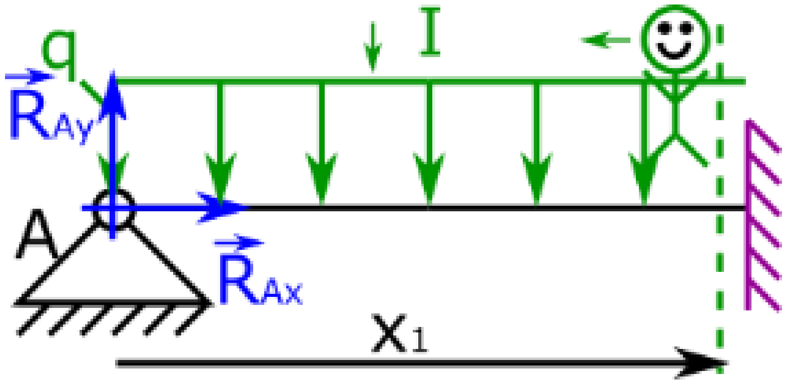
13. After these considerations, we can write the first part of equation for bending moments in the this section.

$$M(x_1) = R_{Ay} * x_1 \ldots$$

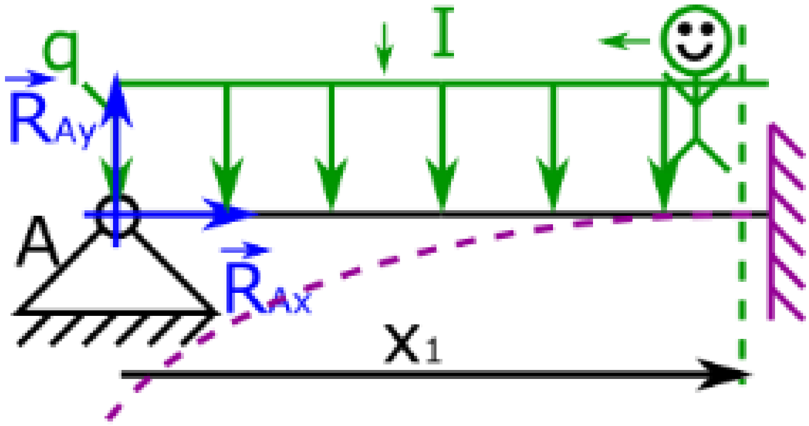
14. Now we move to the second force. We see that at the beginning of the beam there is next force that causes the beam to bend, it is distributed force q which starts at the beginning of the beam. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



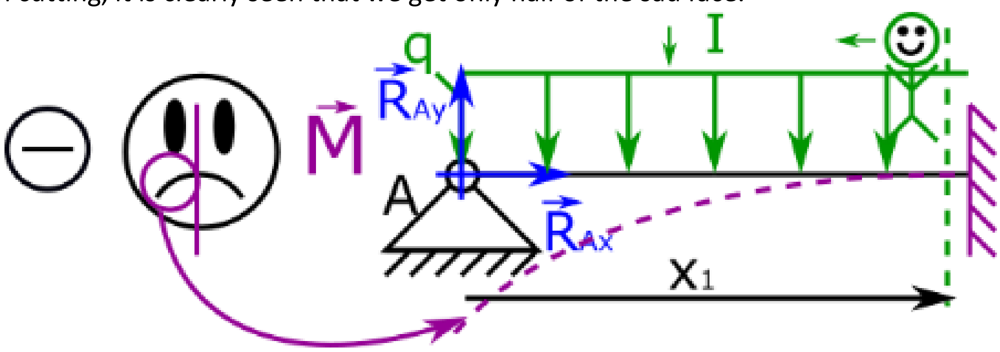
15. Now, to determine how the force bends the beam, we assume that at the place where we stand (where the cut), we catch the beam rigidly, the same way as we did for force R_{Ay}.



16. At this point, when one end of the beam is fixed, we imagine how the distributed force q tries to bend the beam. You can see clearly in this situation, under the influence of this force, the beam would bend, as shown by the purple dashed line.



17. According to the notation introduced earlier, if the beam under the influence of force is sad, then we assume that the bending moment from such force is a negative sign. Of course, when cutting, it is clearly seen that we get only half of the sad face.

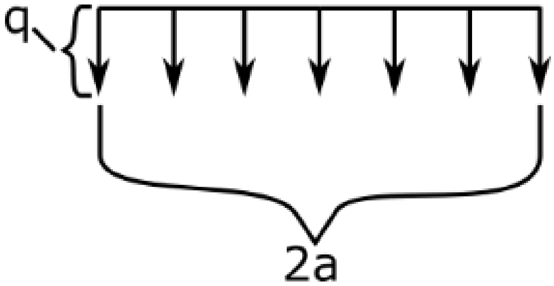


18. Now we should write second part of our bending moments equation.

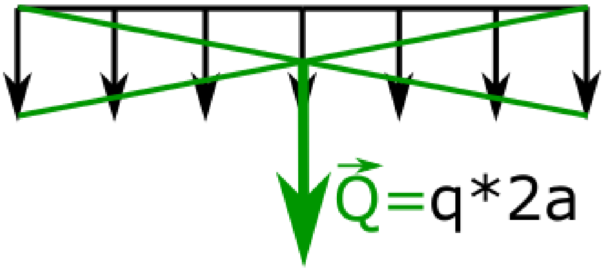
$$M(x_1) = R_{Ay} * x_1 \ldots$$

However, in order to makes everything clear it will be done step by step.

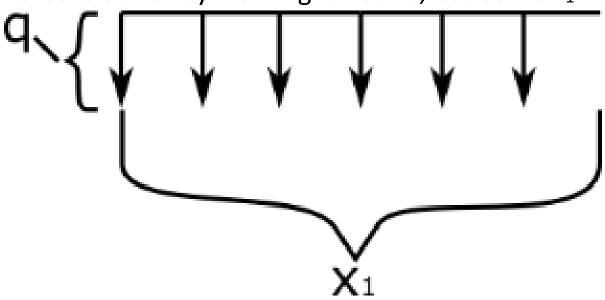
19. We know that our distributed force has a value of q and extends over a length of 2a..



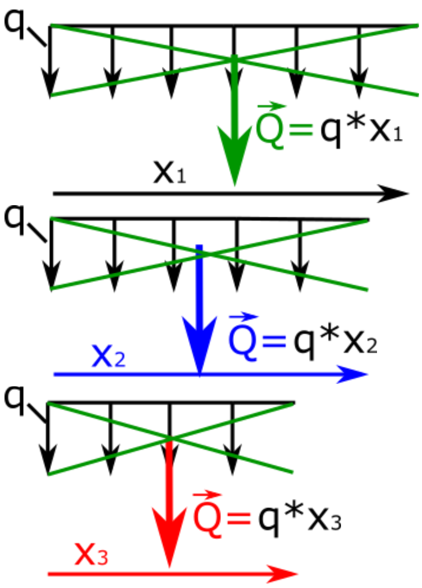
20. This load can be reduced to one force that will work at the intersection of the diagonals of the rectangle, as shown in the figure. It is clear that the value of the reduced force $Q = q * 2a$.



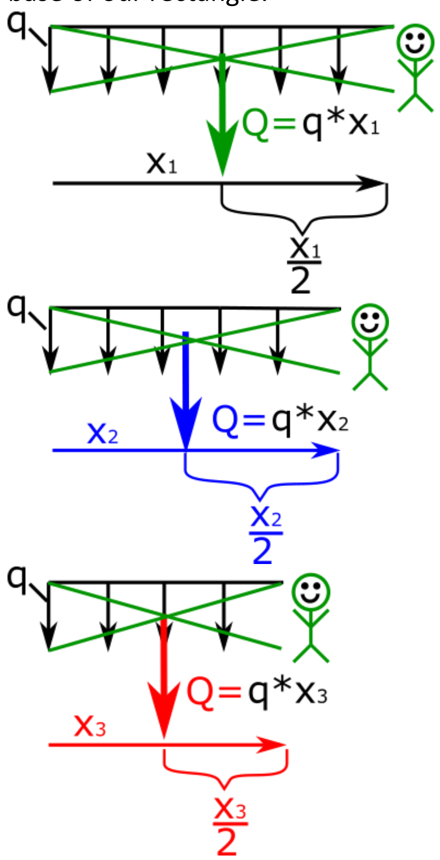
21. We already know the situation for the whole distributed force. However, in our case, when we are standing at the end of the beam, we have cut the force just before its end and the distance at which the force is currently working is not 2a, but some x_1.



22. Therefore, in this case, the value of the reduced force, which also acts at the intersection of the diagonals of such a rectangle will be $Q = q * x_1$. At the same time, it can be seen that this situation will be true for every distance we will be looking from the beginning of distributed force.



23. We already know how the reduced force from a distributed force looks like. However, we must write the moment equation. In order to receive the moment we have to multiply our force by the arm. We must remember that we will be interested in the distance from the place where we stand on the beam (cut end of the beam) to the place where the reduced force acts. It is clear that in the case of a rectangle, the reduced force will always work in the middle of the rectangle's base. Thanks to this, we can easily determine the moment arm, which in our case will be $\frac{x_1}{2}$. Such dependence will be true, regardless of the length of the base of our rectangle.

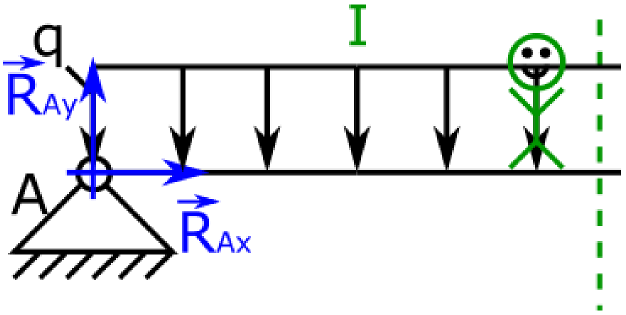


24. We can write final of equation for bending moments in this section. Where $q*x_1$ is our reduced force and $\frac{x_1}{2}$ is our arm.

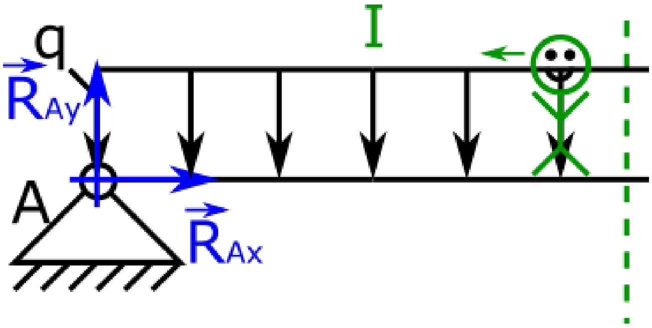
$$M(x_1) = R_{Ay} * x_1 - q * x_1 * \frac{x_1}{2}$$

CUTTING FORCES

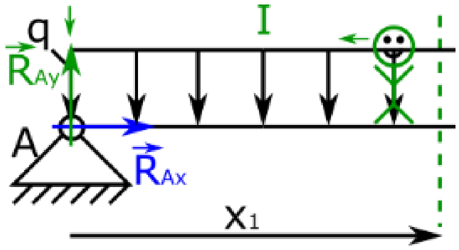
25. Now we can move to the next internal forces, i.e. the cutting forces.

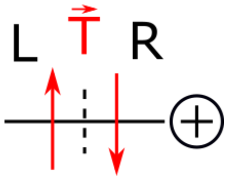


26. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



27. We see that at the beginning of the beam there is one force that seems to cut the beam, R_{Ay} force. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ay}'s is up, which means that we will take this force with a positive sign.

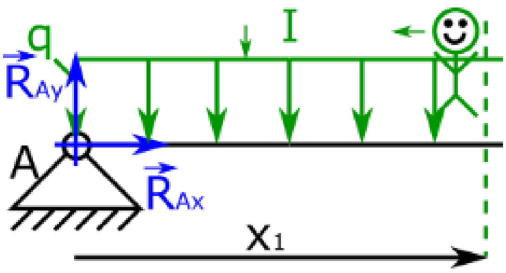




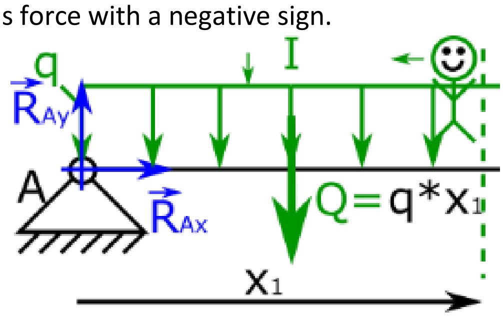
28. After these considerations, we can write the first part of equation for cutting forces in the section.

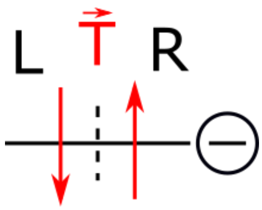
$$T(x_1) = R_{Ay} \dots$$

29. Now we move to the second force which will be our distributed force.



30. Considering information regarding considerations about the bending moment, we can immediately write that the cutting force from the distributed force will be q * x_1. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of Q's is down, which means that we will take this force with a negative sign.



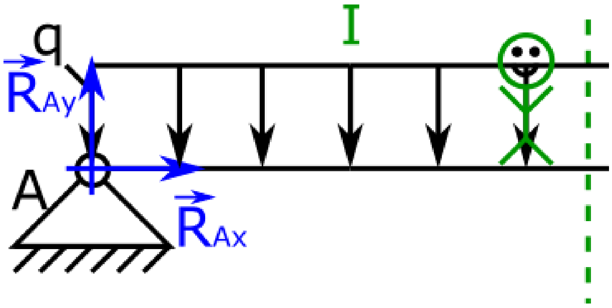


31. We can write final of equation for cutting forces in the second section. We need to take first part and add second part.

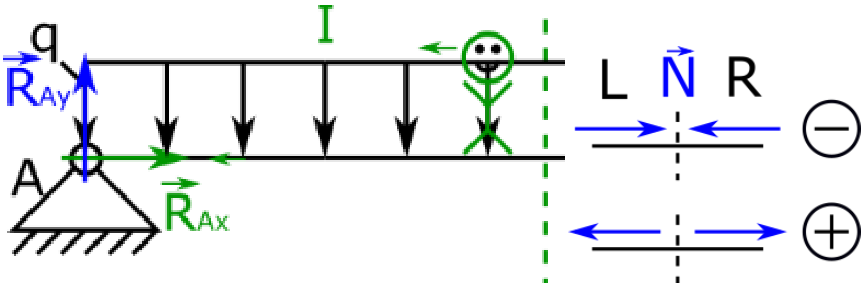
$$T(x_1) = R_{Ay} - q * x_1$$

NORMAL FORCES

32. Now we can move to the final internal forces, i.e. the normal forces.



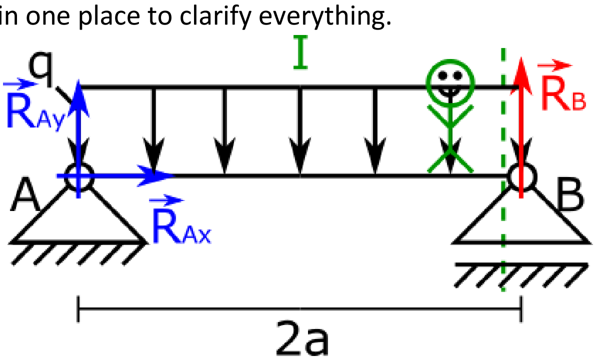
33. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look). We see that at the beginning of the beam there is one force acting along the beam, R_{Ax} force. Now let's get back to our assumptions, from the beginning, for normal forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ax}'s is into the cut, which means that we will take this force with a negative sign.



34. After these considerations, we can write the first equation for cutting forces in the first section.

$$N(x_1) = -R_{Ax}$$

35. Let's write equations in one place to clarify everything.



I Section $0 \leq x_1 < 2a$

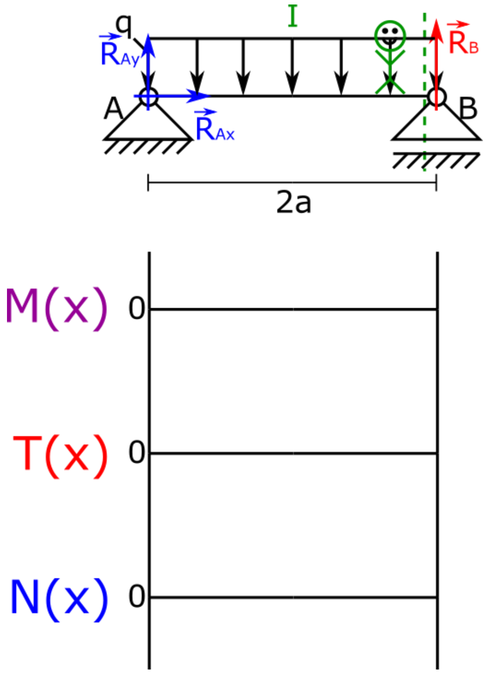
$$M(x_1) = R_{Ay} * x_1 - q * x_1 * \frac{x_1}{2}$$

$$T(x_1) = R_{Ay} - q * x_1$$

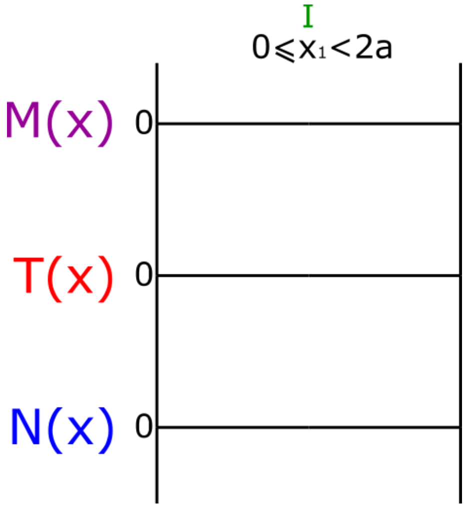
$$N(x_1) = -R_{Ax}$$

CHARTS

36. The last part related to solving beams – charts. To easily draw charts, it is best to draw them under the beam, which we solve, as shown in the figure.



37. In this example, the charts will be drawn step by step so that you can understand their creation. We will start from the bending moments graph.



38. We will need the equation of bending moments for the first section.

$$M(x_1) = R_{Ay} * x_1 - q * x_1 * \frac{x_1}{2}$$

We know the limits of the first section.

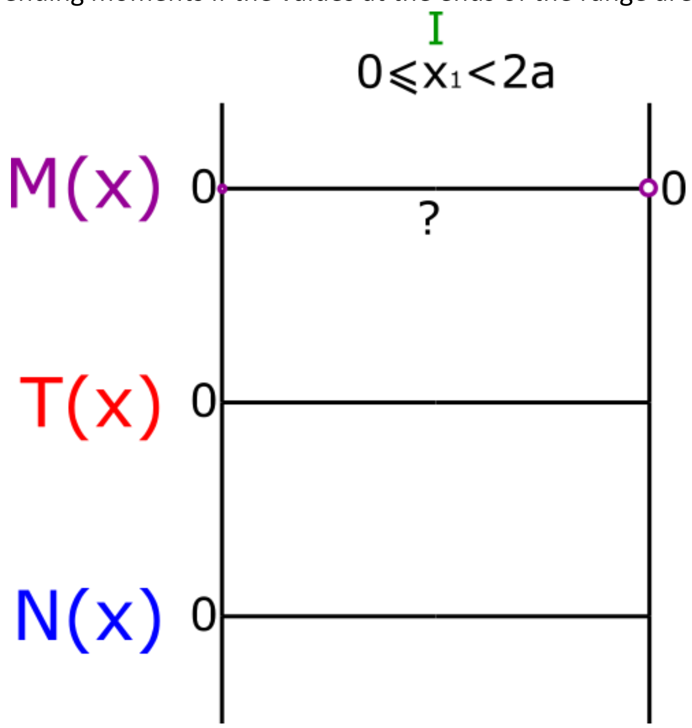
$$0 \leq x < 2a$$

We substitute the boundary values into our equation (for x_1).

$$M(0) = R_{Ay} * 0 - q * 0 * \frac{0}{2} = 0$$

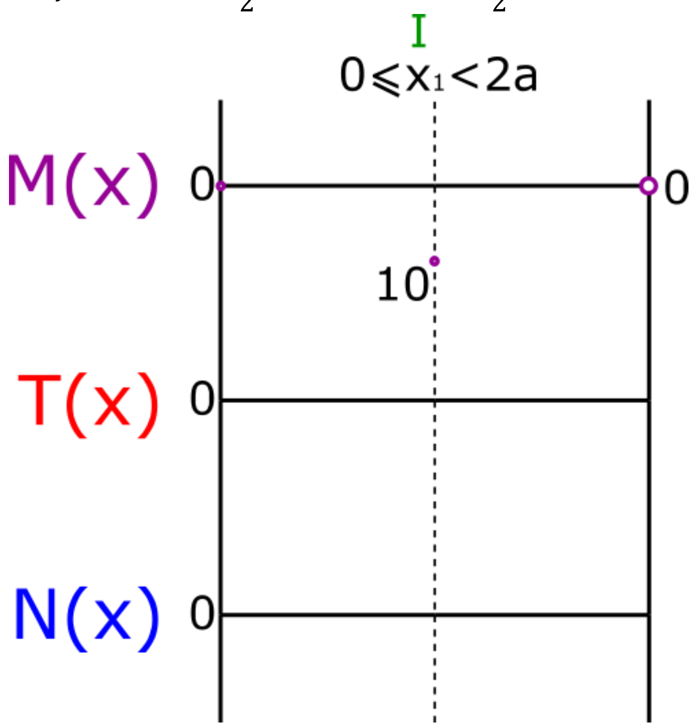
$$M(2a) = R_{Ay} * 2a - q * 2a * \frac{2a}{2} = 10 * 4 - 5 * 4 * \frac{4}{2} = 40 - 40 = 0kNm$$

39. When we know the values of the bending moment at the ends of the section we put these values on the graph. **Important information is that we will write positive values at the bottom of the chart and negative values at the top.** At this point, the question arises how to draw a graph of bending moments if the values at the ends of the range are equal to 0.



40. To solve the problem, we will recalculate the bending moment by selecting the next value from the range we are in. Let's take the value from the middle of the range, i.e. a.

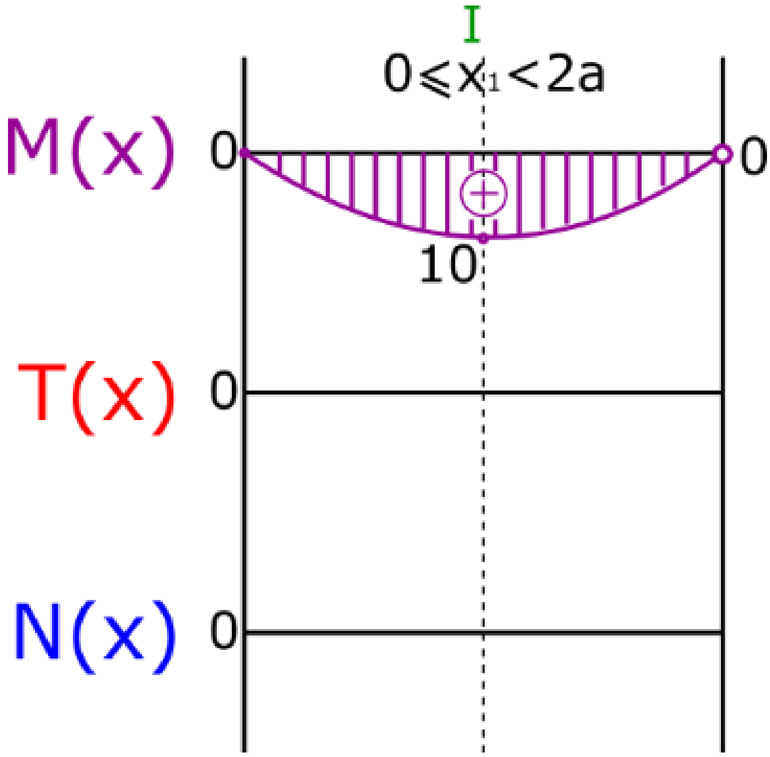
$$M(a) = R_{Ay} * a - q * a * \frac{a}{2} = 10 * 2 - 5 * 2 * \frac{2}{2} = 20 - 10 = 10kNm$$



41. Now that we know which side of the chart we are on, we can draw the chart. Of course, if we still have doubts, we can take further values from the range and calculate the bending moment for them. At this point, we need to think about what shape our chart will have. If we take a good look at the bending moments equation, we notice that we have a quadratic equation, so the graph of our function should take the form of a parabola.

$$M(x_1) = R_{Ay} * x_1 - q * x_1 * \frac{x_1}{2} = R_{Ay} * x_1 - q * \frac{x_1^2}{2}$$

We further mark that we are on the positive side. Finally, we dash this chart. **Important information that the lines must be vertical, any other hatching, painting the chart will be incorrect.**



42. We will need the equation of cutting forces.

$$T(x_1) = R_{Ay} - q * x_1$$

We know the limits of the section.

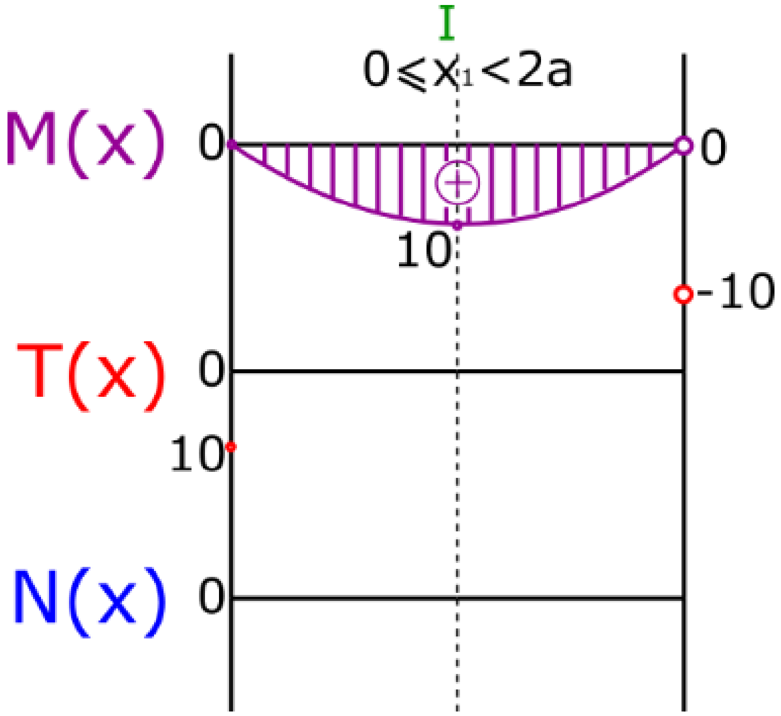
$$0 \leq x < 2a$$

We substitute the boundary values into our equation (for x_1).

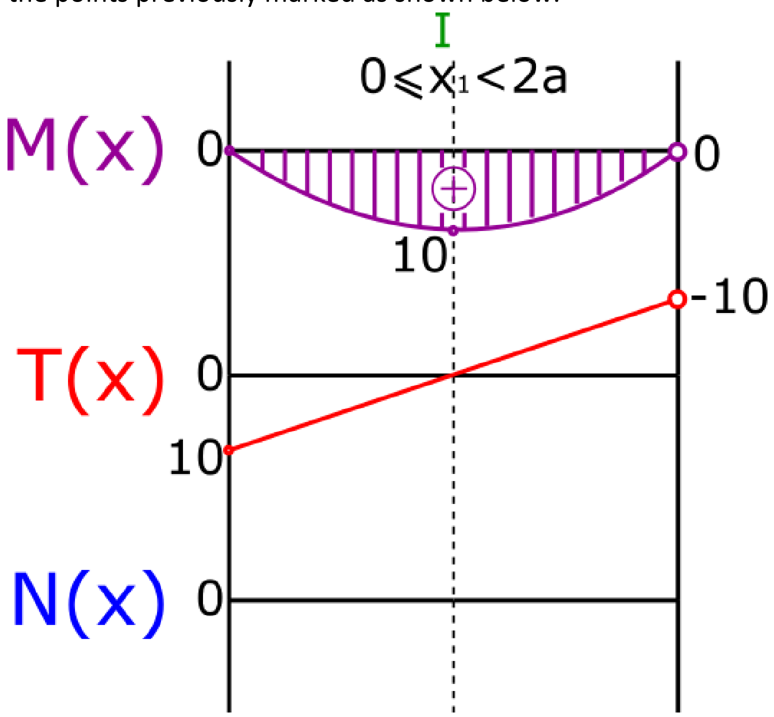
$$T(0) = R_{Ay} - q * x_1 = 10 - q * 0 = 10kN$$

$$T(2a) = R_{Ay} - q * x_1 = 10 - 5 * 4 = -10kN$$

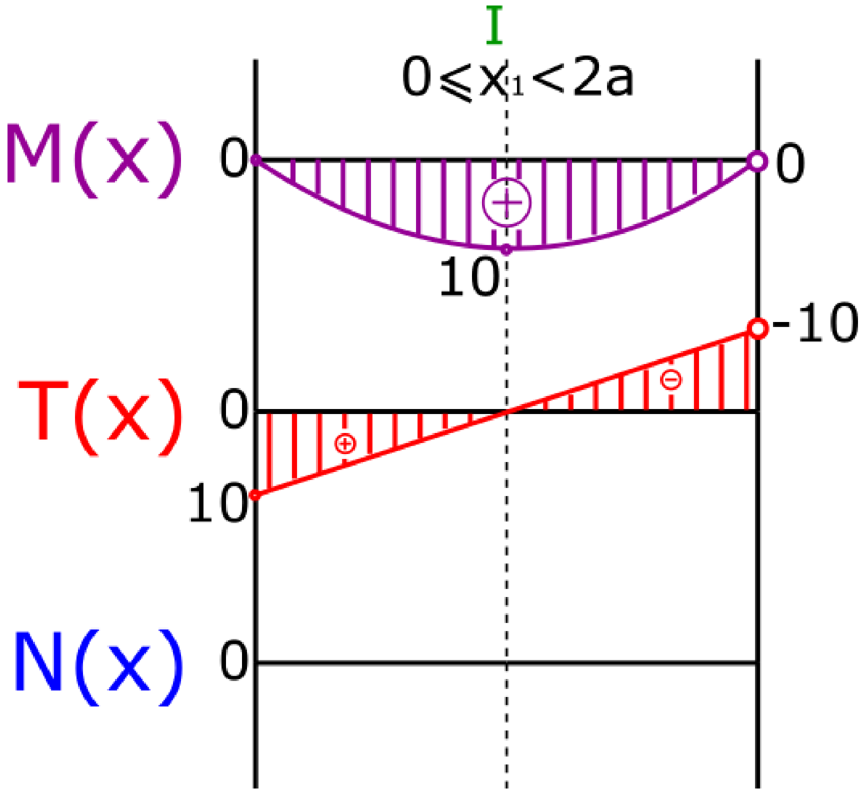
43. When we know the values of the cutting forces at the ends of the section we put these values on the graph.



44. Then we consider what kind of function describes the cutting forces equation we wrote. It can be clearly seen that this is a linear equation. Based on this information, we can connect the points previously marked as shown below.



45. We further mark on which side we are (positive or negative) and finally, we dash this chart.



46. We will need the equation of normal forces.

$$N(x_1) = -R_{Ax}$$

We know the limits of the first section.

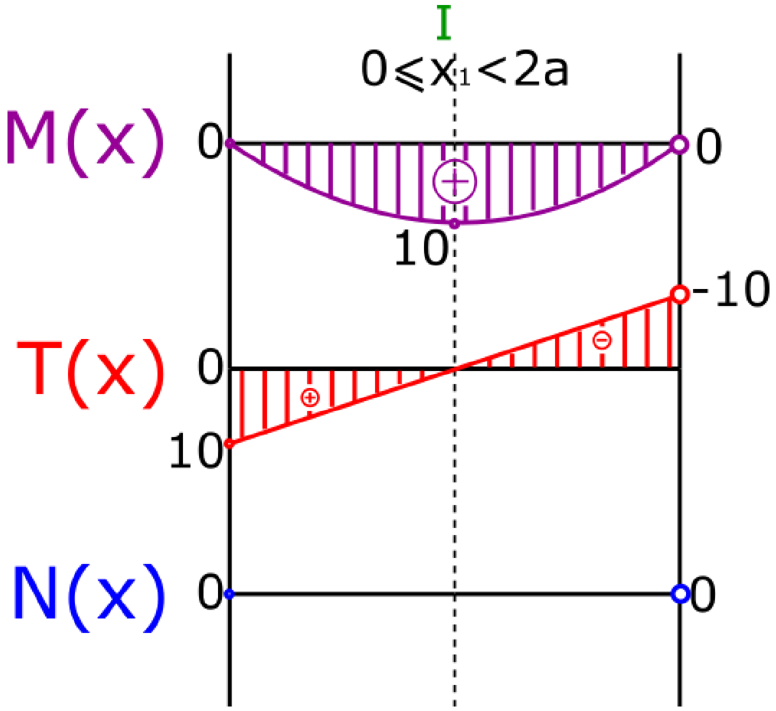
$$0 \leq x < 2a$$

We substitute the boundary values into our equation (for x_1).

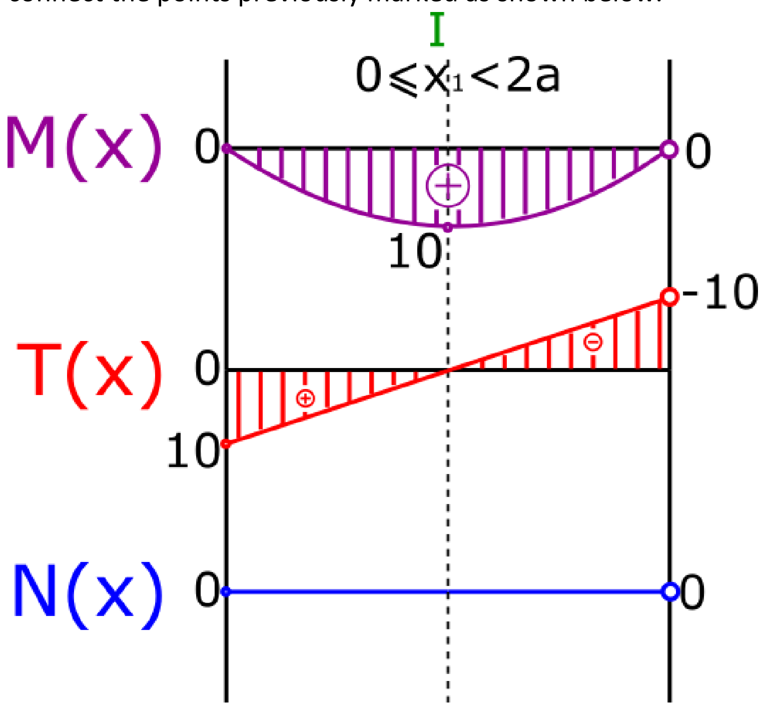
$$N(0) = -R_{Ax} = 0kN$$

$$N(2a) = -R_{Ax} = 0kN$$

47. When we know the values of the normal forces at the ends of the section we put these values on the graph.



48. Then we consider what kind of function describes the normal forces equation we wrote. It can be clearly seen that this is a constant equation. Based on this information, we can connect the points previously marked as shown below.



49. Looking at the charts above, we can pull out some information that will also allow us to check whether we have solved our beam well.
    - First of all, when we look at the bending moments chart, we see that in the place where we have the joints, the moments values are equal to 0. What's more, the

moments function, when the beam is loaded by distributed force, is in parabola shape.

- Secondly, looking at the shear forces graph, it is clear that in the middle of the beam the graph goes through zero. What's more, it is exactly the point where the bending moment parabola reaches its extreme. In addition, it can be clearly seen that the function of shearing forces is a function one order smaller than the function of bending moments. Here we have a linear function there we have a quadratic equation. The following conclusion can be drawn from this.

  The function of shear forces is derived from the function of bending moments

$$\frac{dM(x)}{dx} = T(x)$$

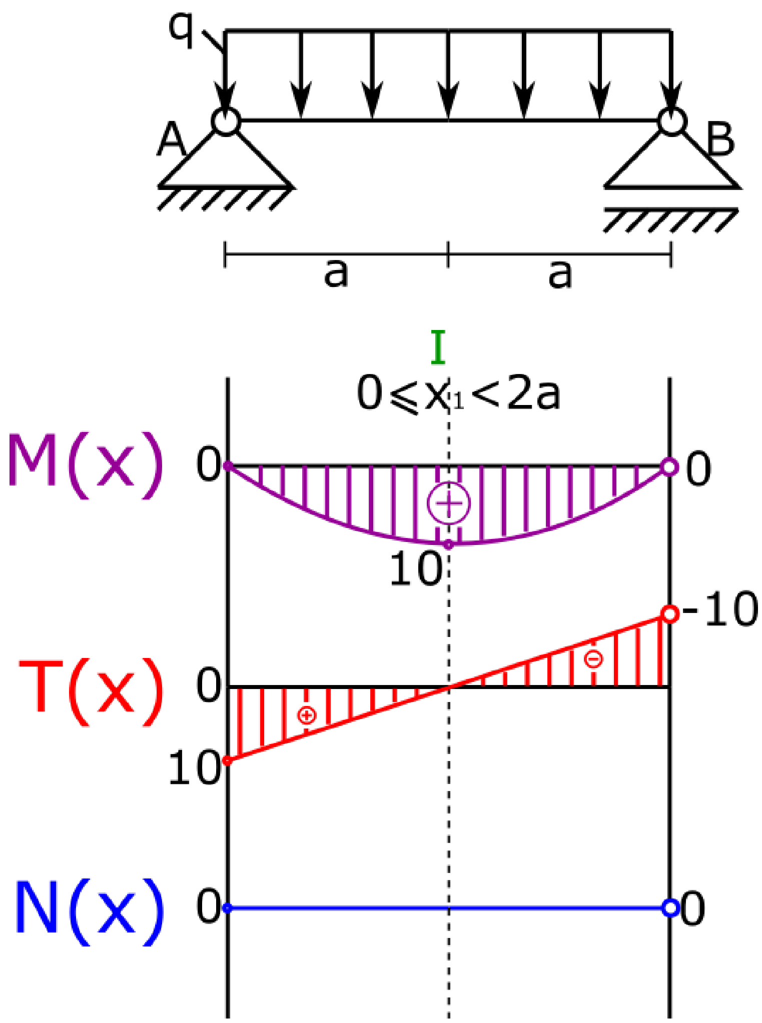
  ATTENTION

  By entering the beam on the right, the function of shear forces is a derivative of the function of bending moments with the opposite sign

$$\frac{dM(x)}{dx} = -T(x)$$

- Finally, normal forces. It is clear that if there are no forces acting along the beam, then the normal force must be zero.

50. Finally a beam with calculated reactions, internal force equations and graphs of these forces.



$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - q * 2a + R_B \rightarrow R_{Ay} = q * 2a - R_B = 20 - 10 = 10kN$$

$$\sum_{i=1}^{n} M_A = 0 = -q * 2a * a + R_B * 2a \rightarrow R_B = \frac{q * 2a * a}{2a} = 10kN$$

I Section $0 \leq x_1 < 2a$

$$M(x_1) = R_{Ay} * x_1 - q * x_1 * \frac{x_1}{2}$$

$$T(x_1) = R_{Ay} - q * x_1$$

$$N(x_1) = -R_{Ax}$$