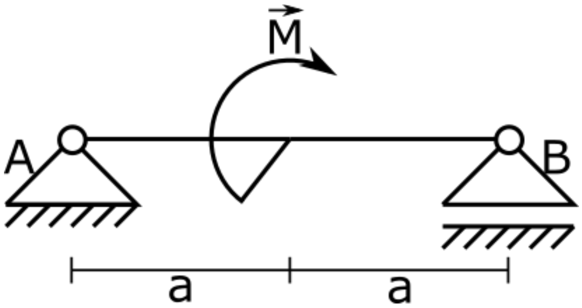
Beams - moment

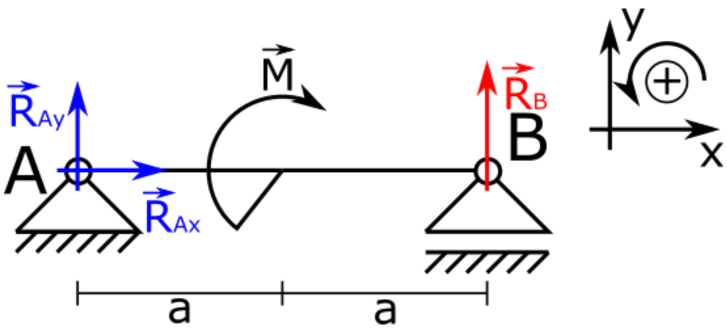
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam. An important information, the beam solution also includes drawing internal force diagrams.

Ex.1

For the beam shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a = 2 [m], M = 20 [kNm].



1. The first thing to do when solving beams is to determine the number of supporting unknowns, as was the case with trusses. It is clear that in this case we have three unknown supports. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, anticlockwise turning moments will be positive.



2. Now we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

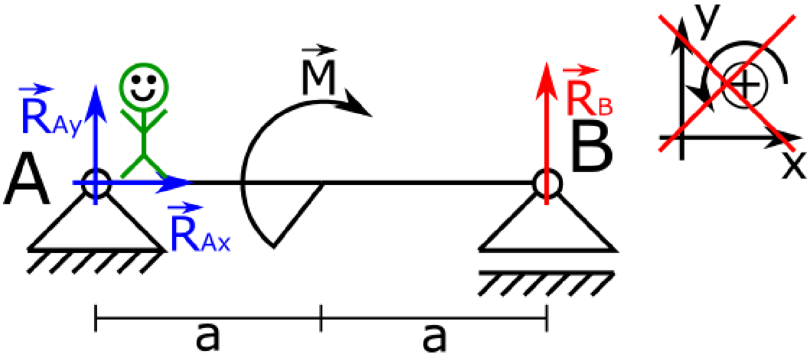
$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

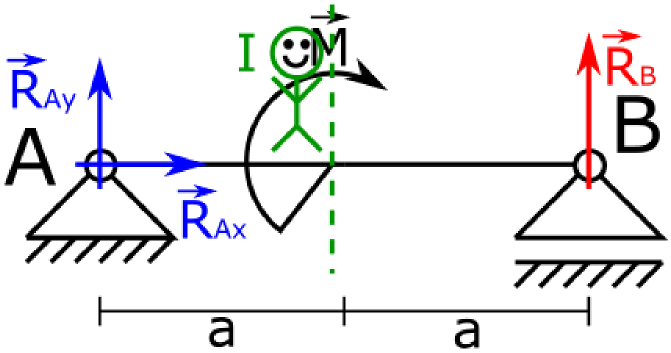
$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} + R_B \rightarrow R_{Ay} = -R_B = -5kN$$

$$\sum_{i=1}^{n} M_A = 0 = -M + R_B * 2a \rightarrow R_B = \frac{M}{2a} = \frac{20}{4} = 5kN$$

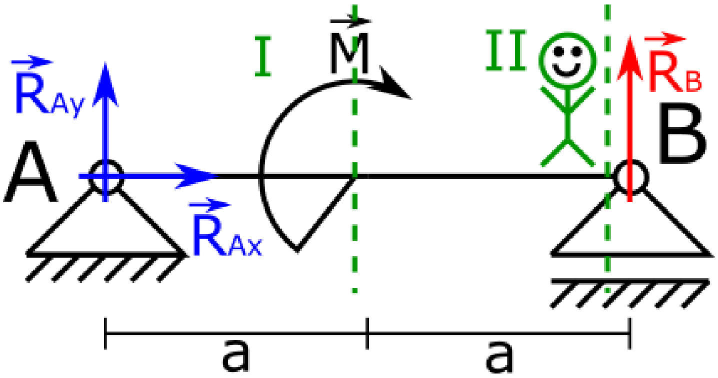
3. This is where the similarity to solving trusses ends.
4. After determining the reaction in the supports, in the next step we need to determine how many cuts the beam needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces that was introduced at the beginning. We enter the beam from its left (you can also enter the beam from the right).



5. Cuts will be made on the beam in places where something happens on the beam (something appears or disappears). The important information is that we always cut before something on the beam has happened.
We see that when we enter the beam from its left, forces R_{Ax} and R_{Ay} appear. However, we cannot make the cut before these forces, because then we are not yet on the beam.
We go further along the beam and come across moment M. Something happened, moment appeared. So we know that in this place just before the appearance of moment M should be cut.



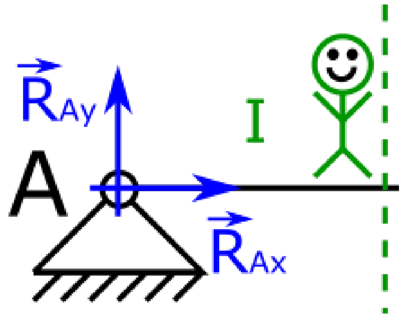
6. After making this cut, we bypass the M moment and continue until we come to the R_B force. Again, something is happening, force comes to our way, so at this point just before the R_B force we make the cut again.



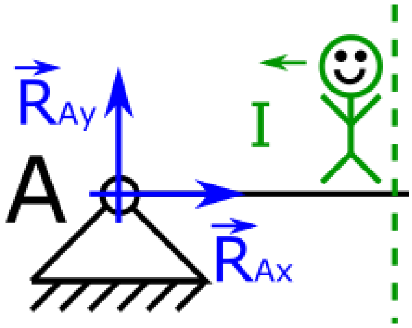
7. In this way, we divided the beam into two sections I and II. The first section within $0 \leq x < a$ second section within $a \leq x < 2a$.

BENDING MOMENTS

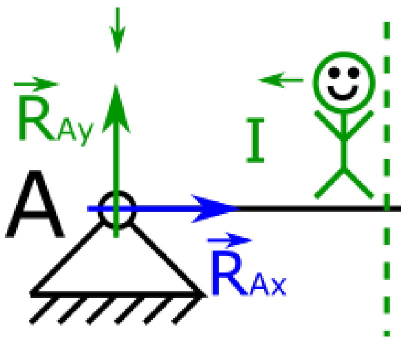
8. At this point, we can move on to determining internal forces. To do this we have to go through each of the sections. We assume that we enter the beam on the left, as before and reach the end of the first section. We will begin to determine internal forces from bending moments, then cutting forces and finally normal forces.



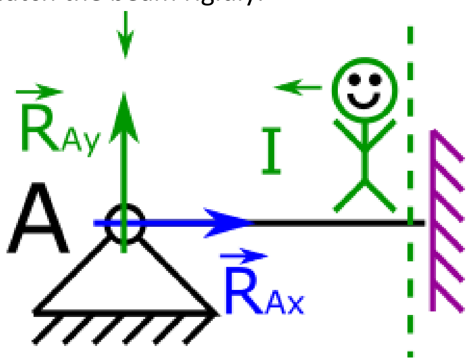
9. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



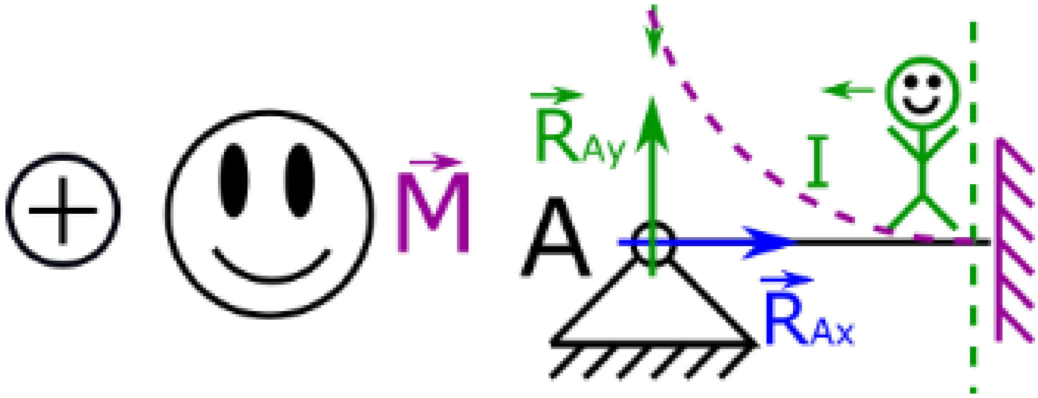
10. We see that at the beginning of the beam there is one force that causes the beam to bend, R_{Ay} force.



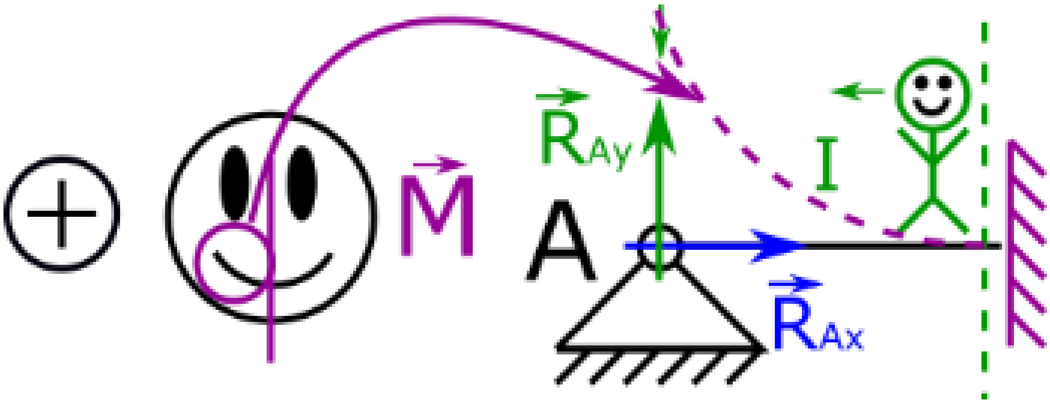
11. Now, to determine how the force bends the beam, we assume that at the place where we stand (where the cut), we catch the beam rigidly.



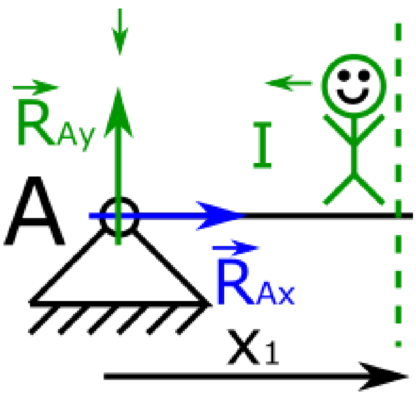
12. At this point, when one end of the beam is fixed, we imagine how the force R_{Ay} tries to bend the beam. You can see clearly in this situation, under the influence of this force, the beam would bend, as shown by the purple dashed line.



13. According to the notation introduced earlier, if the beam under the influence of force smiles, then we assume that the bending moment from such force is a positive sign. Of course, when cutting, it is clearly seen that we get only half of the smile.



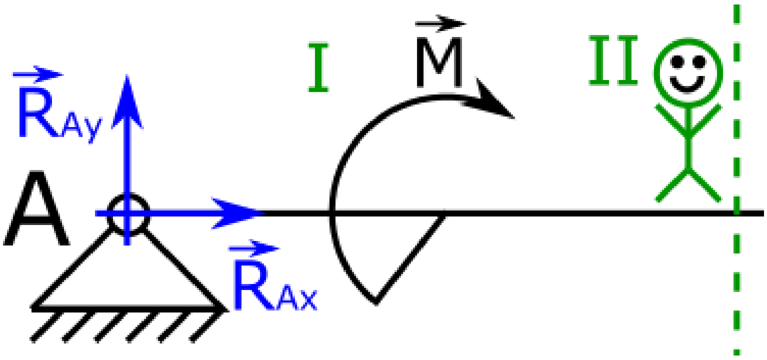
14. We already know what sign the bending moment from R_{Ay} will have, now the arm of this moment should be determined. Because we are standing just before the appearance of the force F, it only means that we are at a distance of some x from the beginning of the beam. Let's call this distance x_1, since we're in the first section.



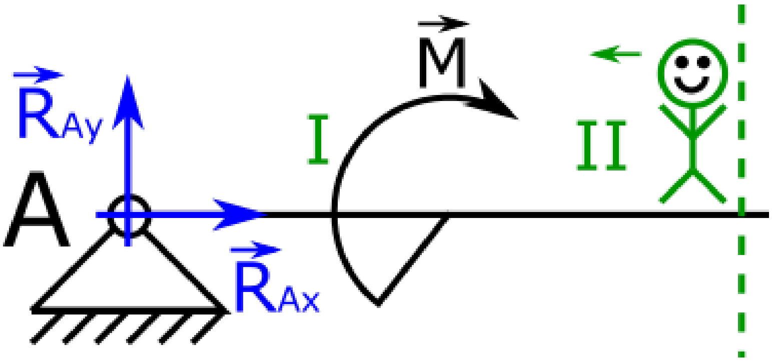
15. After these considerations, we can write the first equation for bending moments in the first section.

$$M(x_1) = R_{Ay} * x_1$$

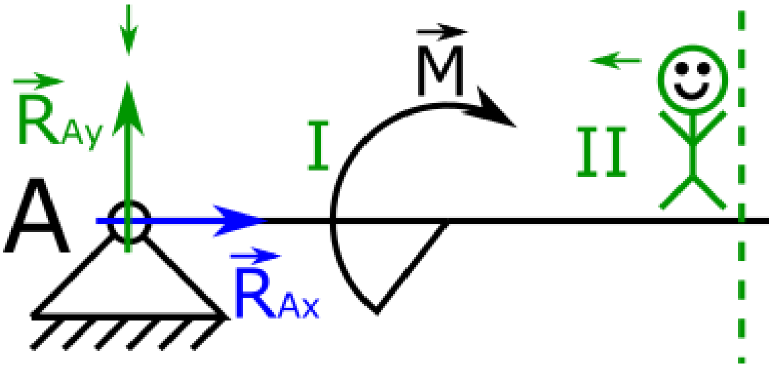
16. Now we move to the second section. The important thing is that the first section remains. You can imagine that we just walk along the beam and we can't cut what is behind us, and the beam just lengthens.



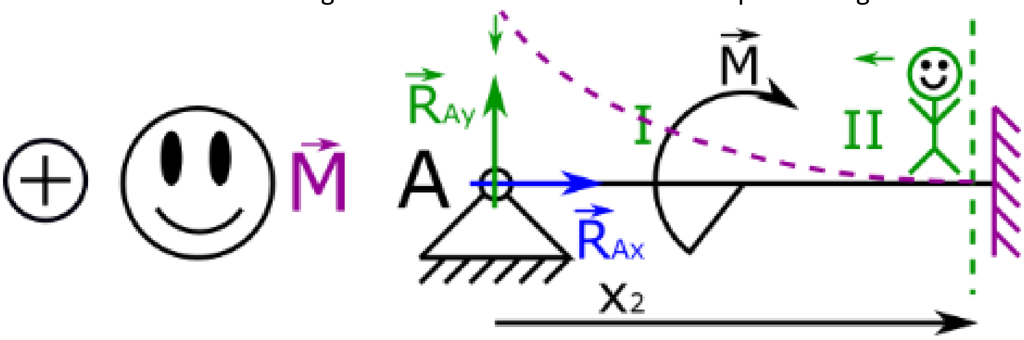
17. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



18. We see that at the beginning of the beam there is one force that causes the beam to bend, R_{Ay} force.



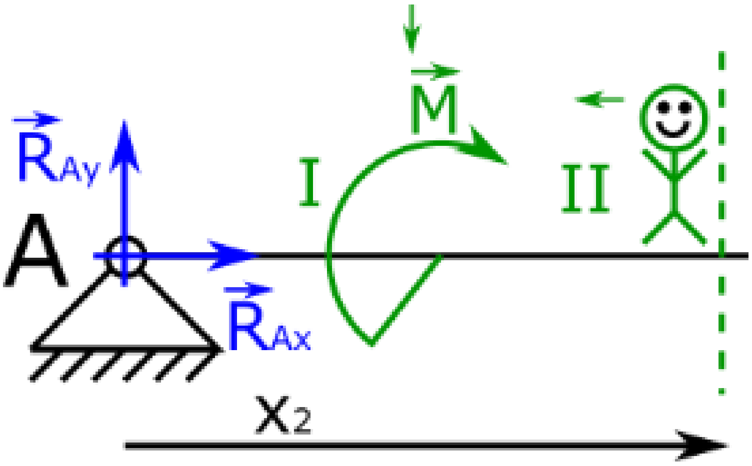
19. Based on the previous section, we can see that this force still bends the beam upwards, hence the moment bending from this force will still be with a positive sign.



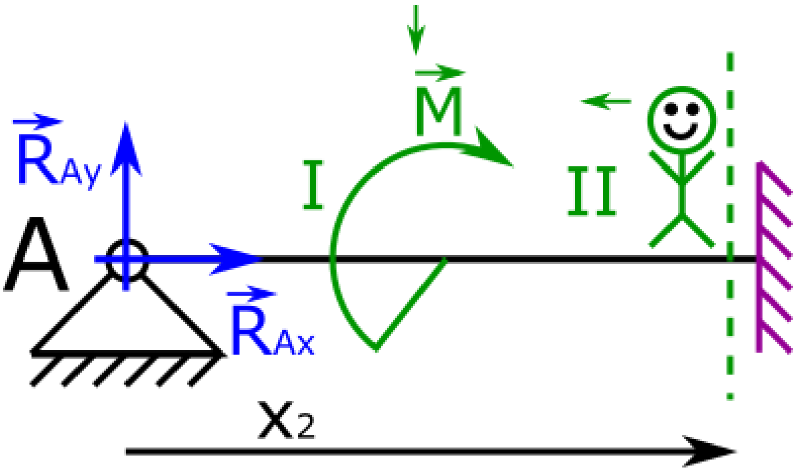
20. We can write first part of equation for bending moments in the second section. We will write the next part after further considerations.

$$M(x_2) = R_{Ay} * x_2 \dots$$

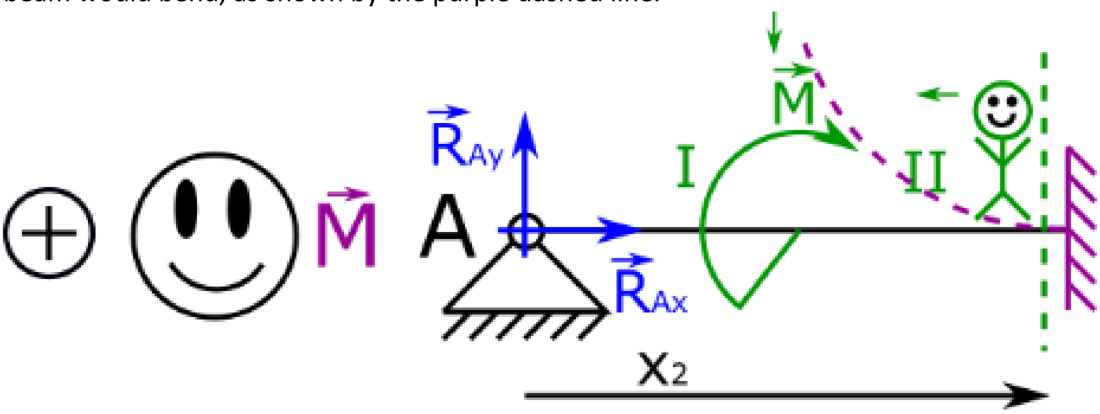
21. We see that next thing on the beam that causes the beam to bend is moment M.



22. Now, to determine how the moment bends the beam, we assume that at the place where we stand (where the cut), we catch the beam rigidly.



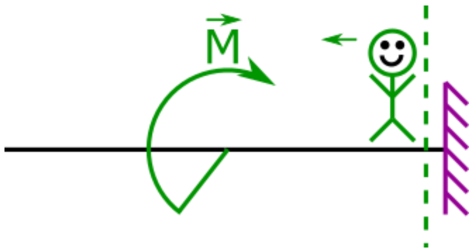
23. At this point, when one end of the beam is fixed, we imagine how the moment M tries to bend the beam. You can see clearly in this situation, under the influence of this moment, the beam would bend, as shown by the purple dashed line.

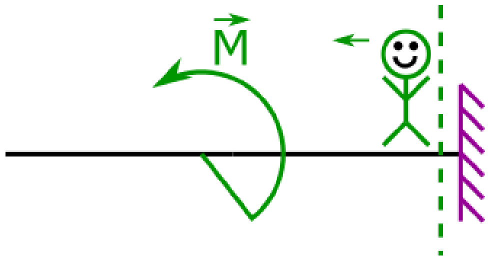


24. We can write final of equation for bending moments in the second section. We need to take first part and add second part. **Important information. In the case of a concentrated moment, we do not multiply its value by the arm, but we always place such a moment in the equation of moments.**

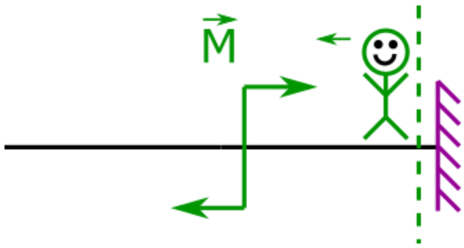
$$M(x_2) = R_{Ay} * x_2 + M$$

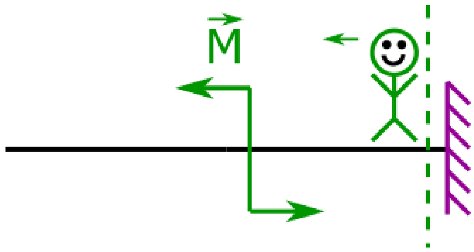
25. Due to the fact that in the case of concentrated moments there are problems with determining how they bend the beam. Therefore, below you will find a description of how to make it easier to understand this phenomenon. It will be presented in a situation when the moment is turning clockwise and counterclockwise.



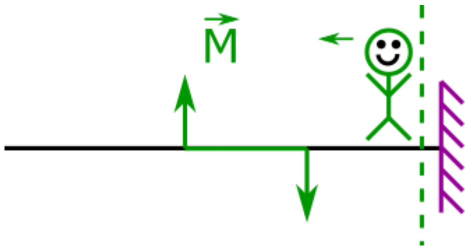


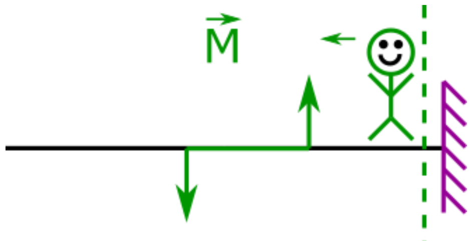
26. We know that a focused moment can be replaced by a pair of parallel forces of the same value and opposite senses that causes rotation around a specific point. As shown in the picture.



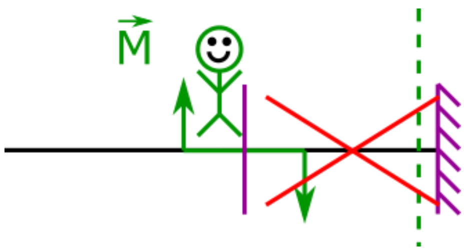


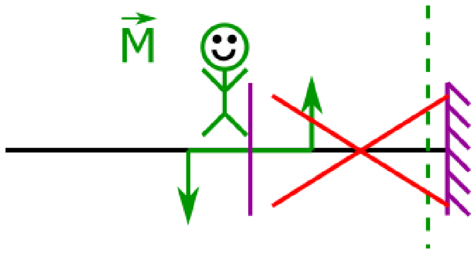
27. Because this pair of forces causes rotation, so let's turn these forces so that their directions are perpendicular to the beam.



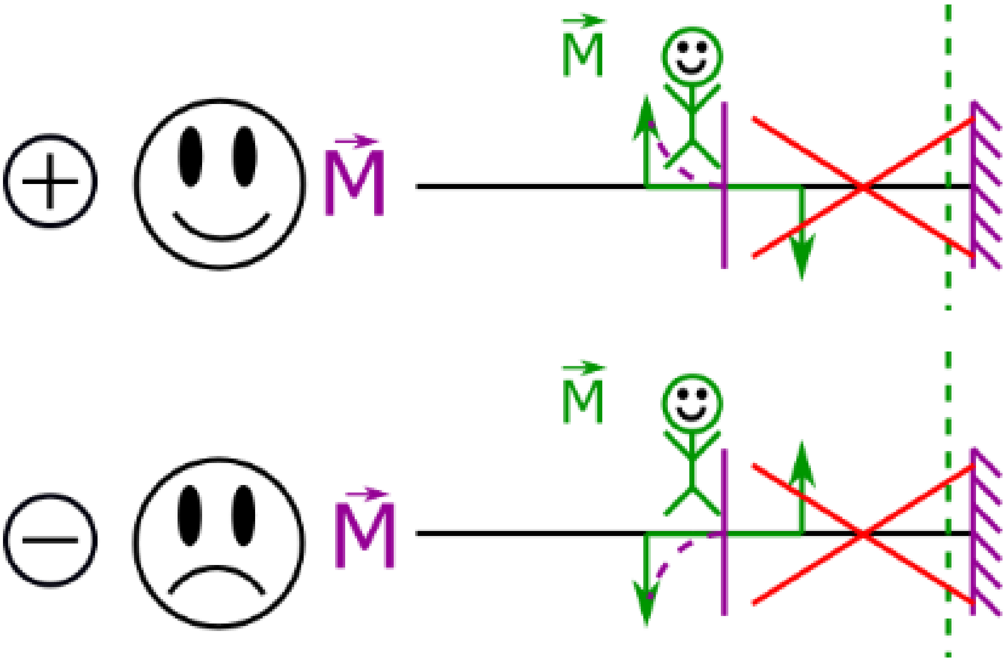


28. At this point, we can assume that we will take only one part of the pair of forces into our considerations. It will be the part from which we entered the beam. In our case, the left part.



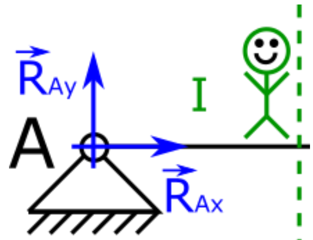


29. If we now assume that we are dealing simply with a force that acts on our beam. Then we look at how this force will bend the beam and thus we get the answer to the question of how our moment bends the beam. As shown below.

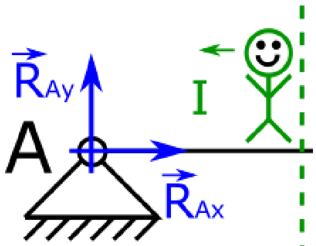


CUTTING FORCES

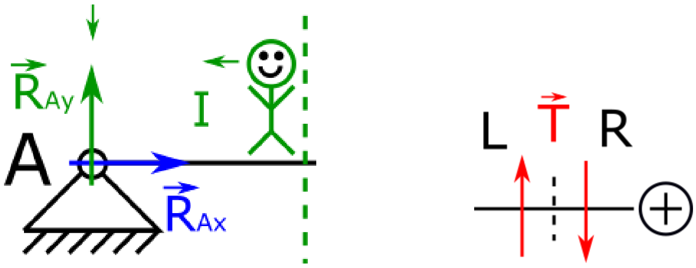
30. Now we can move to the next internal forces, i.e. the cutting forces. We will consider these forces similarly to the previous section after the section.



31. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



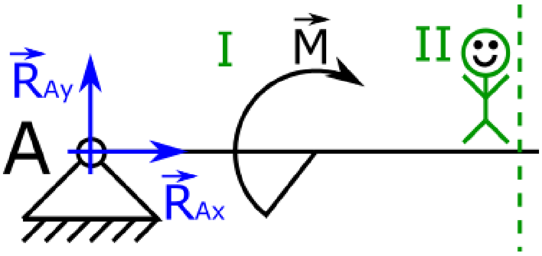
32. We see that at the beginning of the beam there is one force that seems to cut the beam, R_{Ay} force. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ay}'s is up, which means that we will take this force with a positive sign.



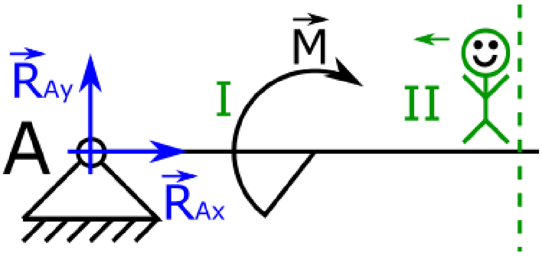
33. After these considerations, we can write the first equation for cutting forces in the first section.

$$T(x_1) = R_{Ay}$$

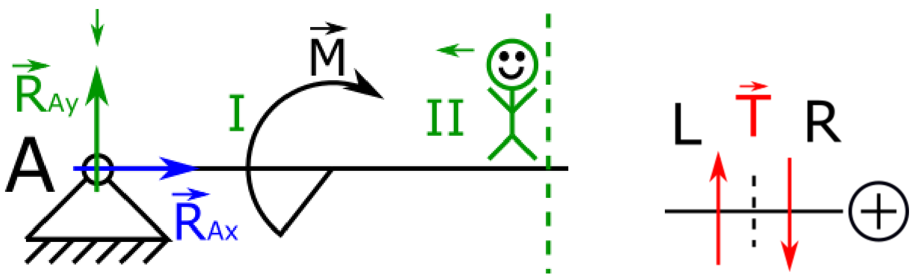
34. Now we move to the second section. The important thing is that the first section remains. You can imagine that we just walk along the beam and we can't cut what is behind us, and the beam just lengthens.



35. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look).



36. We see that at the beginning of the beam there is one force that seems to cut the beam, R_{Ay} force. Now let's get back to our assumptions, from the beginning, for cutting forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ay}'s is up, which means that we will take this force with a positive sign. So it is the same, as it was for the first section.

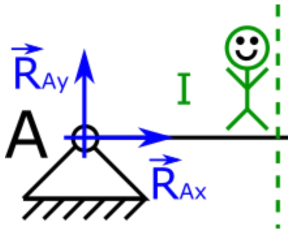


37. We can write equation for cutting forces in the second section. There are no other forces that trying to cut our beam, the only thing left **is moment which is not a force** this is why we do not include it in the **equation of cutting forces**.

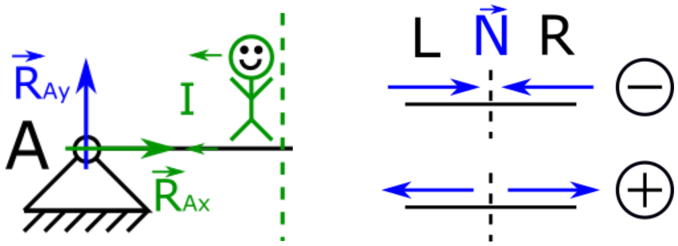
$$T(x_2) = R_{Ay}$$

NORMAL FORCES

38. Now we can move to the final internal forces, i.e. the normal forces. We will consider these forces similarly to the previous section after the section.



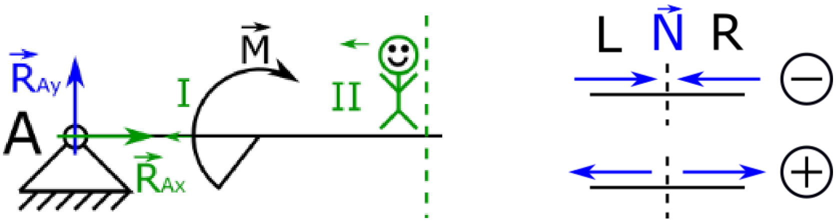
39. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look). We see that at the beginning of the beam there is one force acting along the beam, R_{Ax} force. Now let's get back to our assumptions, from the beginning, for normal forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ax}'s is into the cut, which means that we will take this force with a negative sign.



40. After these considerations, we can write the first equation for cutting forces in the first section.

$$N(x_1) = -R_{Ax}$$

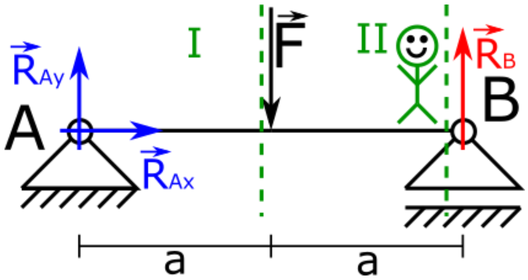
41. Now we move to the second section. The important thing is that the first section remains. You can imagine that we just walk along the beam and we can't cut what is behind us, and the beam just lengthens. Standing at the end of this section, we turn around and look at the beginning of the beam (the green arrow indicates how we look). We see that at the beginning of the beam there is one force acting along the beam, R_{Ax} force, what is more this is the only force which is acting along the beam. Now let's get back to our assumptions, from the beginning, for normal forces. It can be clearly seen that we are on the left side of the cut, and the sense of R_{Ax}'s is into the cut, which means that we will take this force with a negative sign.



42. After these considerations, we can write the first equation for cutting forces in the first section.

$$N(x_2) = -R_{Ax}$$

43. Let's write equations for both of sections in one place to clarify everything.



I Section $0 \leq x_1 < a$

$$M(x_1) = R_{Ay} * x_1$$

$$T(x_1) = R_{Ay}$$

$$N(x_1) = -R_{Ax}$$

II Section $a \leq x_2 < 2a$

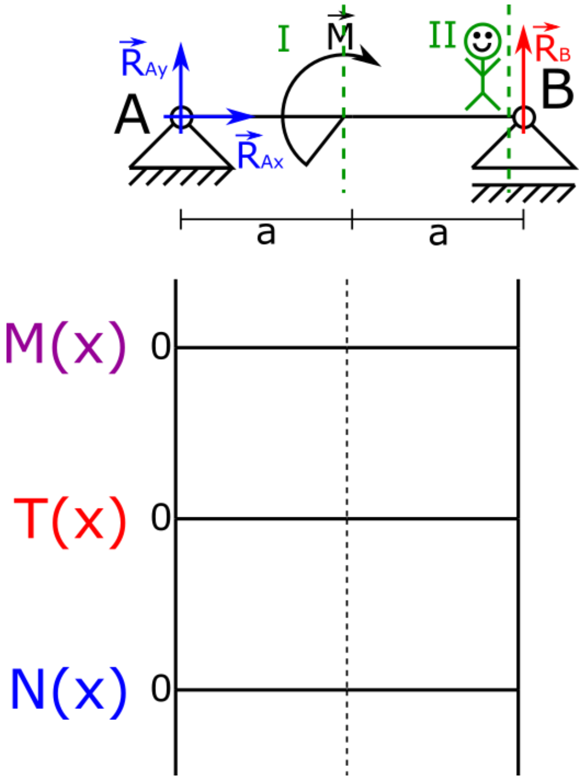
$$M(x_2) = R_{Ay} * x_1 + M$$

$$T(x_2) = R_{Ay}$$

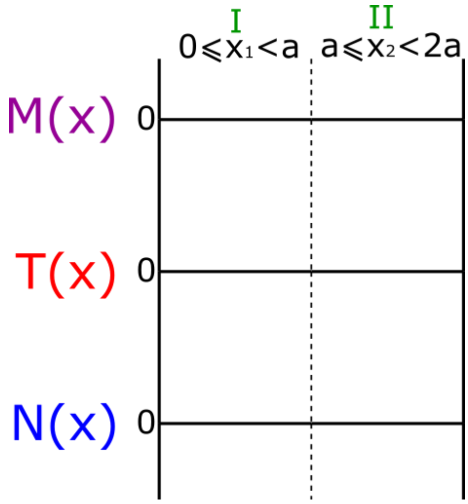
$$N(x_2) = -R_{Ax}$$

CHARTS

44. The last part related to solving beams – charts. To easily draw charts, it is best to draw them under the beam, which we solve, as shown in the figure.



45. In this example, the charts will be drawn step by step so that you can understand their creation. The first graphs will be made for the first section starting from the bending moments graph.



46. We will need the equation of bending moments for the first section.

$$M(x_1) = R_{Ay} * x_1$$

We know the limits of the first section.

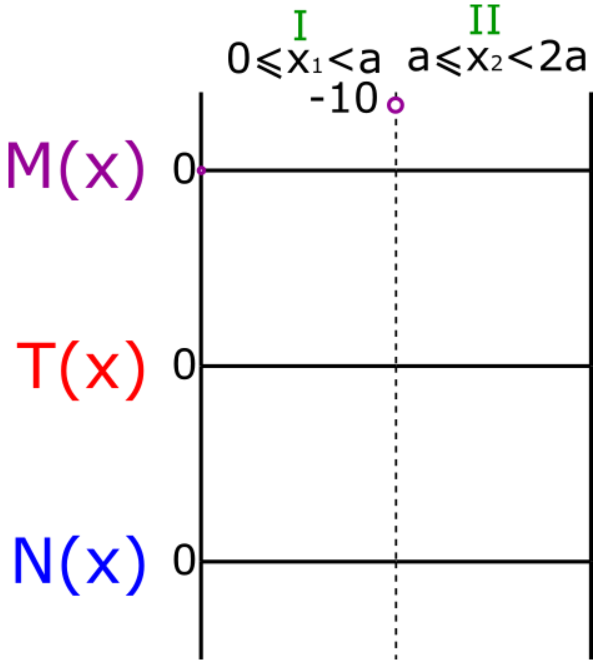
$$0 \leq x < a$$

We substitute the boundary values into our equation (for x_1).

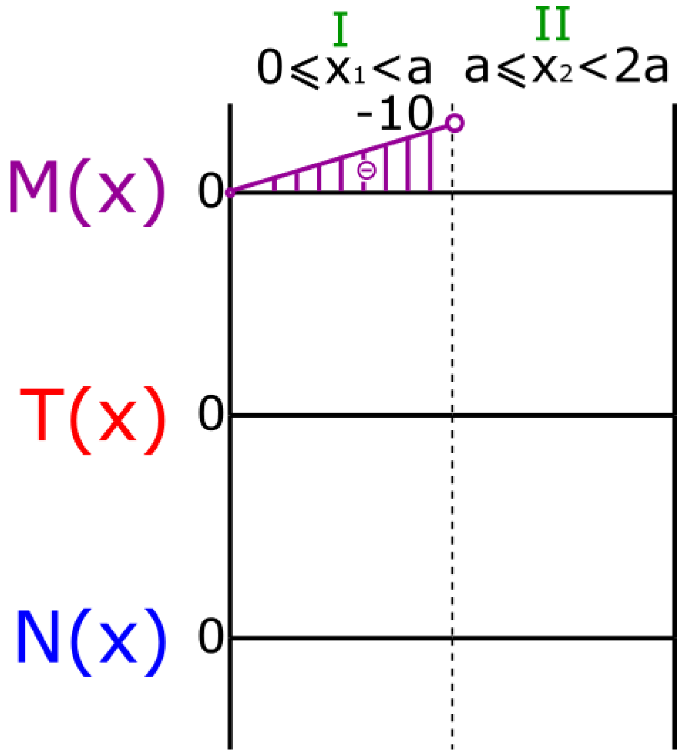
$$M(0) = R_{Ay} * 0 = 0$$

$$M(a) = R_{Ay} * a = -5 * 2 = -10kNm$$

47. When we know the values of the bending moment at the ends of the section we put these values on the graph. **Important information is that we will write positive values at the bottom of the chart and negative values at the top.**



48. Then we consider what kind of function describes the bending moment equation we wrote. It can be clearly seen that this is also a linear equation, what's more it is a descending function. Based on this information, we can connect the points previously marked as shown below. We further mark that we are on the negative side. Finally, we dash this chart. **Important information that the lines must be vertical, any other hatching, painting the chart will be incorrect.**



49. Now, unlike the previous examples, we will deal with the equation of bending moments in the second section. And we will consider how the bending moment in the second section changes.

We will need the equation of bending moments for the second section.

$$M(x_2) = R_{Ay} * x_2 + M$$

We know the limits of the second section.

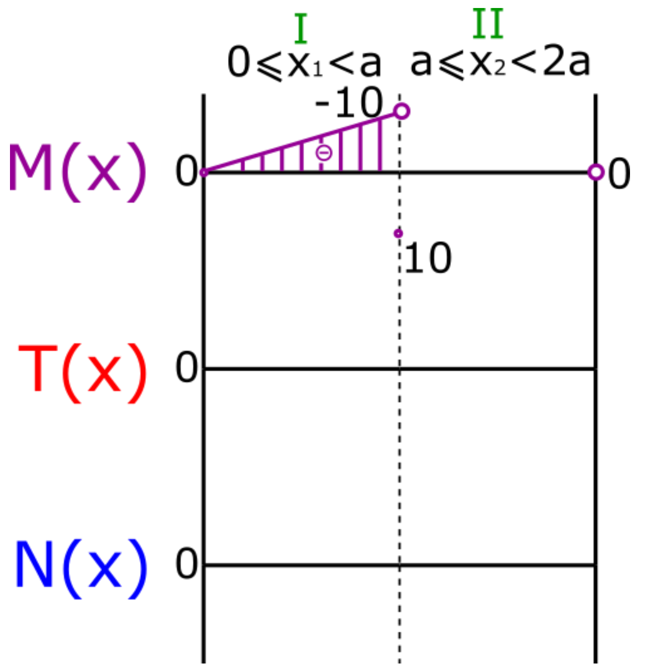
$$a \leq x < 2a$$

We substitute the boundary values into our equation (for x_2).

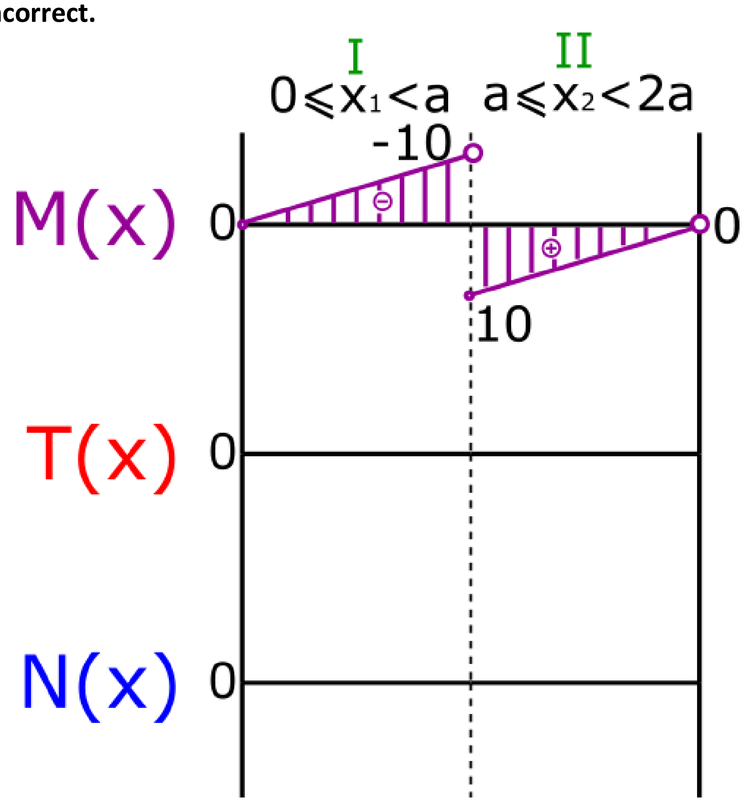
$$M(a) = R_{Ay} * a + M = -5 * 2 + 20 = -10 + 20 = 10kNm$$

$$M(2a) = R_{Ay} * 2a + M = 5 * 4 - 20 = 0kNm$$

50. When we know the values of the bending moment at the ends of the section we put these values on the graph.



51. Then we consider what kind of function describes the bending moment equation we wrote. It can be clearly seen that this is also a linear equation, what's more it is a descending function. Based on this information, we can connect the points previously marked as shown below. We further mark that we are on the negative side. Finally, we dash this chart. **Important information that the lines must be vertical, any other hatching, painting the chart will be incorrect.**



52. We will need the equation of cutting forces for the first section.

$$T(x_1) = R_{Ay}$$

We know the limits of the first section.

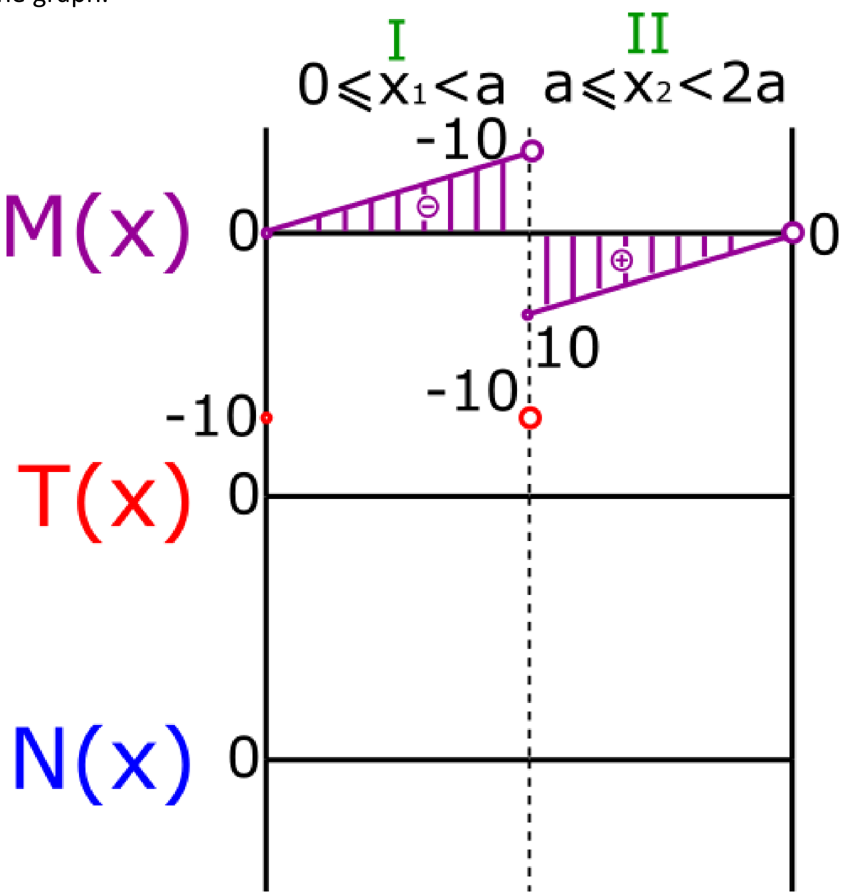
$$0 \leq x < a$$

We substitute the boundary values into our equation (for x_1).

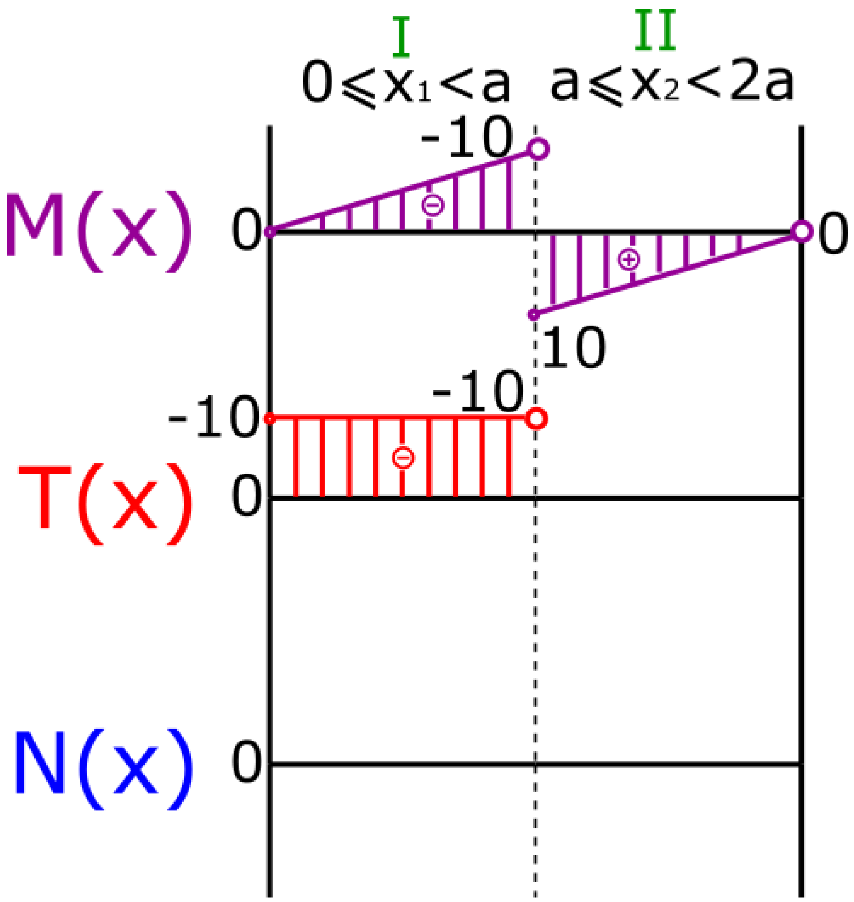
$$T(0) = R_{Ay} = -10kN$$

$$T(a) = R_{Ay} = -10kN$$

53. When we know the values of the cutting forces at the ends of the section we put these values on the graph.



54. Then we consider what kind of function describes the cutting forces equation we wrote. It can be clearly seen that this is a constant equation. Based on this information, we can connect the points previously marked as shown below. We further mark that we are on the negative side. Finally, we dash this chart.



55. Now, unlike the previous examples, we will deal with the equation of cutting forces in the second section. And we will consider how the cutting forces in the second section changes.

We will need the equation of cutting forces for the second section.

$$T(x_2) = R_{Ay}$$

We know the limits of the second section.

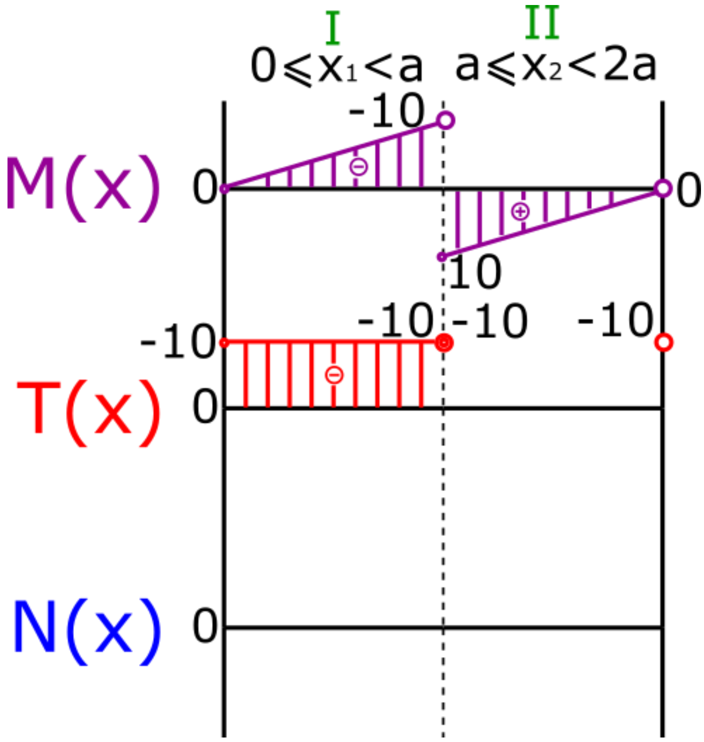
$$a \leq x < 2a$$

We substitute the boundary values into our equation (for x_1).

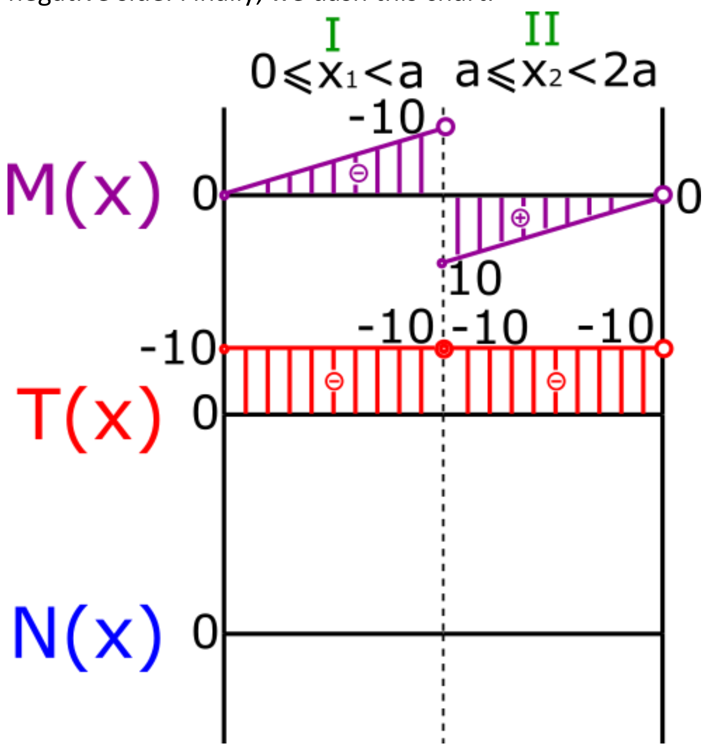
$$T(a) = R_{Ay} = -10kN$$

$$T(2a) = R_{Ay} = -10kN$$

56. When we know the values of the cutting forces at the ends of the section we put these values on the graph.



57. Then we consider what kind of function describes the cutting forces equation we wrote. It can be clearly seen that this is a constant equation. Based on this information, we can connect the points previously marked as shown below. We further mark that we are on the negative side. Finally, we dash this chart.



58. Będziemy potrzebować równania sił normalnych dla pierwszej i drugiej sekcji. Widzimy, że oba te równania są takie same.

$$N(x_1) = -R_{Ax}$$

We know the limits of the first section.

$$0 \leq x < a$$

We know the limits of the second section.

$$a \leq x < 2a$$

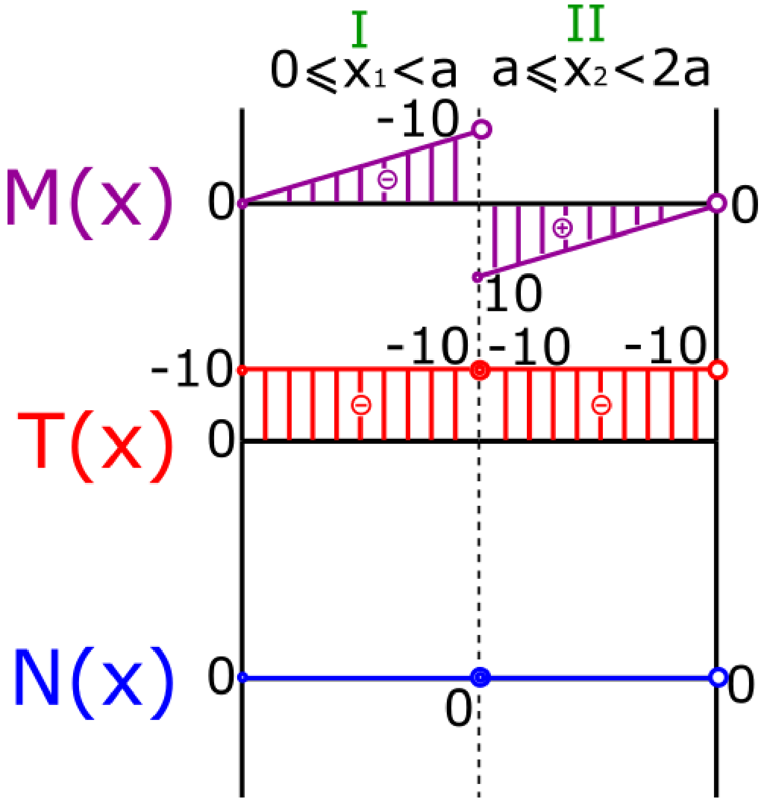
We substitute the boundary values into our equation (for x_1 and x_2).

$$N(0) = -R_{Ax} = 0kN$$

$$N(a) = -R_{Ax} = 0kN$$

$$N(2a) = -R_{Ax} = 0kN$$

59. When we know the values of the normal forces at the ends of the sections we put these values on the graph. Then we consider what kind of function describes the normal forces equation we wrote. It can be clearly seen that this is a constant equation. Based on this information, we can connect the points previously marked as shown below.



60. Looking at the charts above, we can pull out some information that will also allow us to check whether we have solved our beam well.
    - First of all, when we look at the bending moments chart, we see that in the place where we have the joints, the moments values are equal to 0. What's more, the moments values, when the beam is by the moment, are not equal on the place

where concentrated moment is applied, as it is between the sections I and section II in our case. It can be seen that at the connection of sections I and II there is a pitch in the value of the bending moment. This pitch is exactly equal to the value of the concentrated moment M present at this place on the beam. If there was no pitch on the chart, or the pitch value was different than the value of the concentrated moment, it would mean that something is wrong in the calculations and the chart.

- Secondly, looking at the cutting forces graph, it is clear that there are no changes throughout the entire length of the beam. This is because there are no shear forces. The only forces that occur are the forces in the supports and they affect the plot of cutting forces.

The function of shear forces is derived from the function of bending moments

$$\frac{dM(x)}{dx} = T(x)$$

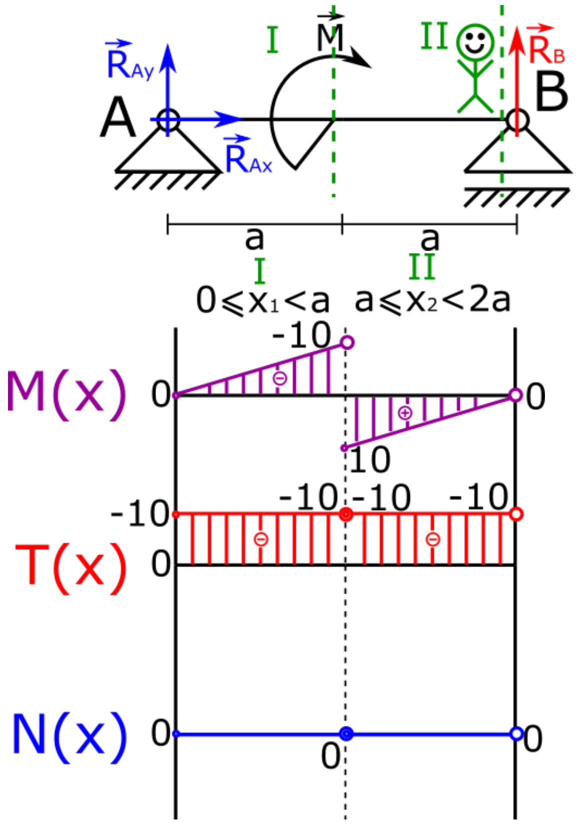
ATTENTION

By entering the beam on the right, the function of shear forces is a derivative of the function of bending moments with the opposite sign

$$\frac{dM(x)}{dx} = -T(x)$$

- Finally, normal forces. It is clear that if there are no forces acting along the beam, then the normal force must be zero.

61. Finally a beam with calculated reactions, internal force equations and graphs of these forces.



$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} + R_B \rightarrow R_{Ay} = -R_B = -5kN$$

$$\sum_{i=1}^{n} M_A = 0 = -M + R_B * 2a \rightarrow R_B = \frac{M}{2a} = \frac{20}{4} = 5kN$$

I Section $0 \leq x_1 < a$

$$M(x_1) = R_{Ay} * x_1$$

$$T(x_1) = R_{Ay}$$

$$N(x_1) = -R_{Ax}$$

II Section $a \leq x_2 < 2a$

$$M(x_2) = R_{Ay} * x_1 + M$$

$$T(x_2) = R_{Ay}$$

$$N(x_2) = -R_{Ax}$$