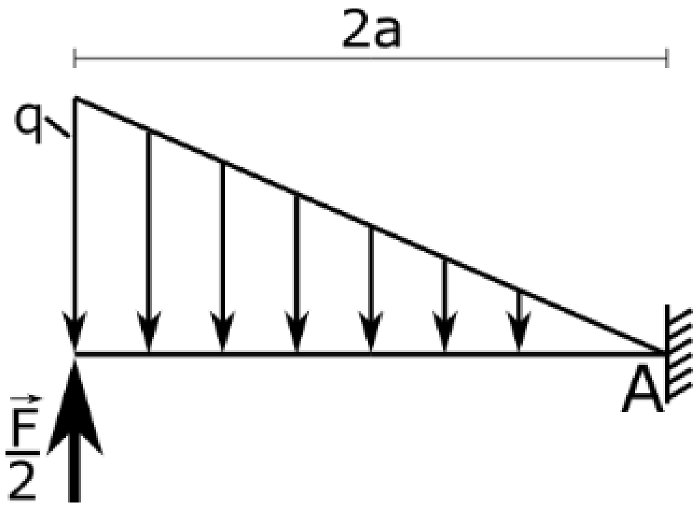
Beams – triangular distributed force

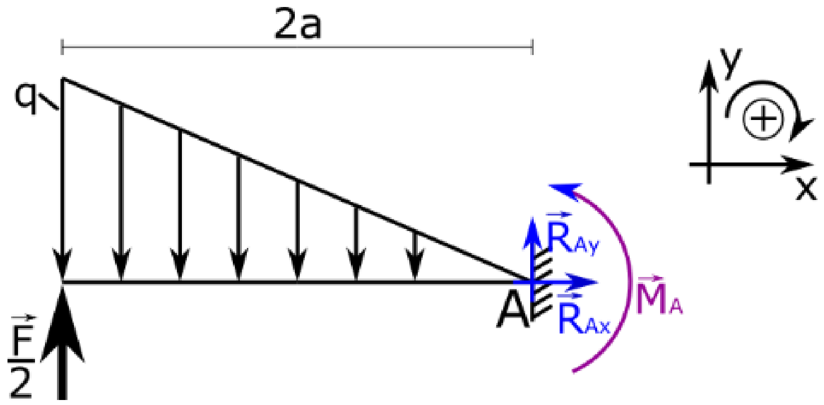
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam with all types of loads. An important information, the beam solution also includes drawing internal force diagrams.

Ex.6

For the beam shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a, F = q*a, q.



1. The first thing to do when solving beams is to determine the number of supporting unknowns, as was the case with trusses. It is clear that in this case we have three unknown supports. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, clockwise turning moments will be positive.

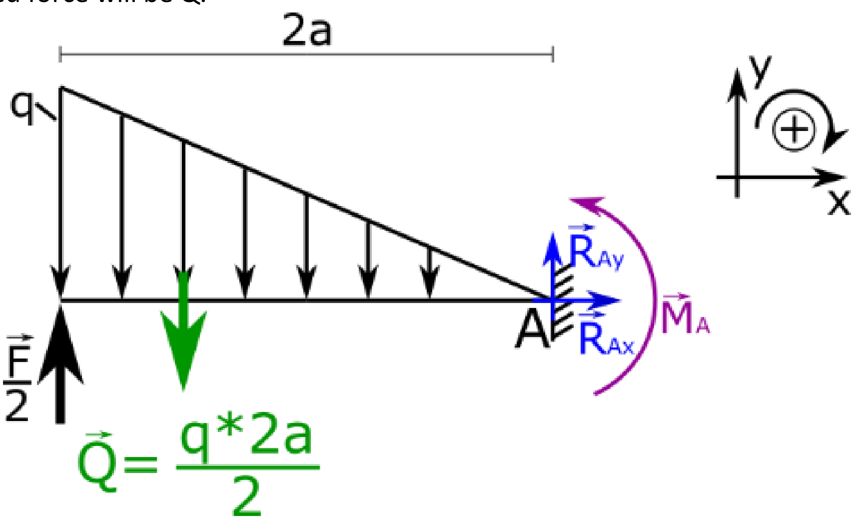


2. Now we can find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

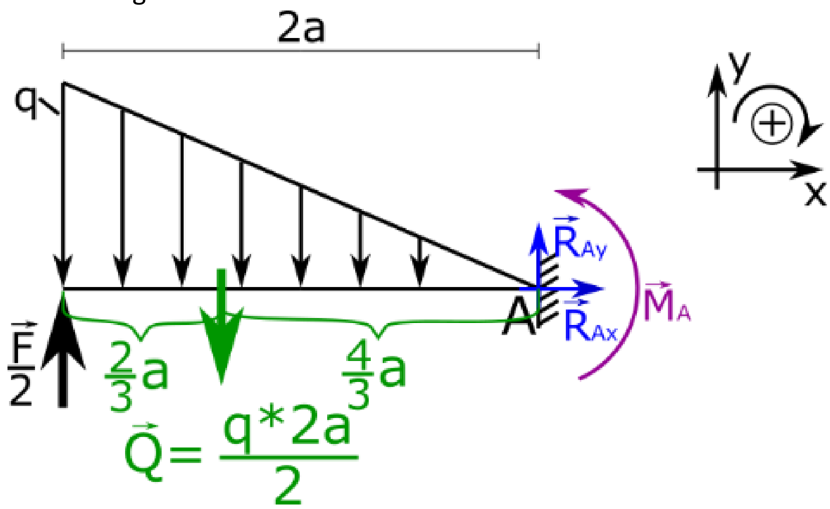
$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

3. However, before calculating the unknown reactions, I will show you how you can reduce the force in the case of a triangular load. First of all, as in the case of a rectangular load, we must

calculate the surface area of our figure formed by the distributed load. In this case, a triangle. Our reduced force will be Q.



4. In the next step, we need to determine where this reduced force is located. In the case of a rectangle, the matter was simple, because we put the reduced force at the intersection of the diagonals. In this case, our figure is a triangle. In a right-angled triangle, the reduced force will always be a third of the distance from the long side and two-thirds from the short side, as shown in the figure below.



5. Now that we know where the reduced force from the continuous load is, we can calculate the reactions (we will know the distance needed in the equation of moments).

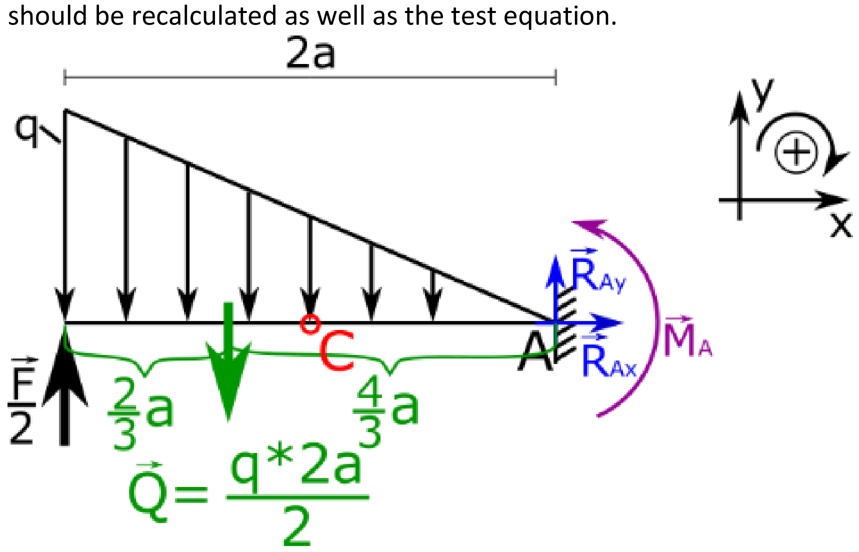
$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \to R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} + \frac{F}{2} - \frac{q*2a}{2} \to R_{Ay} = \frac{q*2a}{2} - \frac{qa}{2} = \frac{qa}{2}$$

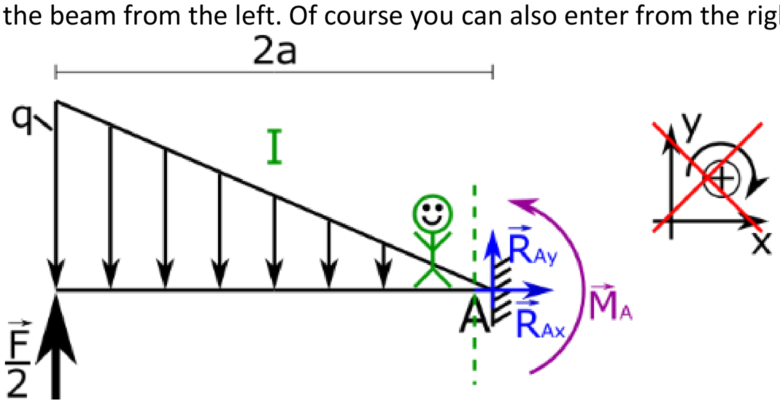
$$\sum_{i=1}^{n} M_A = 0 = \frac{F}{2} * 2a - \frac{q*2a}{2} * \frac{4}{3}a - M_A \to M_A = \frac{qa}{2} * 2a - \frac{q*2a}{2} * \frac{4}{3}a = -\frac{1}{3}qa^2$$

6. **Important information**. Here it will be shown how one can check if the values of unknown reactions have been counted well. To do this, select a point on the system through which the directions of previously found reactions do not pass (the exception is the reaction along the X axis where it is difficult to make a mistake). After choosing a point - in this case it will be a point marked as C - you should count the sum of moments relative to this point by inserting the previously calculated reaction values. After conversion at the end we should get zero. If the value is different, it means that an error has been made somewhere and the reaction should be recalculated as well as the test equation.



$$\sum_{i=1}^{n} M_C = \frac{F}{2} * a - \frac{q * 2a}{2} * \left(a - \frac{2}{3}a\right) - M_A - R_{AY} * a \rightarrow M_A$$
$$= \frac{qa^2}{2} - qa * \frac{a}{3} + \frac{1}{3}qa^2 - a * \frac{qa}{2} = \frac{qa^2}{2} * \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{2}\right) = 0$$

7. After determining the reaction in the supports, in the next step we need to determine how many cuts the beam needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces that was introduced at the beginning. In this case, I will show a more complicated version of the calculations for a triangular load, that's why we enter the beam from the left. Of course you can also enter from the right.



8. Cuts will be made on the beam in places where something happens on the beam (something appears or disappears). The important information is that we always cut before something on the beam has happened.

We see that when we enter the beam from its left, forces F/2 and triangular distributed force appear. However, we cannot make the cut before these forces, because then we are not yet on the beam. We go further along the beam and come across force R_{AY} and moment M_A. Something happened, force and moment appeared. So we know that in this place just before the appearance of force R_{AY} and moment M_A should be first cut.

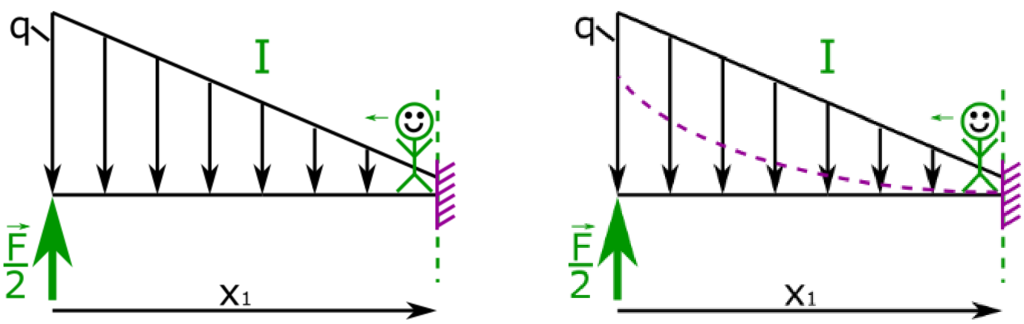
9. In this way, we divided the beam into one section I. The first section within $0 \leq x < 2a$.

BENDING MOMENTS, CUTTING FORCES, NORMAL FORCES

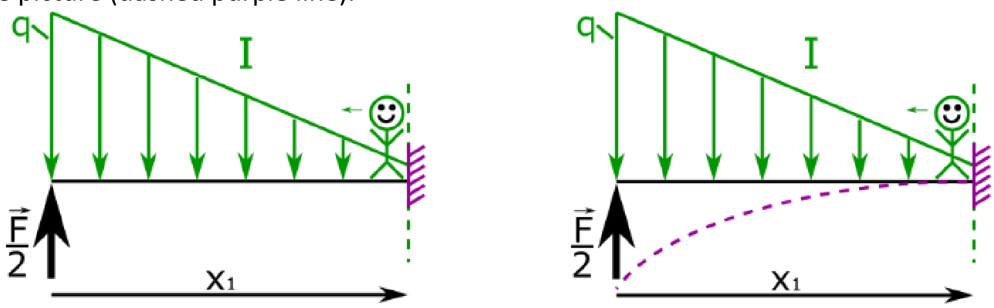
10. At this point, we can move on to determining internal forces. To do this we have to go through the section. In this example, at the beginning we will determine what sign will have the bending moment from each of the forces in our section.

I section within $0 \leq x < 2a$.

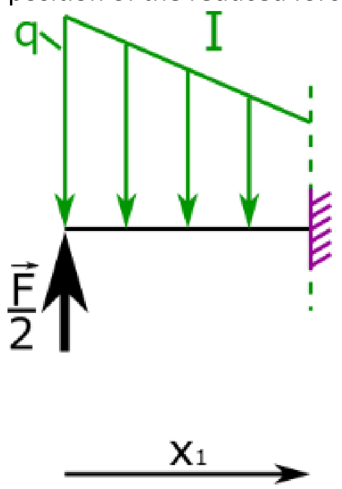
The bending moment will be from the F/2 force and will look like in the picture (dashed purple line).



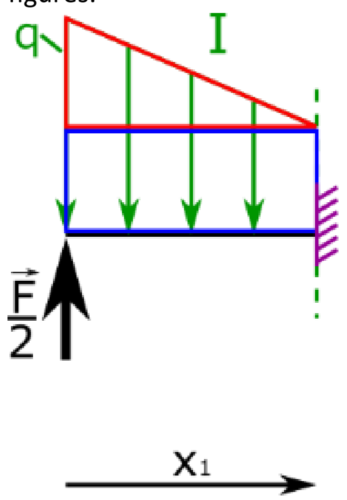
And also the bending moment will be from the triangular distributed force and will look like in the picture (dashed purple line).



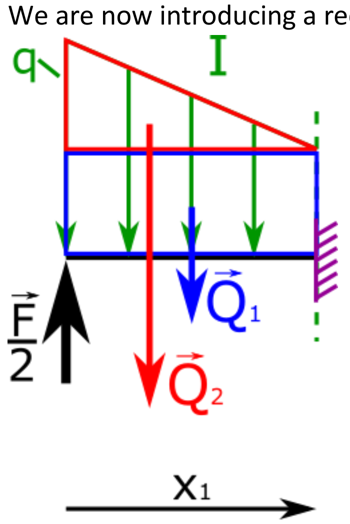
11. In order to overcome the difficulty that is associated with a triangular load in this case, let's move slightly beyond half the beam. It can be clearly seen that from our triangle load we have actually got a trapezoid. And this figure is the problem here. Because I know where the reduced force will be for the rectangle and the triangle, but it is difficult to determine the position of the reduced force for the trapezoid.



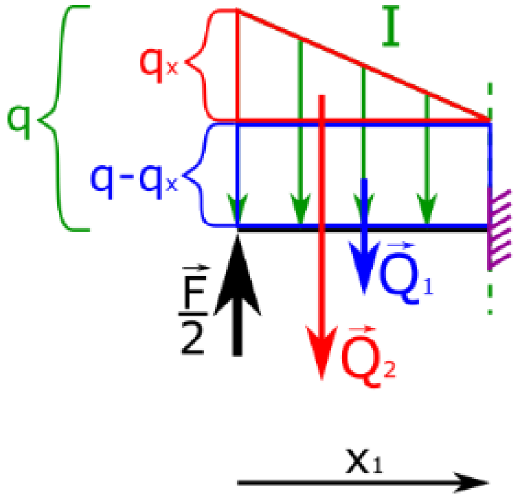
12. To deal with this problem, let's replace our trapezoid with two known triangle and rectangle figures.



13. We are now introducing a reduced force from each load into our system.



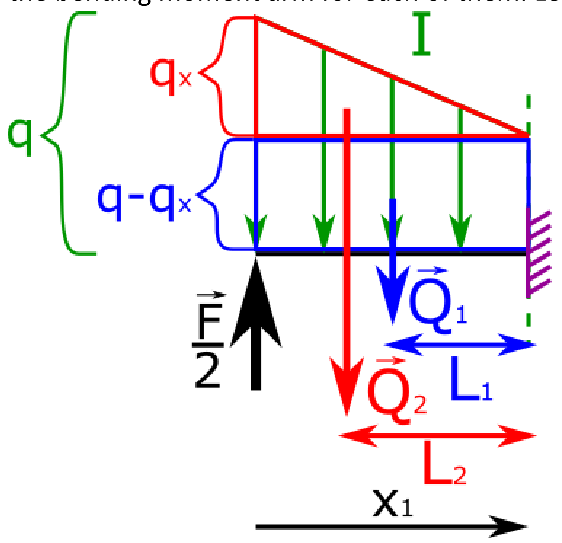
14. Next, determine what value each of the reduced forces will have. It is clear that the size of the triangle and the rectangle will depend on how far on the beam we will be. Therefore, the height of the triangle and rectangle will also vary, as shown in the figure.



$$\vec{Q}_1 = (q - q_x) * x_1$$

$$\vec{Q}_2 = \frac{q_x * x_1}{2}$$

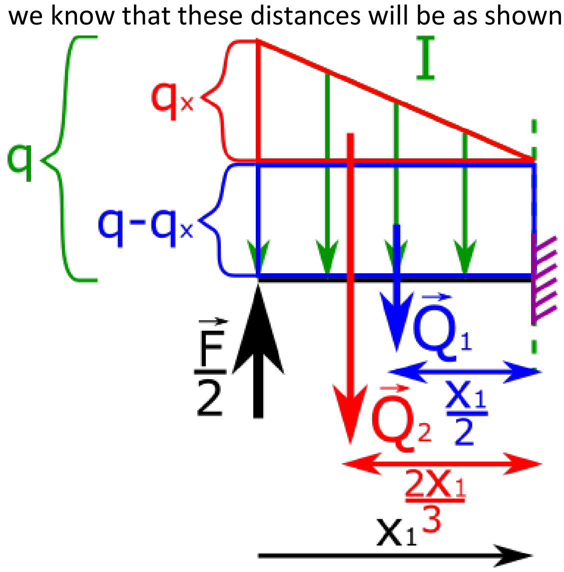
15. Now when we know how the forces change from each of the distributed forces, we must find the bending moment arm for each of them. Let's denote these lengths as L_1 and L_2.



$$\vec{M}_1 = \vec{Q}_1 * L_1$$

$$\vec{M}_2 = \vec{Q}_2 * L_2$$

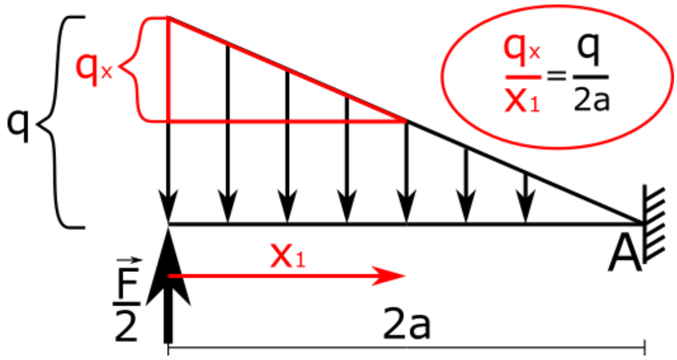
16. From the previous example for a rectangular load and from the information described above, we know that these distances will be as shown in the figure below.



$$\vec{M}_1 = ((q - q_x) * x_1) * \frac{x_1}{2}$$

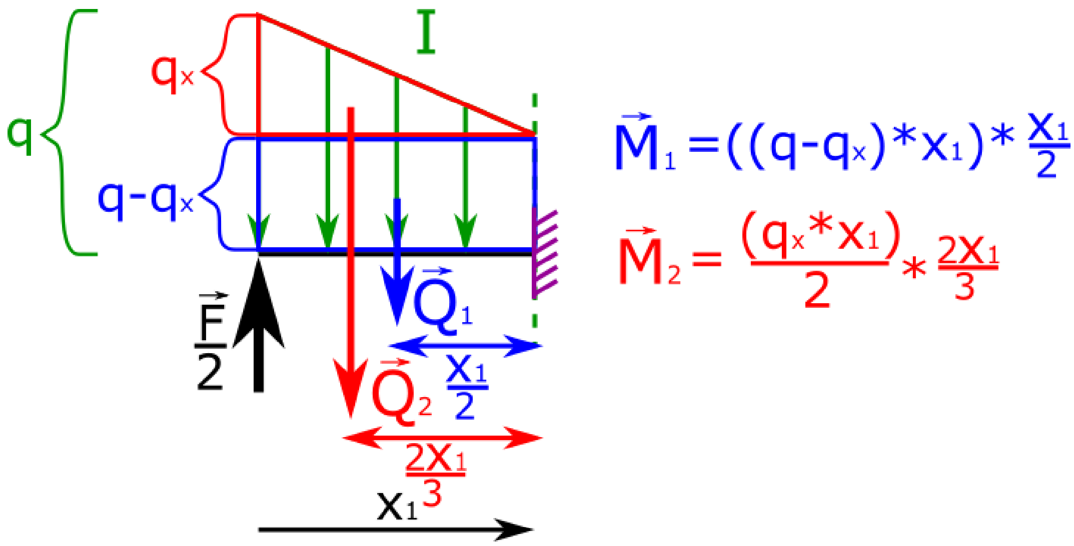
$$\vec{M}_2 = \frac{(q_x * x_1)}{2} * \frac{2x_1}{3}$$

17. The only unknown is how the height of each figure will depend on the distance, so we really need to know what the value of q_x will be. If we look at the drawing below, we can see that the small triangle and the large triangle are similar, therefore, taking into account the similarity of the triangles, we can determine how the q_x parameter changes.



$$q_x = \frac{q * x}{2a}$$

18. At this point, when we have all the information we need, we can write the equations of all internal forces: bending moments and cutting forces. In this case, normal forces are omitted, because it can be seen immediately that they will be zero.



Equations for all types of internal forces

$$M(x_1) = \frac{F}{2} * x_1 - Q_1 * L_1 - Q_2 * L_2 = \frac{qa}{2} * x_1 - x_1(q - q_x) * \frac{x_1}{2} - \frac{q_x * x_1}{2} * \frac{2x_1}{3}$$

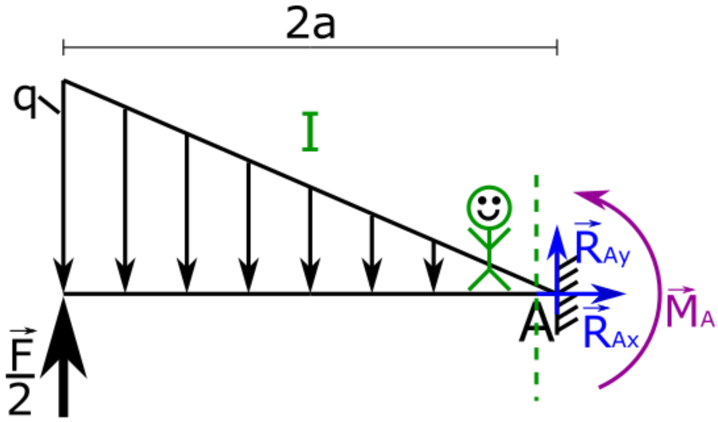
$$= \frac{qa}{2} * x_1 - x_1\left(q - \frac{q * x_1}{2a}\right) * \frac{x_1}{2} - \frac{\frac{q * x_1}{2a} * x_1}{2} * \frac{2x_1}{3}$$

$$= \frac{qa}{2} * x_1 - \frac{qx_1^2}{2} + \frac{qx_1^3}{12a}$$

$$T(x_1) = \frac{qa}{2} - qx_1 + \frac{qx_1^2}{4a}$$

$$N(x_1) = 0$$

19. Let's write equations for the whole beam in one place to clarify everything.



$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} \rightarrow R_{Ax} = 0$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} + \frac{F}{2} - \frac{q*2a}{2} \rightarrow R_{Ay} = \frac{q*2a}{2} - \frac{qa}{2} = \frac{qa}{2}$$

$$\sum_{i=1}^{n} M_A = 0 = \frac{F}{2}*2a - \frac{q*2a}{2}*\frac{4}{3}a - M_A \rightarrow M_A = \frac{qa}{2}*2a - \frac{q*2a}{2}*\frac{4}{3}a = -\frac{1}{3}qa^2$$

I Section $0 \leq x_1 < 2a$

$$M(x_1) = \frac{F}{2}*x_1 - Q_1*L_1 - Q_2*L_2 = \frac{qa}{2}*x_1 - x_1(q-q_x)*\frac{x_1}{2} - \frac{q_x*x_1}{2}*\frac{2x_1}{3}$$

$$= \frac{qa}{2}*x_1 - x_1\left(q - \frac{q*x_1}{2a}\right)*\frac{x_1}{2} - \frac{\frac{q*x_1}{2a}*x_1}{2}*\frac{2x_1}{3}$$

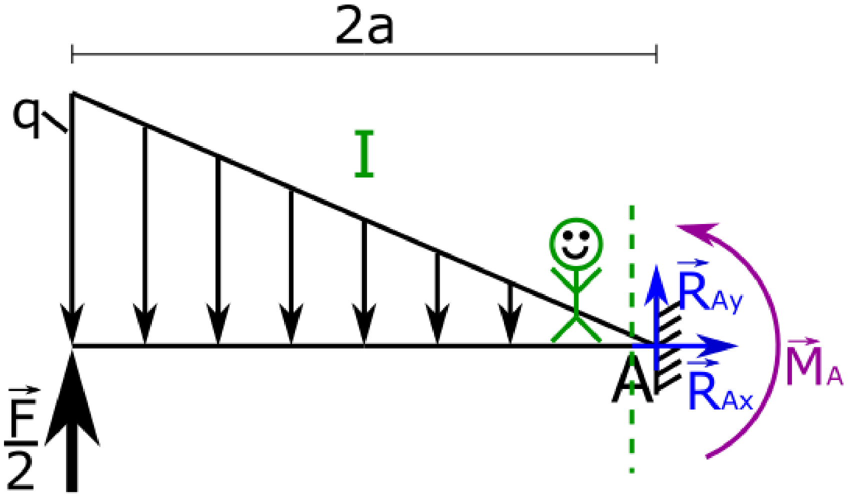
$$= \frac{qa}{2}*x_1 - \frac{qx_1^2}{2} + \frac{qx_1^3}{12a}$$

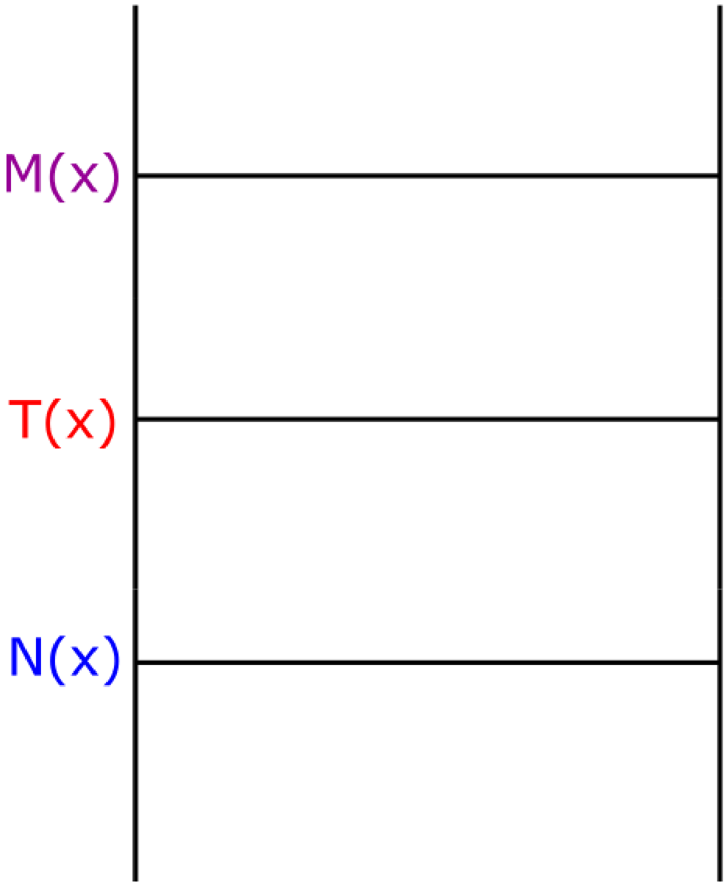
$$T(x_1) = \frac{qa}{2} - qx_1 + \frac{qx_1^2}{4a}$$

$$N(x_1) = 0$$

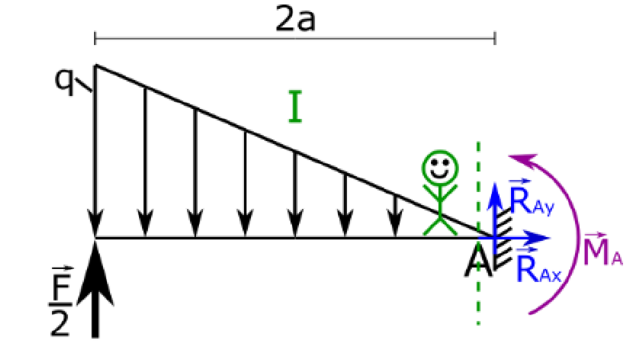
CHARTS

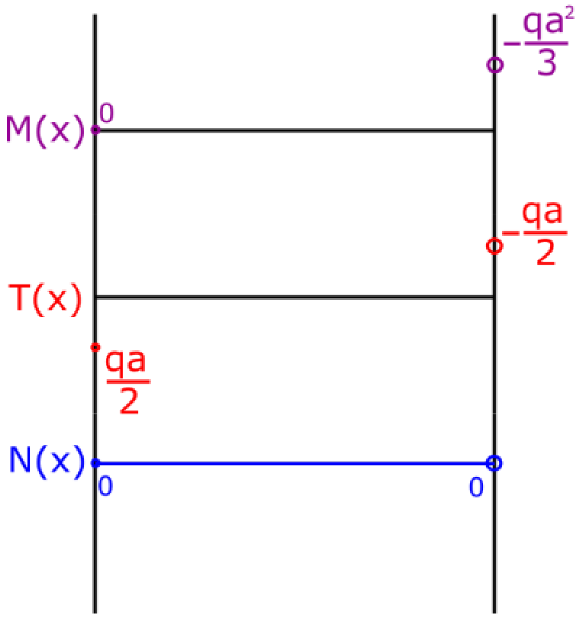
20. The last part related to solving beams – charts. To easily draw charts, it is best to draw them under the beam, which we solve, as shown in the figure.





21. In this example, the charts will be drawn for all internal forces.





We will need the equation for the first section.

$$M(x_1) = \frac{qa}{2} * x_1 - \frac{qx_1^2}{2} + \frac{qx_1^3}{12a}$$
$$T(x_1) = \frac{qa}{2} - qx_1 + \frac{qx_1^2}{4a}$$
$$N(x_1) = 0$$

We know the limits of the first section.

$$0 \leq x < 2a$$

We substitute the boundary values into our equations (for x_1).

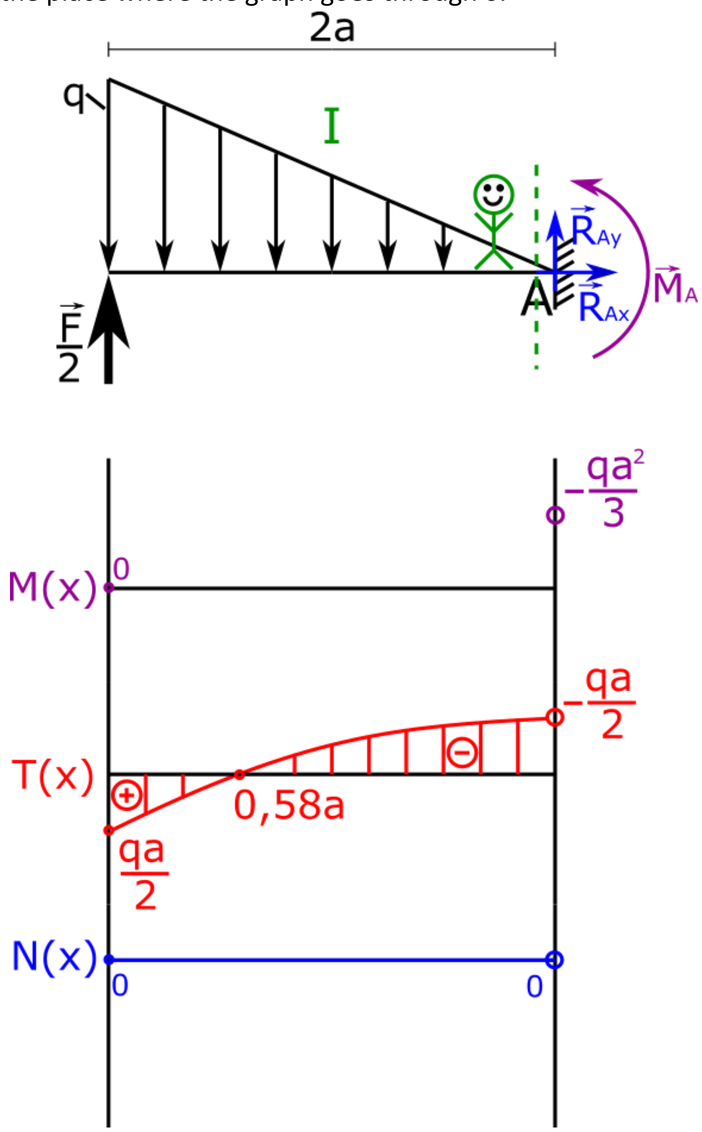
$$M(0) = \frac{qa}{2} * 0 - \frac{q0}{2} + \frac{q0}{12a} = 0$$

$$M(2a) = \frac{qa}{2} * 2a - \frac{q(2a)^2}{2} + \frac{q(2a)^3}{12a} = -\frac{qa^2}{3}$$

$$T(0) = \frac{qa}{2} - q0 + \frac{q0}{4a} = \frac{qa}{2}$$

$$T(2a) = \frac{qa}{2} - q2a + \frac{q(2a)^2}{4a} = -\frac{qa}{2}$$

22. At this point, a problem arises because the bending moment equation is a third order function, and the shear force equation is a quadratic equation. That is why it is difficult to draw a bending moment diagram from scratch. So let's start from the equation of cutting forces. Let's find the place where the graph goes through 0.



$$T = 0$$

$$\frac{qa}{2} - qx + \frac{qx^2}{4a} = 0$$

$$\Delta = q^2 - 4 * \frac{qa}{2} * \frac{q}{4a}$$

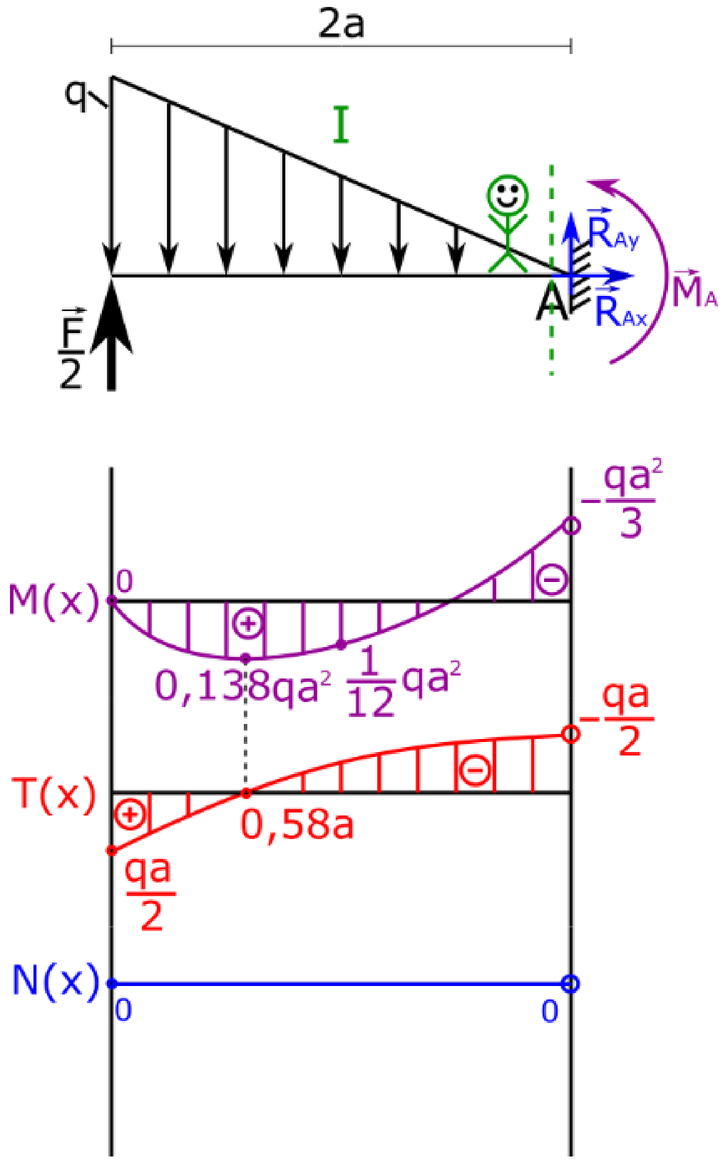
$$\Delta = \frac{q^2}{2}$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-q) - \sqrt{\frac{q^2}{2}}}{2 * \frac{q}{4a}} = 0.58a$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-q) + \sqrt{\frac{q^2}{2}}}{2 * \frac{q}{4a}} = 3.41a$$

We received two roots of the quadratic equation. It is clear that the value of 3.41a is out of our range, hence the cutting force graph will pass through zero at a distance of 0.58a.

23. Knowing where the cutting force graph goes through zero and that the shear force graph is a derivative of the bending moment, we can say that the bending moment function must take the extreme here. Therefore, the bending moment value for this distance should be calculated.



$$M(0.58a) = \frac{q0.58a}{2} * 2a - \frac{q(0.58a)^2}{2} + \frac{q(0.58a)^3}{12a} = 0{,}138\ qa^2$$

In addition, it is also worth calculating the value for half the section.

$$M(a) = \frac{qa}{2} * 2a - \frac{q(a)^2}{2} + \frac{q(a)^3}{12a} = \frac{1}{12a}\ qa^2$$