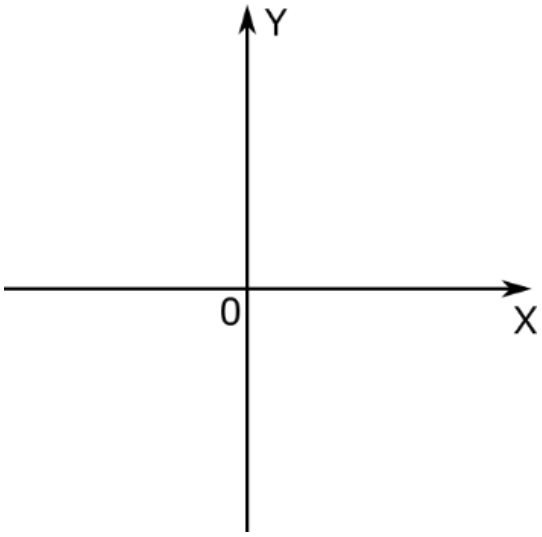
Convergent system of forces.

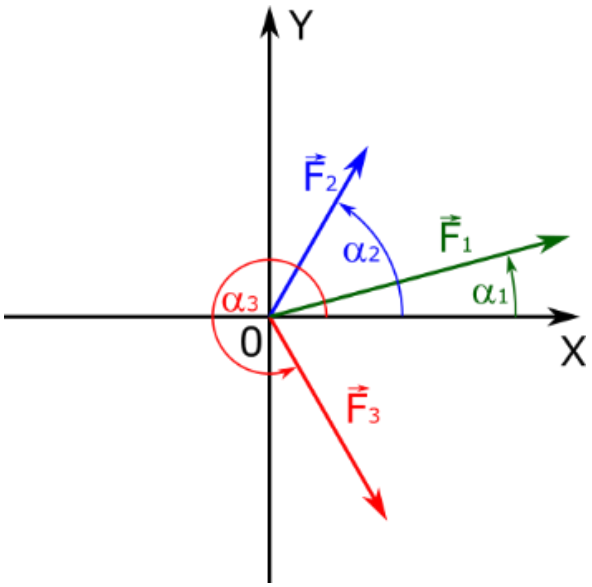
Ex1. Find the resultant force of three given forces F_1=80N, F_2=50N, F_3=60N which are placed on same plane, if it is known that directions of these forces creating with positive sense of axis X such angles, respectively: α_1=15°, α_2=60°, α_3=300°.

Solution

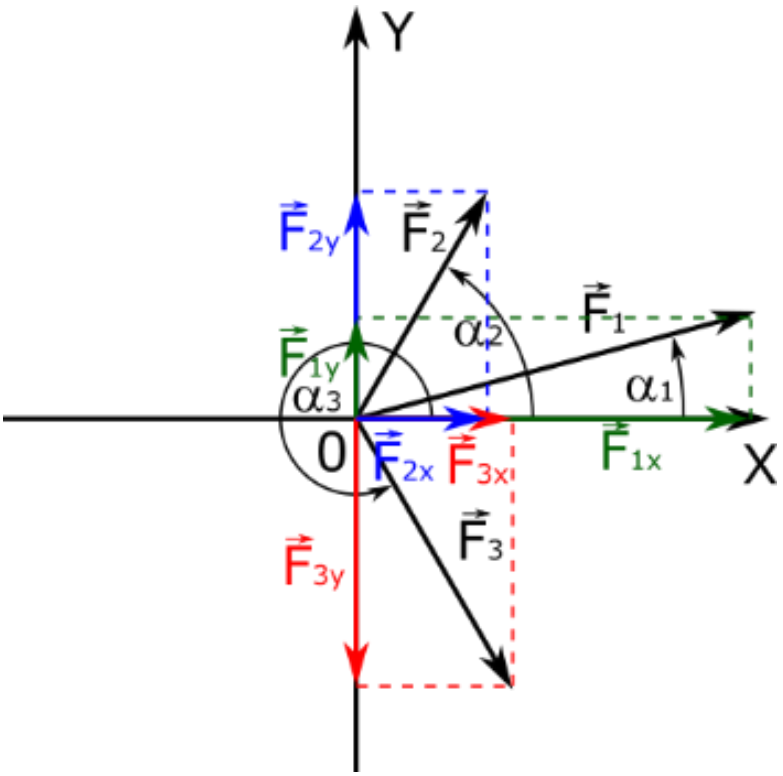
1. First of all you need to draw a coordinate system. (In this case it will be just planar system XY.)



2. We have an information about angles between direction of each of force and direction of axis X, so we can draw forces in the coordinate system. We need to remember to draw angles starting with axis X.



3. When forces are introduced we can start the solution. It could be solved in two ways a) analytically, b) graphically. Let start with the first one. In order to solve this example analytically we need to project our forces on the axises. Actually we need to split our forces to two forces Fx and Fy respectively.



4. When projections are done, we can add values for these projections respect to the axises, according to the formula:

$$F_x = F_{1x} + F_{2x} + F_{3x}$$
$$F_y = F_{1y} + F_{2y} + F_{3y}$$

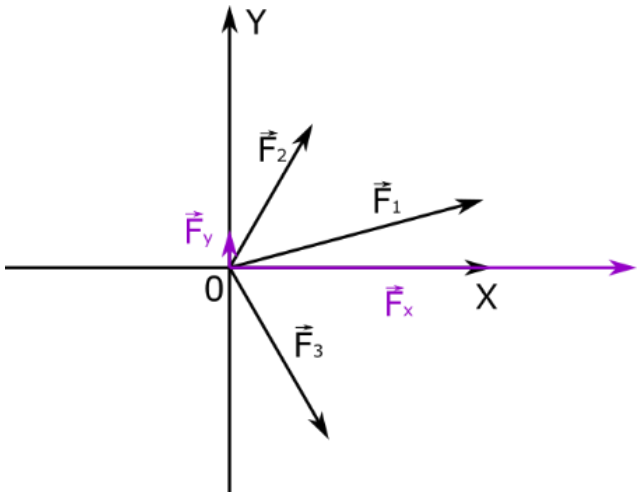
where:

$F_{1x} = F_1\cos(\alpha_1) = 77{,}3N$
$F_{2x} = F_2\cos(\alpha_2) = 25N$
$F_{3x} = F_3\cos(\alpha_3) = 30N$
$F_{1y} = F_1\sin(\alpha_1) = 20{,}7N$
$F_{2y} = F_2\sin(\alpha_2) = 43{,}3N$
$F_{3y} = F_3\sin(\alpha_3) = -51{,}96N$

then

$F_x = 77{,}3N+25N+30N = 132{,}3$
$F_y = 20{,}7N+43{,}3N-51{,}96N = 12{,}04N$

5. Because the resultant force need to be found, we need to find the value and the direction of this force. In order to do this we will use calculated values of F_x and F_y. Projections of resultant force can be now introduced to the coordinate system.



Based on this drawing we can see that in order to calculate value of the resultant force we can use Pythagoras theorem as it is shown below.

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

then

F = 132,85N

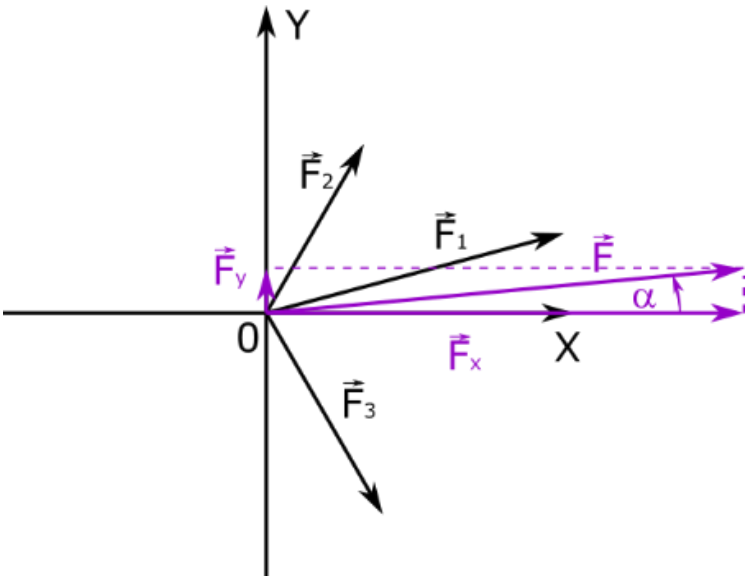
In addition in order to fully determine the position of the resultant force in the coordinate system we need to find the angle between its direction and direction of axis X. It can be done with this formula.

$$\tan \alpha = \frac{F_y}{F_x}$$

then

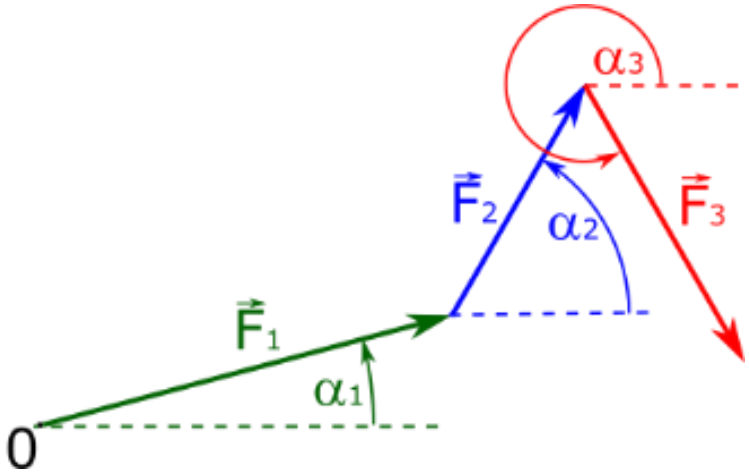
$\tan\alpha$ = 0,09

and $\alpha = \tan^{-1}(0{,}09) = 5^o8''$

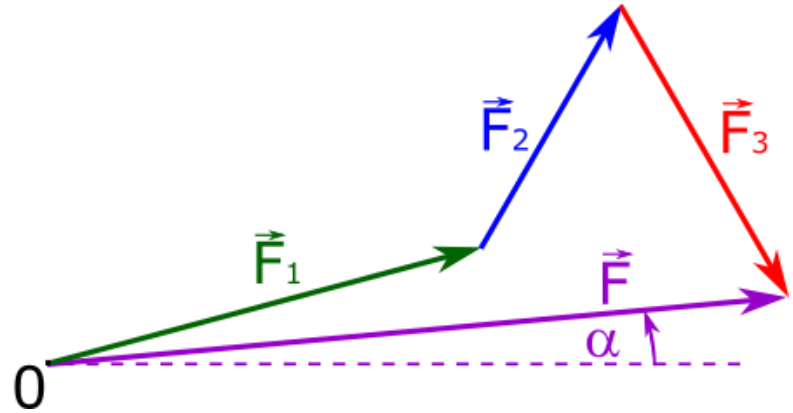


6. Graphical solution. We need to pick a beginning point, where first vector will be attached.

Then we are adding vectors one be one by attaching beginning of one vector with the end of another one. Of course we need to remember about the angles.



Then we can draw the resultant force.



If everything is correctly and precisely draw we can read the value of the force from the drawing however, usually it is hard to draw it in this way. That is why it need to be calculated in analytical way.