Convergent system of forces.

Ex2. The rope is going through the points ABCD. It is attached to the wall at point A and going over the roll C. At the end of the rope at point D, a weight Q = 10kN is hanging. Additionally at point B weight P is hanging also. Determine the force in the rope between points A and B and value of the weight P, if in equilibrium state angles $\alpha$ and $\beta$ are equal $\alpha$ = 45° and $\beta$ = 60° and there is no friction.
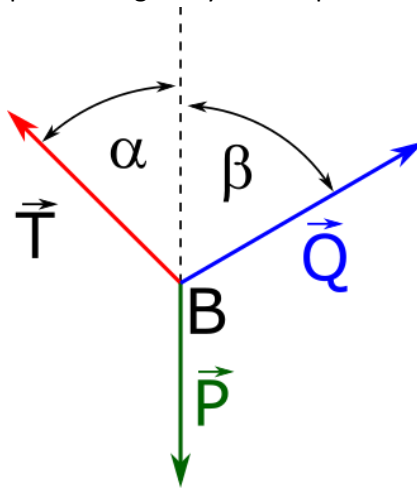
Solution

1. First of all you need to make a drawing of the situation (usually it is already given).
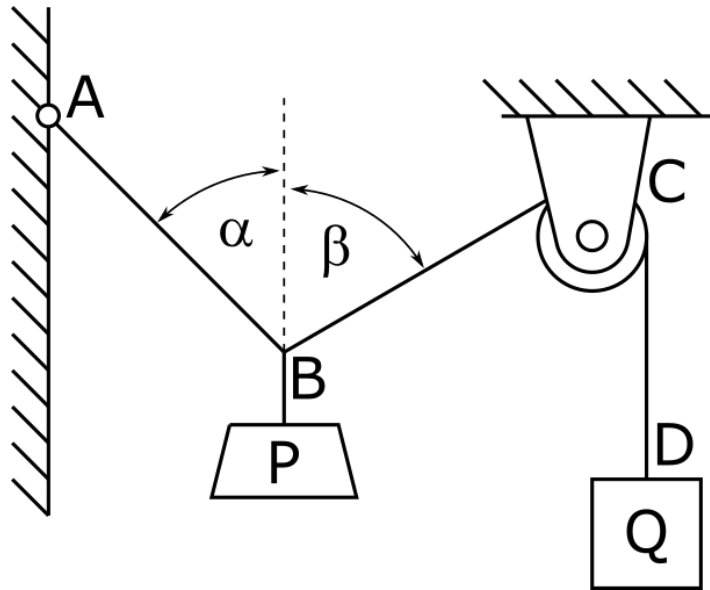


2. Next we can introduce forces that are acting in the system. It is clearly seen that between points A and B must be some tension (force). What is more this force T must acting exactly along direction of AB, what is shown in the picture.
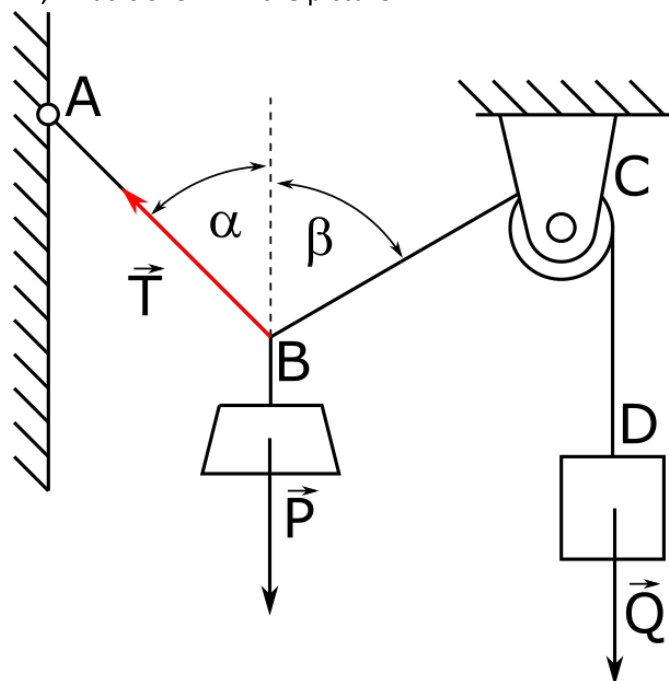
The sense of force T was determined based on the intuition and drawing itself. One can noticed that all forces except of force T are acting downwards, so force T must acting upwards, along its direction. What is more it one can found that force Q are acting not only at point D but actually, at the whole distance from point B through point C up to point D. This is why we can reduce this system to the simple convergent system at point B. As it is shown below.
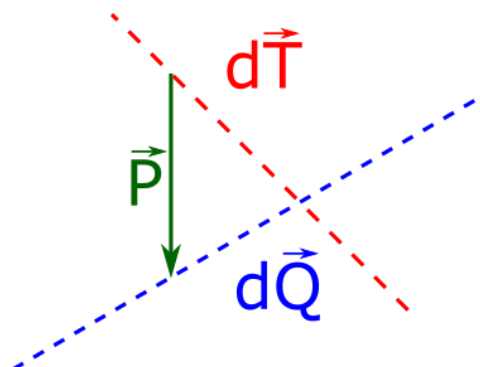


3. Now this system can be solved in two ways analytically and graphically. We will start with graphical solution.

4. First the chain of forces need to be drawn. Assume that the sense of force T is unknown. We will start with force P
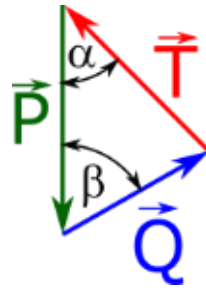


now we will add direction of force Q
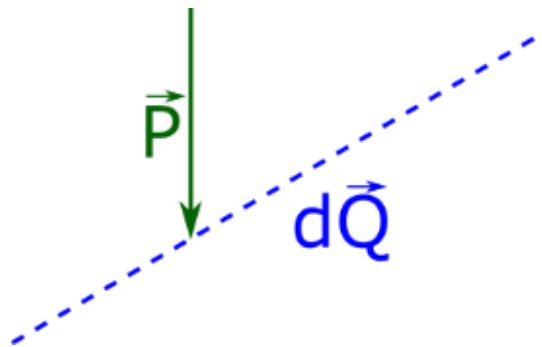


next we need to add direction of force T

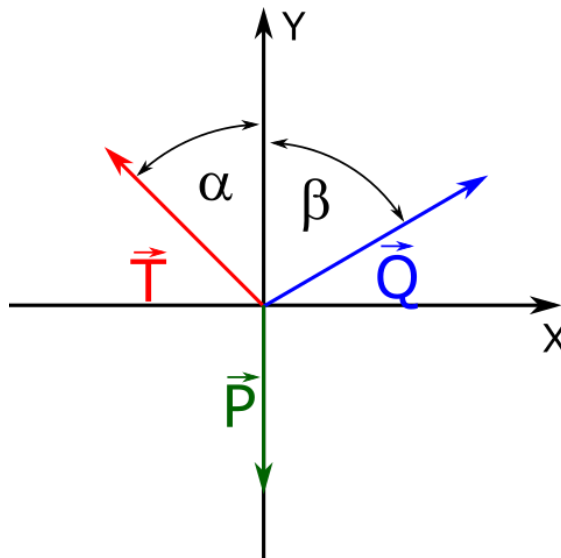Finally we can draw all forces that are acting in this system.



Now with the use of sines theorem all values might be calculated

$$\frac{T}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{P}{\sin(180° - (\alpha + \beta))}$$

$$T = Q\frac{\sin\beta}{\sin\alpha} = 12{,}25kN$$

$$P = Q\frac{\sin(180° - (\alpha + \beta))}{\sin\alpha} = 13{,}66\ kN$$

5. In analytical method we need to back to the basic convergent system drawing and introduce there coordinate system.



Now we can write equation of equilibrium for planar convergent system. It is known that for planar convergent system we can write two equilibrium equations. In this system there are two unknowns so it is possible to solve it.

$\Sigma F_{ix} = 0 = -T \sin\alpha + Q \sin\beta$

$\Sigma F_{iy} = 0 = -P + T \cos\alpha + Q \cos\beta$

From both above equations we will get

$$T = Q\frac{\sin\beta}{\sin\alpha} = 12{,}25kN$$
$$P = T \cos\alpha + Q \cos\beta = 13{,}66kN$$