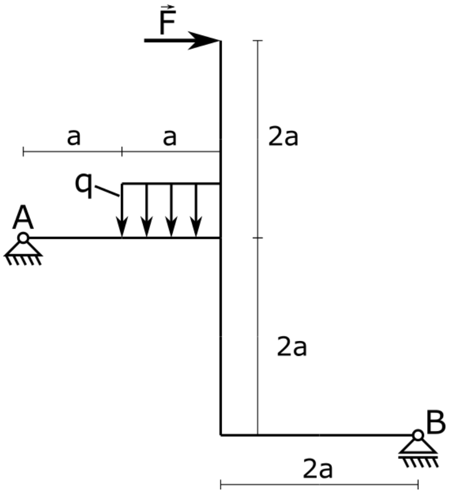
Frames II

Why are the frames related to the topic of beams? Frames are really a system of connected beams forming a rigid structure. This topic is omitted as a separate one because of the difficulties that may arise when solving the framework. Based on the above information, **to be able to solve the frame you must first be able to solve the beams.**

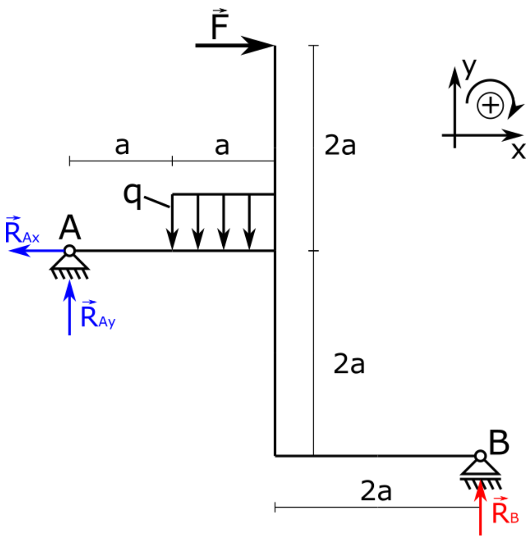
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam with all types of loads. An important information, the beam solution also includes drawing internal force diagrams.

Ex.2

For the frame shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a, F=4qa[kN], q[kN/m].



1. In this case, we have frame with one stable support and one movable support but we also have a place where the frame parts cross. As for beams, we must first identify support unknowns. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, clockwise turning moments will be positive.



2. Now we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating clockwise will be with positive sign.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

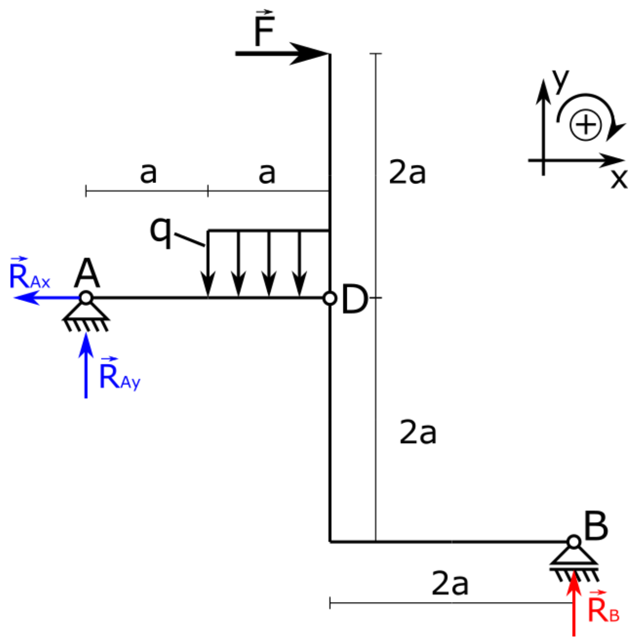
$$\sum_{i=1}^{n} F_{xi} = 0 = -R_{Ax} + F \rightarrow R_{Ax} = F = 4qa[kN]$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - q * a + R_B \rightarrow R_{Ay} = q * a - R_B = qa - \frac{19}{8}qa = -\frac{11}{8}qa[kN]$$

$$\sum_{i=1}^{n} M_A = 0 = q * a * 1{,}5a + F * 2a - R_B * 4a \rightarrow R_B = \frac{F * 2a + q * 1{,}5a^2}{4a} =$$

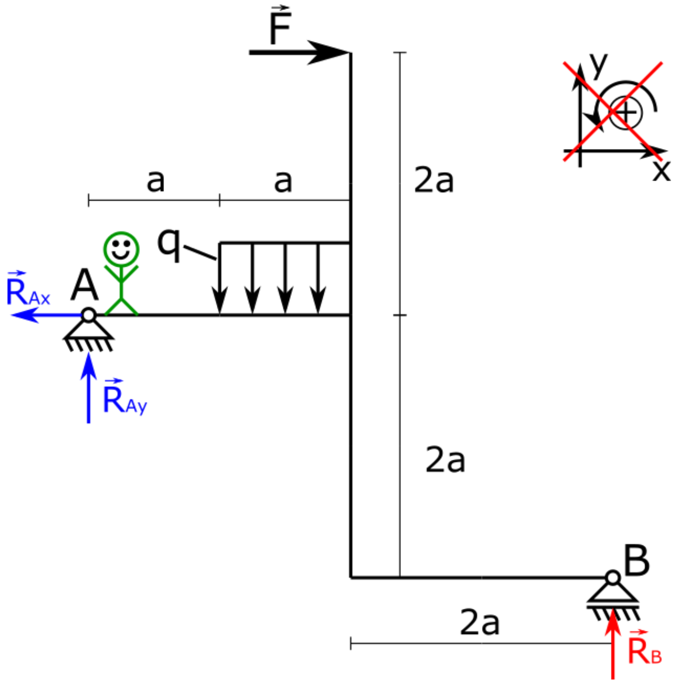
$$= \frac{4qa * 2a + 1{,}5qa^2}{4a} = \frac{19}{8}qa[kN]$$

3. **Important information**. Here it will be shown how one can check if the values of unknown reactions have been counted well. To do this, select a point on the system through which the directions of previously found reactions do not. After choosing a point - in this case it will be a point marked as D - you should count the sum of moments relative to this point by inserting the previously calculated reaction values. After conversion at the end we should get zero. If the value is different, it means that an error has been made somewhere and the reaction should be recalculated as well as the test equation.

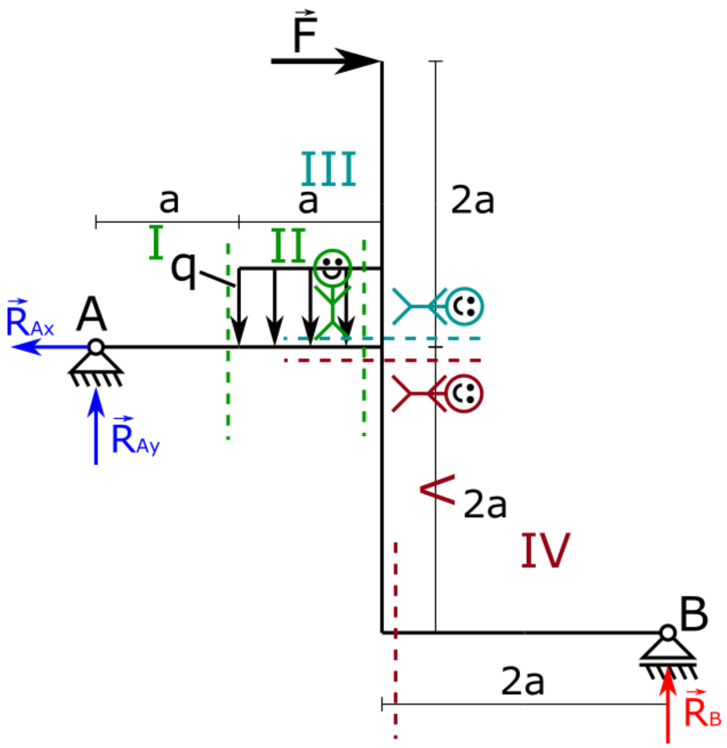


$$\sum_{i=1}^{n} M_D = -R_B * 2a + R_{Ay} * 2a + F * 2a - qa * 0{,}5a = -\frac{19}{4}qa^2 - \frac{11}{4}qa^2 + 8qa^2 - \frac{2}{4}qa^2 = 0$$

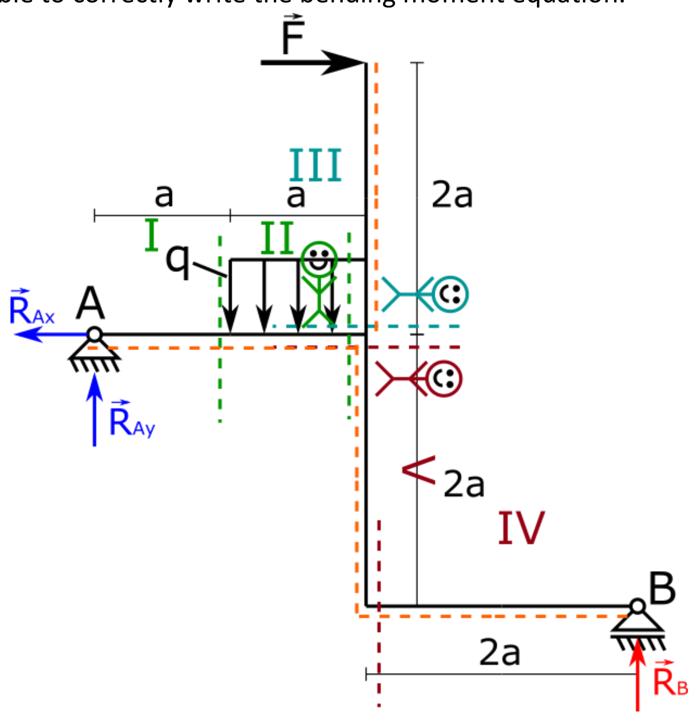
4. After determining the reaction in the supports, in the next step we need to determine how many cuts the frame needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces as it was for the beams. In the case of this frame you will notice that we will enter it from all sides. Thanks to this approach, it will be easier to write equations at a later stage and there will be less chance of making a mistake.



5. Cuts will be made on the frame in places where something happens on the frame (something appears or disappears). The important information is that we always cut before something on the frame has happened.
We see that when we enter the frame from its left, forces R_{Ax} and R_{Ay} appears. However, we cannot make the cut before these forces, because then we are not yet on the frame. We go further along the frame and come across distributed force q. Something happened. So we know that in this place just before the appearance of distributed force q should be first cut. Going further we reach the place where frame cross with other parts of it. Something happened. So we know that in this place just before intersection should be second cut. At this point, we end the journey through this part of the frame and enter the frame from top. We see that when we enter the frame from its top, force F appears. However, we cannot make the cut before this force, because then we are not yet on the frame. Going further we reach the place where frame cross with other parts of it. Something happened. So we know that in this place just before intersection should be third cut. Finally, we will enter the frame on its right side. We see that when we enter the frame from its right, force R_B appears. However, we cannot make the cut before this force, because then we are not yet on the frame. Going further we reach the place where frame turn (changes its direction). Something happened. So we know that in this place just before change in direction should be fourth cut. After changing the direction, we going further to the next place where the frame cross with other parts of it. Something happened. So we know that in this place just before intersection should be the final cut, the fifth cut.



6. The last thing to do before starting to write equations in each of the intervals is to **mark where on each part of the frame we will take its bottom**. In this case it has been marked with an orange dashed line. This step is very important because it allows us to tell if a given part of the frame is smiling or has a sad face as it was in the case of beams. This information is needed to be able to correctly write the bending moment equation.



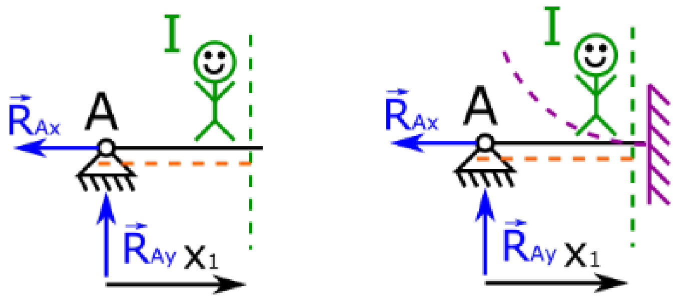
7. In this way, we divided the beam into five sections. The first section within $0 \leq x < a$, second section within $a \leq x < 2a$, third section within $0 \leq \mathrm{x} < 2\mathrm{a}$, fourth section within $0 \leq x < 2a$, fifth section within $0 \leq x < 2a$.

BENDING MOMENTS, CUTTING FORCES, NORMAL FORCES

8. At this point, we can move on to determining internal forces. To do this we have to go through each of the sections. In this example, we will determine equations for all internal forces section by section. In addition, information about the bending moment will appear at each of the section.

   I section within $0 \le x < a$.

   The bending moment will be from the R_{Ay} force and will look like in the pictures (dashed purple line). Cutting force from R_{Ay}. Normal force R_{Ax}.

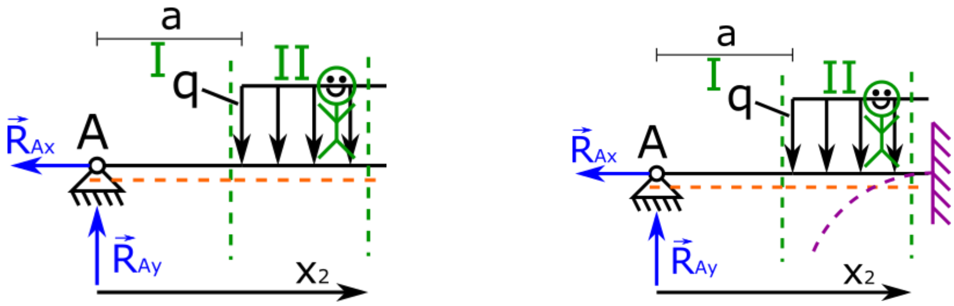


   Equations for all types of internal forces

$$M(x_1) = R_{Ay} * x_1$$
$$T(x_1) = R_{Ay}$$
$$N(x_1) = R_{Ax}$$

9. II section within $a \le x < 2a$.

   The bending moment will be from the R_{Ay} force and distributed force a. The bending from R_{Ay} will be the same as it was in previous section. Bending moment from distributed force q will look like in the picture (dashed purple line). Cutting forces from R_{Ay} and distributed force q. Normal force R_{Ax}.



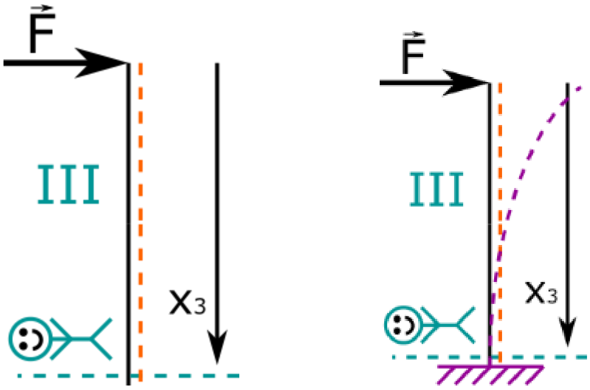
   Equations for all types of internal forces

$$M(x_2) = R_{Ay} * x_2 - q * (x_2 - a) * \frac{(x_2 - a)}{2}$$
$$T(x_2) = R_{Ay} - q * (x_2 - a)$$
$$N(x_2) = R_{Ax}$$

10. III section within $0 \leq x < 2a$ from right side **it is important to mark that we are calculating sections from the right side.**

In this case, we enter the frame from top. However, in reality, if we consider individual parts of the frame as beams and turn the head, we see that it looks as if we were entering a normal beam from its right side.

The bending moment will be from force F and will look like in the pictures (dashed purple line). Cutting forces from F. There will be no normal forces.

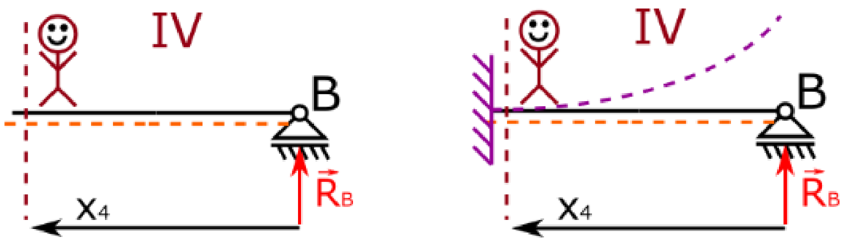


Equations for all types of internal forces

$$M(x_3) = -F * x_3$$
$$T(x_3) = F$$
$$N(x_3) = 0$$

11. IV section within $0 \leq x < 2a$ from right side **it is important to mark that we are calculating sections from the right side.**

The bending moment will be only from the R_B force and will look like in the picture (dashed purple line). Cutting force only from R_B. There will be no normal forces.

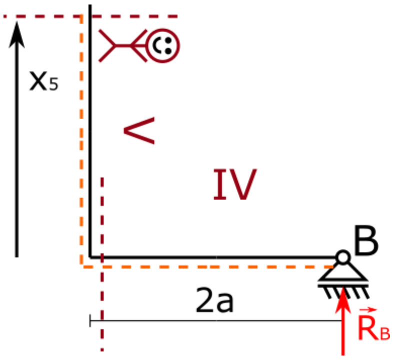


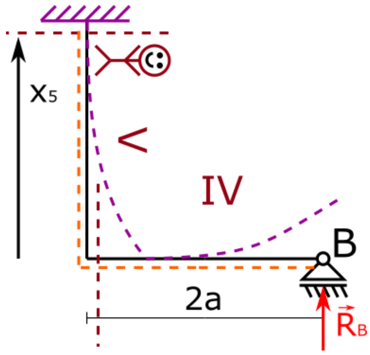
Equations for all types of internal forces

$$M(x_4) = R_B * x_4$$
$$T(x_4) = -R_B$$
$$N(x_4) = 0$$

12. V section within $0 \le x < 2a$ from right side **it is important to mark that we are calculating sections from the right side.**

The bending moment will be from the R_B force. The bending from R_B will look like in the picture (dashed purple line). There will be no cutting forces. Normal force $-R_B$.

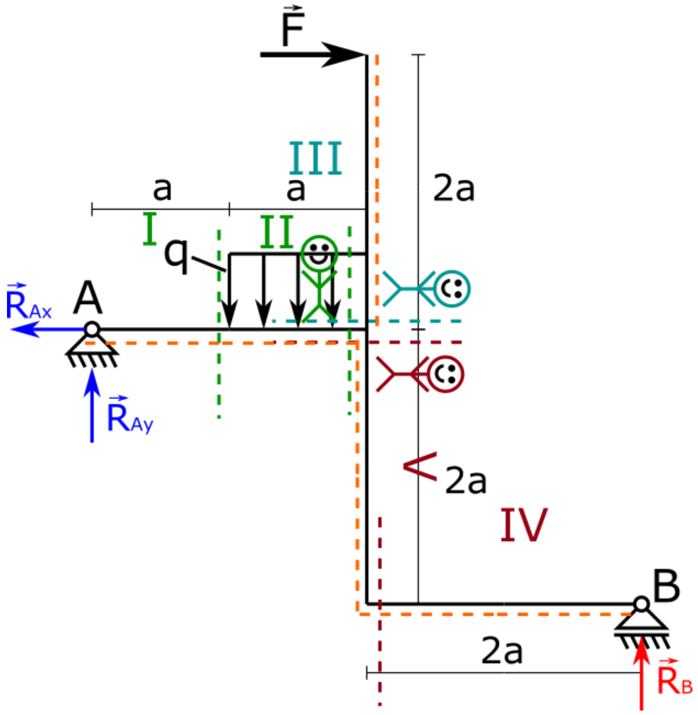




Equations for all types of internal forces

$$M(x_5) = R_B * 2a$$
$$T(x_2) = 0$$
$$N(x_2) = -R_B$$

13. Let's write equations for all sections in one place to clarify everything.



I Section $0 \leq x_1 < a$

$$M(x_1) = R_{Ay} * x_1$$
$$T(x_1) = R_{Ay}$$
$$N(x_1) = R_{Ax}$$

II Section $a \leq x_1 < 2a$

$$M(x_2) = R_{Ay} * x_2 - q * (x_2 - a) * \frac{(x_2 - a)}{2}$$
$$T(x_2) = R_{Ay} - q * (x_2 - a)$$
$$N(x_2) = R_{Ax}$$

III Section $0 \leq x_1 < 2a$ right side

$$M(x_3) = -F * x_3$$
$$T(x_3) = F$$
$$N(x_3) = 0$$

IV Section $0 \leq x_1 < 2a$ right side

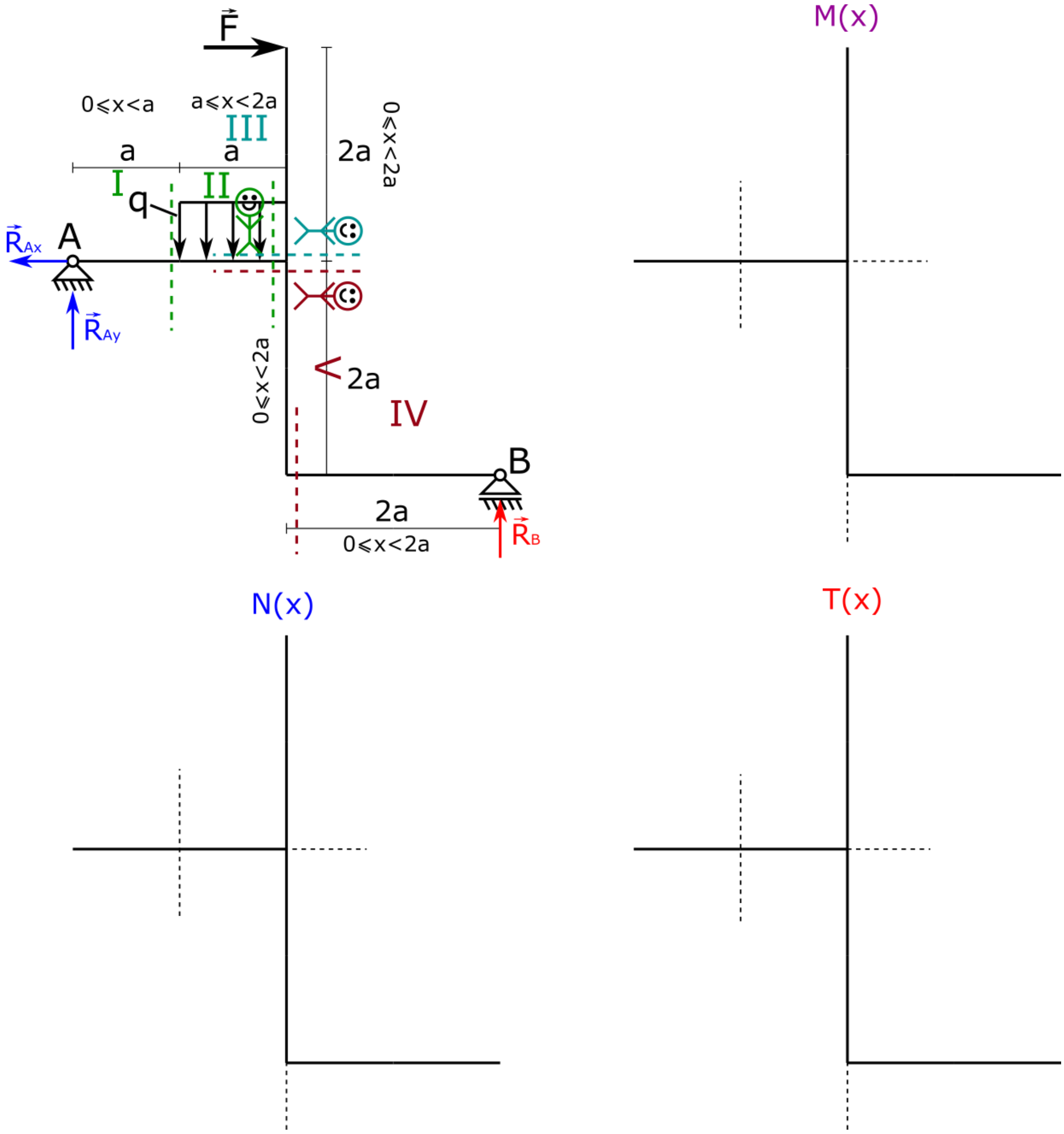
$$M(x_4) = R_B * x_4$$
$$T(x_4) = -R_B$$
$$N(x_4) = 0$$

V Section $a \leq x_1 < 2a$ right side

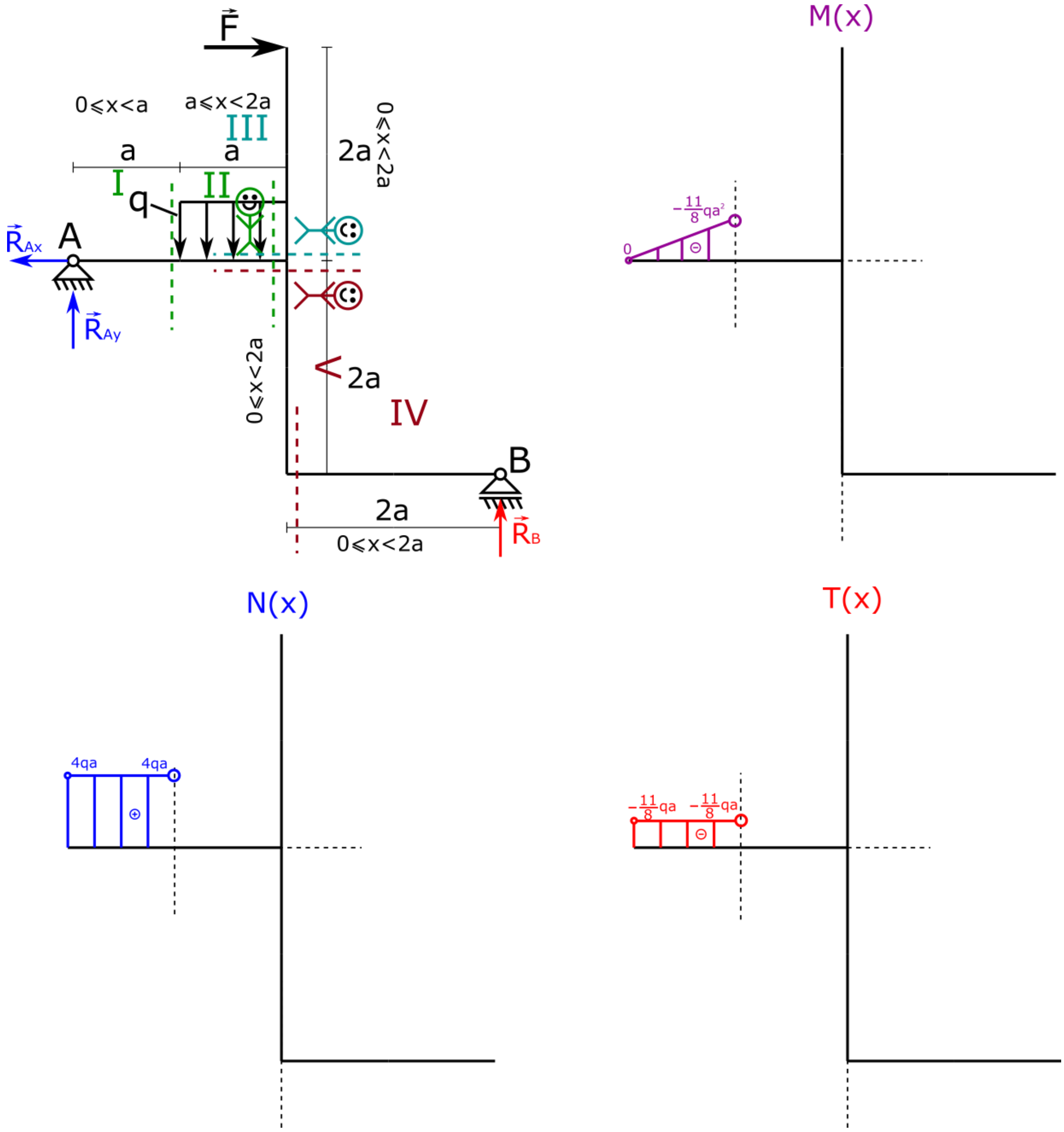
$$M(x_5) = R_B * 2a$$
$$T(x_2) = 0$$
$$N(x_2) = -R_B$$

CHARTS

14. The last part related to solving frames – charts. To easily draw charts, it is best to draw them next to the frame as a frame with the same shape as given frame, as shown in the figure.



15. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the first section.

$$M(x_1) = R_{Ay} * x_1$$
$$T(x_1) = R_{Ay}$$
$$N(x_1) = R_{Ax}$$

We know the limits of the first section.

$$0 \leq x < a$$

We substitute the boundary values into our equations (for x_1).

$$M(0) = R_{Ay} * 0 = 0$$

$$M(a) = R_{Ay} * a = -\frac{11}{8}qa * a = -\frac{11}{8}qa^2[kNm]$$

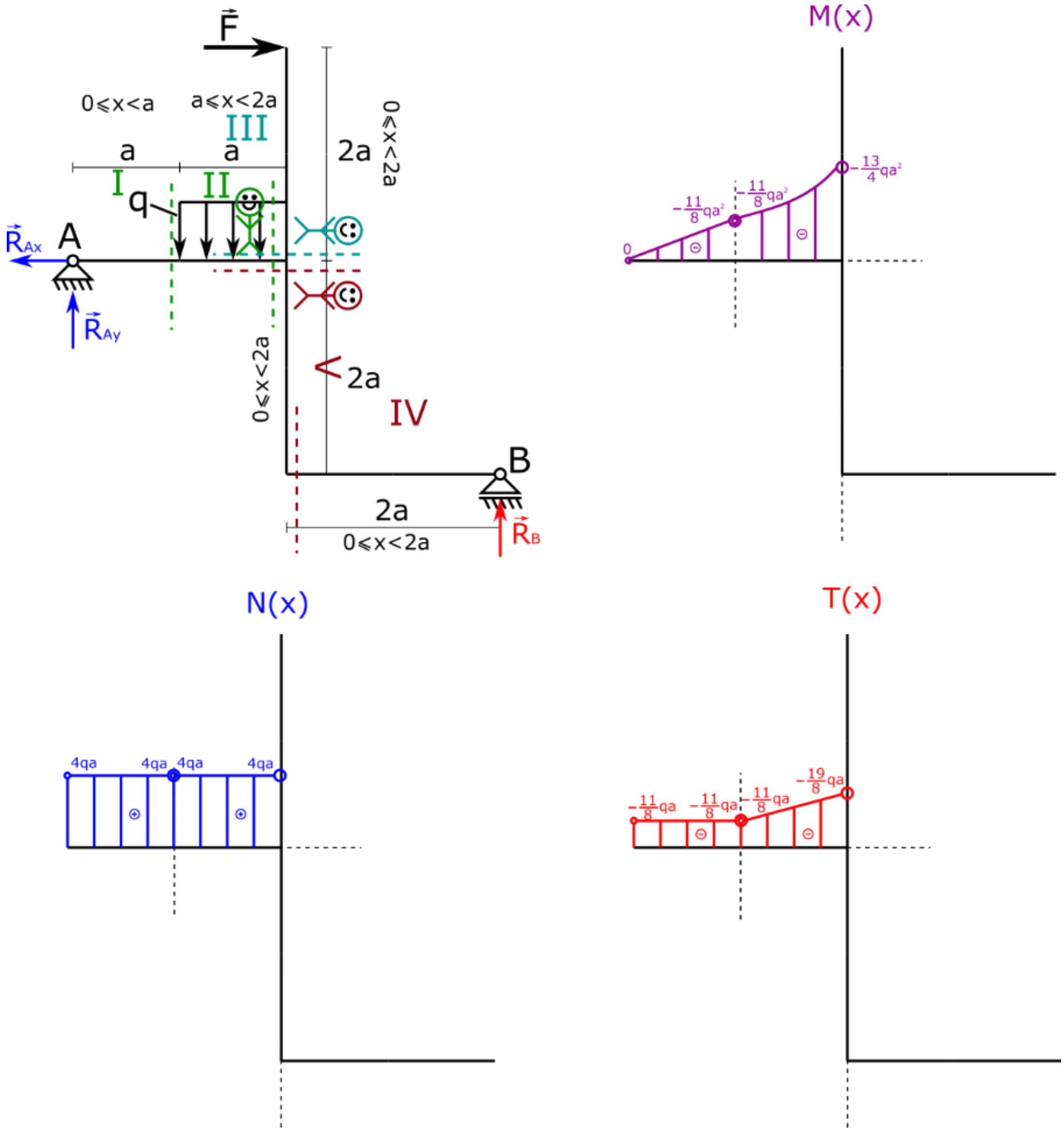
$$T(0) = R_{Ay} = -\frac{11}{8}qa[kN]$$

$$T(a) = R_{Ay} = -\frac{11}{8}qa[kN]$$

$$N(0) = R_{Ax} = 4qa$$

$$N(a) = R_{Ax} = 4qa$$

16. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the second section.

$$M(x_2) = R_{Ay} * x_2 - q * (x_2 - a) * \frac{(x_2 - a)}{2}$$
$$T(x_2) = R_{Ay} - q * (x_2 - a)$$
$$N(x_2) = R_{Ax}$$

We know the limits of the second section.

$$a \leq x < 2a$$

We substitute the boundary values into our equations (for x_2).

$$M(a) = R_{Ay} * a - q * (a - a) * \frac{(a - a)}{2} = -\frac{11}{8} qa^2 [kNm]$$

$$M(2a) = R_{Ay} * 2a - q * (2a - a) * \frac{(2a - a)}{2} = -\frac{13}{4} qa^2 [kNm]$$

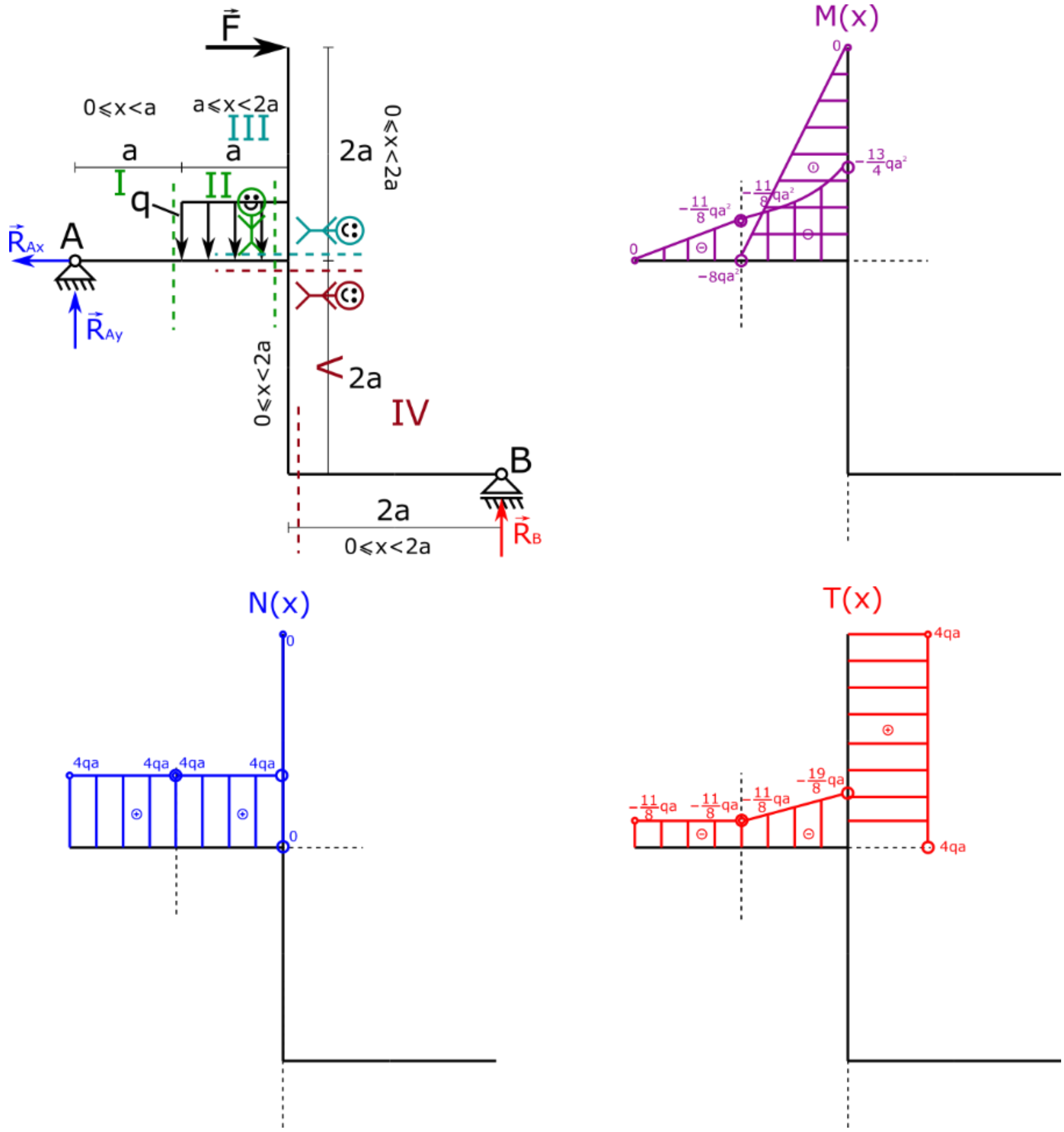
$$T(a) = R_{Ay} - q * (a - a) = -\frac{11}{8}qa[kN]$$

$$T(2a) = R_{Ay} - q * (2a - a) = -\frac{19}{8}qa[kN]$$

$$N(a) = R_{Ax} = 4qa$$

$$N(2a) = R_{Ax} = 4qa$$

17. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the third section.

$$M(x_3) = -F * x_3$$

$$T(x_3) = F$$

$$N(x_3) = 0$$

We know the limits of the third section.

$$0 \leq x < 2a$$

We substitute the boundary values into our equations (for x_3).

$$M(0) = -F * 0 = 0[kNm]$$

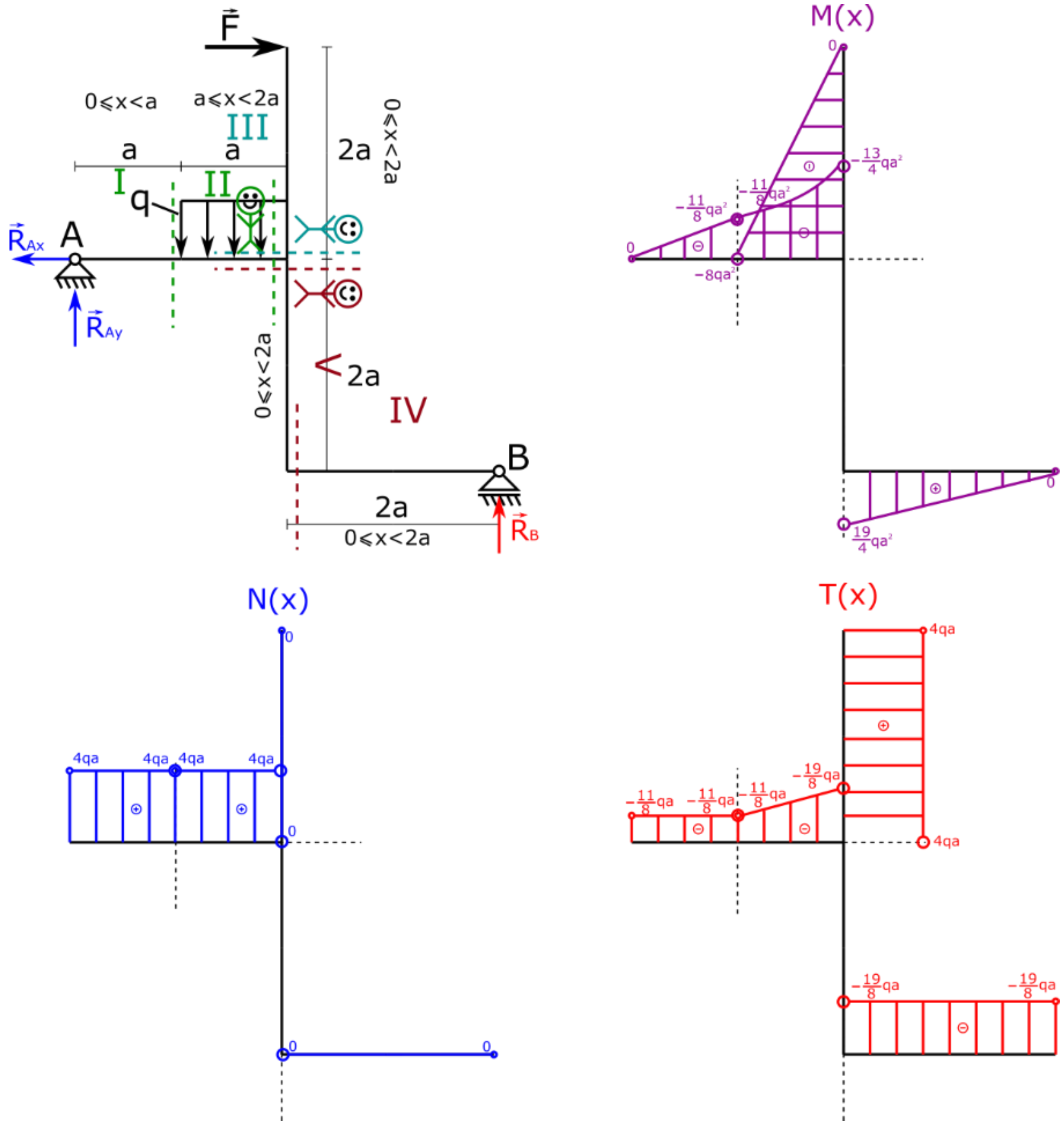
$$M(2a) = -F * 2a = -8qa^2[kNm]$$

$$T(0) = F = 4qa[kN]$$

$$T(2a) = F = 4qa[kN]$$

$$N(0) = N(2a) = 0[kN]$$

18. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the fourth section.

$$M(x_4) = R_B * x_4$$
$$T(x_4) = -R_B$$
$$N(x_4) = 0$$

We know the limits of the fourth section.

$$0 \le x < 2a$$

We substitute the boundary values into our equations (for x_4).

$$M(0) = R_B * 0 = 0$$

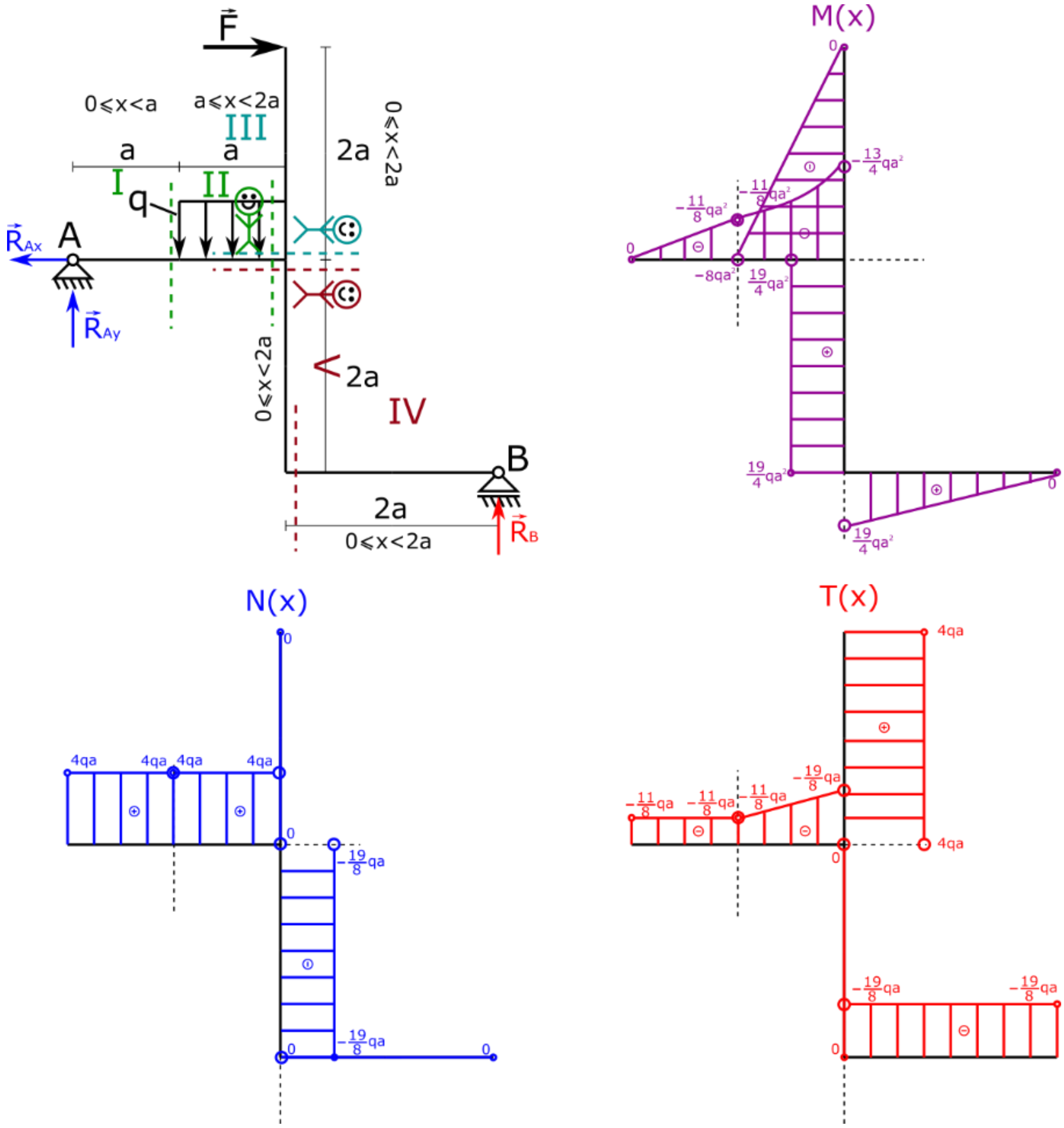
$$M(2a) = R_B * a = \frac{19}{8} qa^2 [kNm]$$

$$T(0) = -R_B = -\frac{19}{8} qa [kN]$$

$$T(2a) = -R_B = -\frac{19}{8}qa[kN]$$

$$N(0) = N(2a) = 0[kN]$$

19. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the fifth section.

$$M(x_5) = R_B * 2a$$
$$T(x_2) = 0$$
$$N(x_2) = -R_B$$

We know the limits of the fifth section.

$$0 \leq x < 2a$$

We substitute the boundary values into our equations (for x_5).

$$M(0) = R_B * 2a = \frac{19}{4} qa^2 [kNm]$$

$$M(2a) = R_B * 2a = \frac{19}{4} qa^2 [kNm]$$

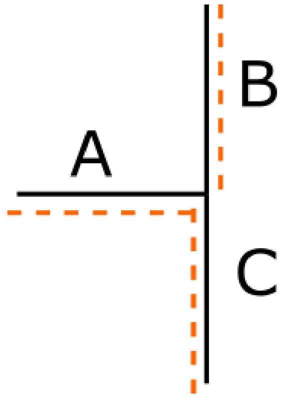
$$T(0) = 0[kN]$$

$$T(2a) = 0[kN]$$

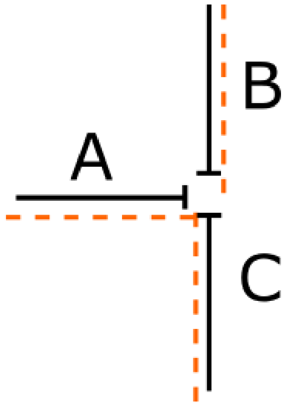
$$N(a) = -R_B = -\frac{19}{8}qa[kN]$$

$$N(2a) = -R_B = -\frac{19}{8}qa[kN]$$

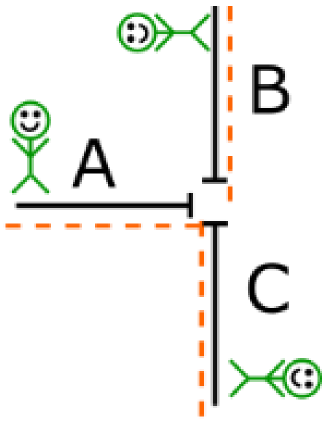
20. **Important information**. Below it will be shown how to check if the values of bending moments obtained on individual parts of the frame intersection are correct.
21. We will focus on the very place where the three parts of the frame meet. The method is also suitable for larger quantities. Let's call our parts A, B and C, respectively. It is important to mark as at the beginning, where the bottom for individual parts was adopted (measuring dashed line).



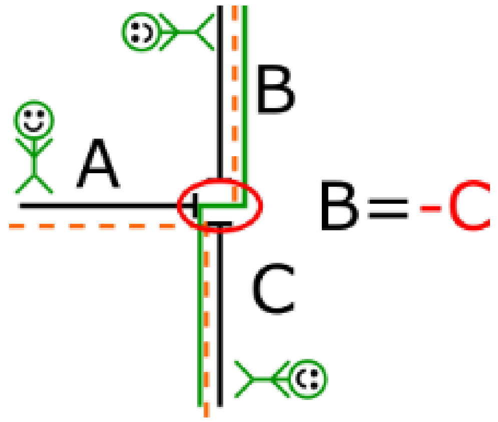
22. Now, in a place where individual parts come to the intersection, we make a cut.



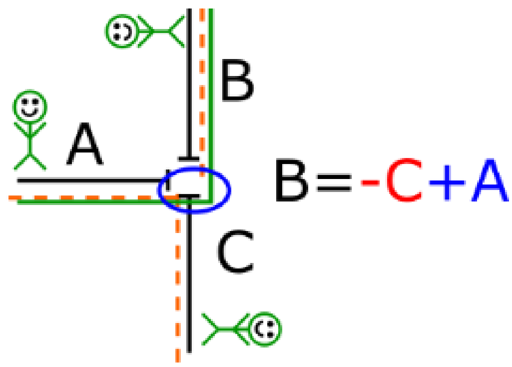
23. Then we go to each of the beams at its top (the man symbolizes the top of the individual beams - part of the frame).



24. In the next step, we wonder which part of the frame we want to check the correctness of the charts. We will assume that our initial part will be part B. So we want to check if the moments from the other parts of the frame will allow us to obtain the value we achieved for part B at the intersection. Let's start with part C. From part C we want to go to part B, in such a way that the upper part of beam C and beam B are on the same side (as the green line leads). We can see that to do this I will have to go to the other side of beam C, as shown in the figure (marked in red). This means that in the equation we have to change the sign of our bending moment. You can see it in the equation.



25. Then go to part A and perform a similar operation. We want to go to beam B so that we are on the upper part (green line). It is clear that we do not cut the beam A or beam B in any way (marked in blue). Therefore, the sign at the moment bending from beam A will remain as it was.



26. Finally, in the resulting equation, we insert the bending moments obtained earlier for each part of the frame in the place of the intersection and check that the left side is equal to the right. If the sides of the equation do not match, it means that in one of the earlier steps we made a mistake.