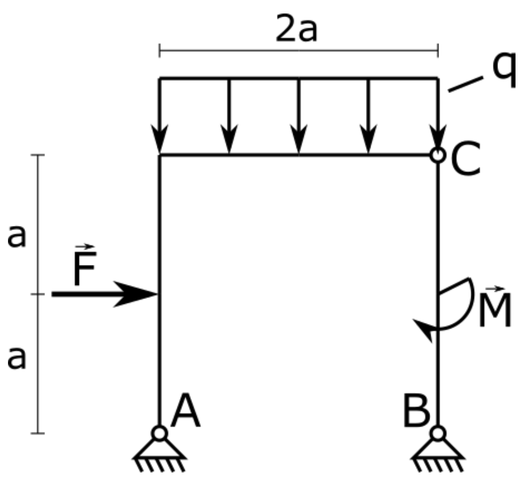
Frames

Why are the frames related to the topic of beams? Frames are really a system of connected beams forming a rigid structure. This topic is omitted as a separate one because of the difficulties that may arise when solving the framework. Based on the above information, **to be able to solve the frame you must first be able to solve the beams.**

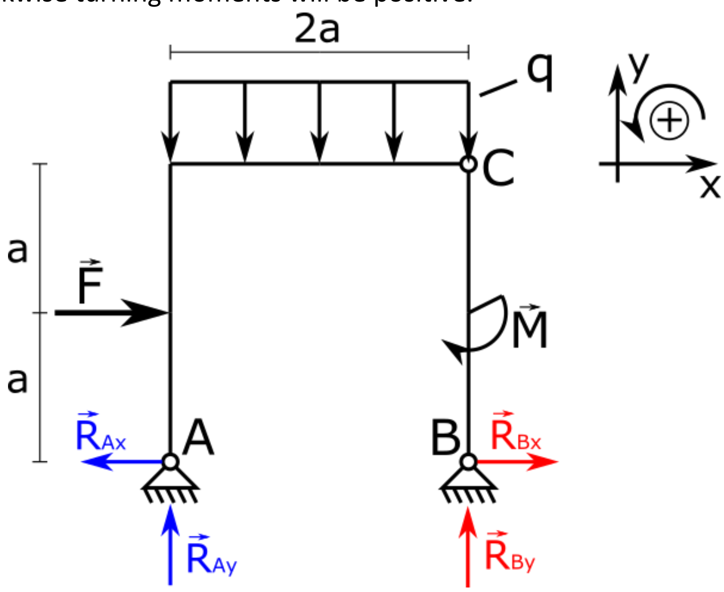
Since the theory is best understood by example, below I will introduce a step-by-step solution to a simple beam with all types of loads. An important information, the beam solution also includes drawing internal force diagrams.

Ex.1

For the frame shown in the drawing determine: reactions in supports, bending moments, cutting forces and normal forces. Draw the graphs of these forces. Data: a, F=qa[kN], M=qa^2[kNm], q[kN/m].



1. In this case, we have frame with two stable supports and on the frame at point C there is a joint. As for beams, we must first identify support unknowns. In addition, you must specify how we will adopt the coordinate system in accordance with which we will determine the values of supporting unknowns and in which direction the moments will have positive values. In this case, clockwise turning moments will be positive.



2. It is clear that in this case we have four supporting unknowns, and we can only write three equations of equilibrium, as below.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

3. In order to find all unknowns, we will do the same as in the case of beams with joints. In this case, I will use the second method, i.e. I will also use moments from one of the sides of our joint. To make it easier to count, we'll use the right side.

$$\sum_{i=1}^{n} F_{xi} = -R_{Ax} + F + R_{Bx} \rightarrow R_{Ax} = F + R_{Bx} = qa + \frac{qa}{2} = 1.5qa[kN]$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - q * 2a + R_{By} \rightarrow R_{Ay} = 2qa - R_{By} = 2qa - 2qa = 0[kN]$$

$$\sum_{i=1}^{n} M_A = 0 = -Fa - 2qa * a - M + R_{By} * 2a \rightarrow R_{By} = \frac{qa^2 + 2qa^2 + qa^2}{2a} = \frac{4qa^2}{2a} = 2qa[kN]$$

$$\sum_{i=1}^{n} M_C^R = 0 = R_{Bx} * 2a - M \rightarrow R_{Bx} = \frac{M}{2a} = \frac{qa}{2} = 0.5qa[kN]$$

Finally, we get the values of our unknown reactions as below

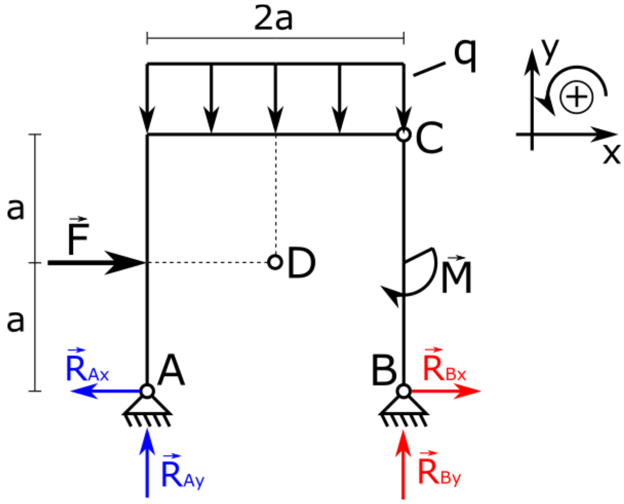
$$R_{Ax} = 1.5qa$$

$$R_{Ay} = 0$$

$$R_{Bx} = 0.5qa$$

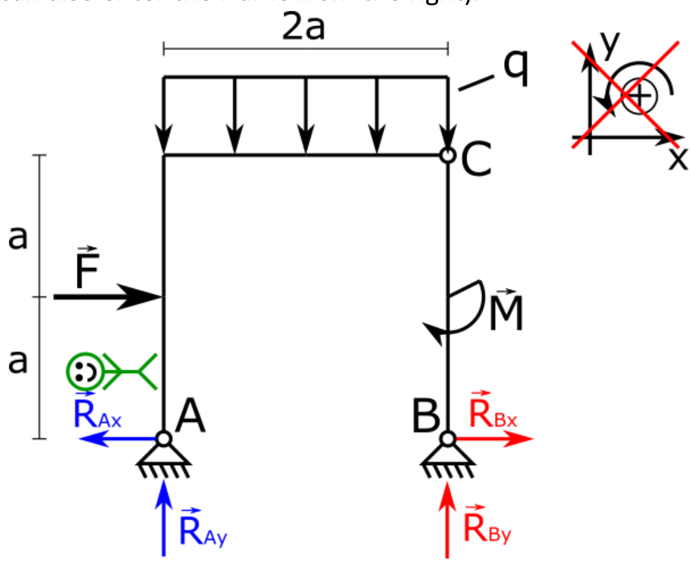
$$R_{By} = 2qa$$

4. **Important information**. Here it will be shown how one can check if the values of unknown reactions have been counted well. To do this, select a point on the system through which the directions of previously found reactions do not. After choosing a point - in this case it will be a point marked as D - you should count the sum of moments relative to this point by inserting the previously calculated reaction values. After conversion at the end we should get zero. If the value is different, it means that an error has been made somewhere and the reaction should be recalculated as well as the test equation.



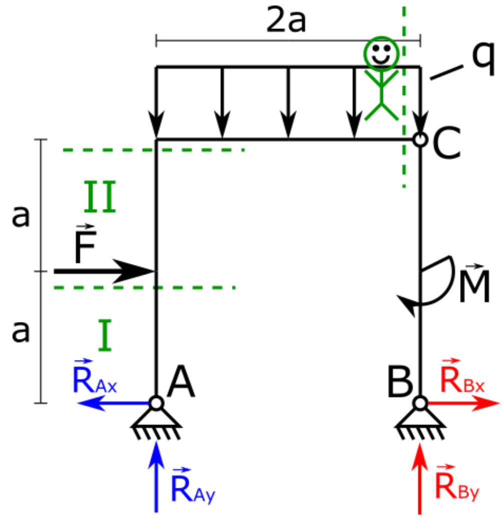
$$\sum_{i=1}^{n} M_D = -R_{Ay} * a - R_{Ax} * a + R_{By} * a + R_{Bx} * a - M$$
$$= -0 * a - 1.5qa * a + 2qa * a + 0.5qa * a - qa^2 = 0$$

5. After determining the reaction in the supports, in the next step we need to determine how many cuts the frame needs to be made to be able to solve it. First of all, we forget about the just introduced coordinate system and how the moment rotate with positive value. From here, we will use the notation for internal forces as it was for the beams. We enter the frame from its left (you can also enter the frame from the right).

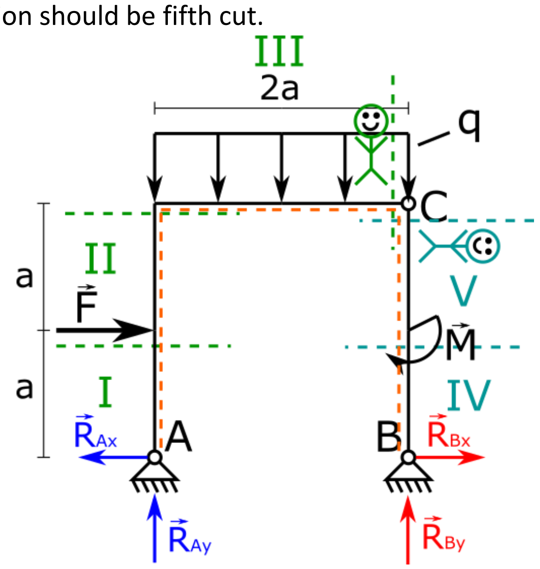


6. Cuts will be made on the frame in places where something happens on the frame (something appears or disappears). The important information is that we always cut before something on the frame has happened.

We see that when we enter the frame from its left, forces R_{Ax} and R_{Ay} appears. However, we cannot make the cut before these forces, because then we are not yet on the frame. We go further along the frame and come across force F. Something happened. So we know that in this place just before the appearance of force F should be first cut. Going further we reach the place where frame turn (changes its direction). Something happened. So we know that in this place just before change in direction should be second cut. After changing the direction, we enter a distributed force and go to the next place where the frame changes direction and ends with a distributed force. Let's place another cut here, just before changing direction.



7. We will enter the rest of the frame from the right. We see that when we enter the frame from its right, forces R_{Bx} and R_{By} appears. However, we cannot make the cut before these forces, because then we are not yet on the frame. We go further along the frame and come across moment M. Something happened. So we know that in this place just before the appearance of moment M should be fourth cut. Going further we reach the place where frame turn (changes its direction). Something happened. So we know that in this place just before change in direction should be fifth cut.



The last thing to do before starting to write equations in each of the intervals is to **mark where on each part of the frame we will take its bottom**. In this case it has been marked with an orange dashed line. This step is very important because it allows us to tell if a given part of the frame is smiling or has a sad face as it was in the case of beams. This information is needed to be able to correctly write the bending moment equation.

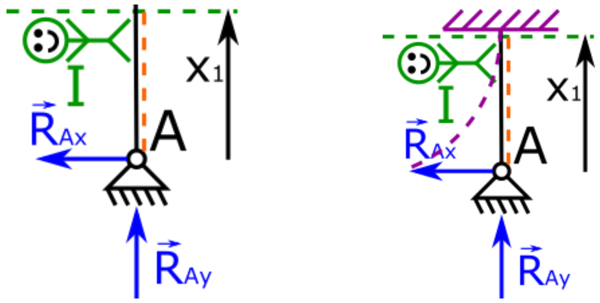
8. In this way, we divided the beam into five sections I, II and III from the left side and IV and V from the right side. The first section within $0 \leq x < a$, second section within $a \leq x < 2a$, **third section within $\mathbf{0} \leq \boldsymbol{x} < \mathbf{2}\boldsymbol{a}$**, fourth section within $0 \leq x < a$, fifth section within $a \leq x < 2a$.

BENDING MOMENTS, CUTTING FORCES, NORMAL FORCES

9. At this point, we can move on to determining internal forces. To do this we have to go through each of the sections. In this example, we will determine equations for all internal forces section by section. In addition, information about the bending moment will appear at each of the section.

   I section within $0 \leq x < a$.

   The bending moment will be from the R_{Ax} force and will look like in the pictures (dashed purple line). Cutting force from R_{Ax}. Normal force R_{Ay}.

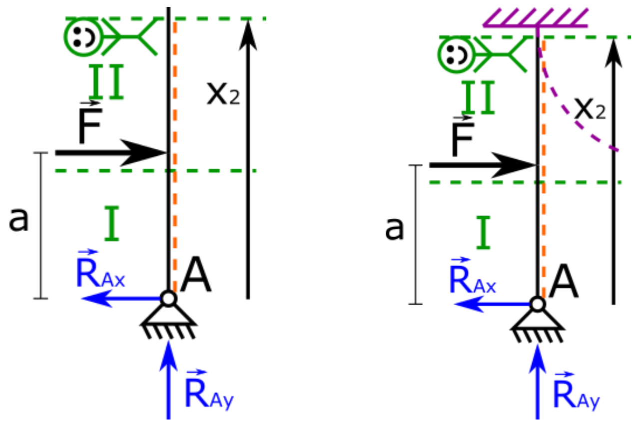


   Equations for all types of internal forces

$$M(x_1) = R_{Ax} * x_1$$
$$T(x_1) = R_{Ax}$$
$$N(x_1) = -R_{Ax}$$

10. II section within $a \leq x < 2a$.

    The bending moment will be from the R_{Ax} force and F force. The bending from R_{AX} will be the same as it was in previous section. Bending moment from F force will look like in the picture (dashed purple line). Cutting forces from R_{Ax} and F. Normal force R_{Ay}.

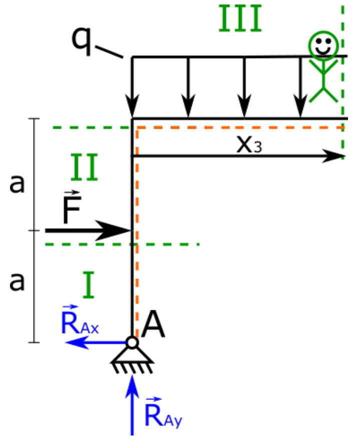


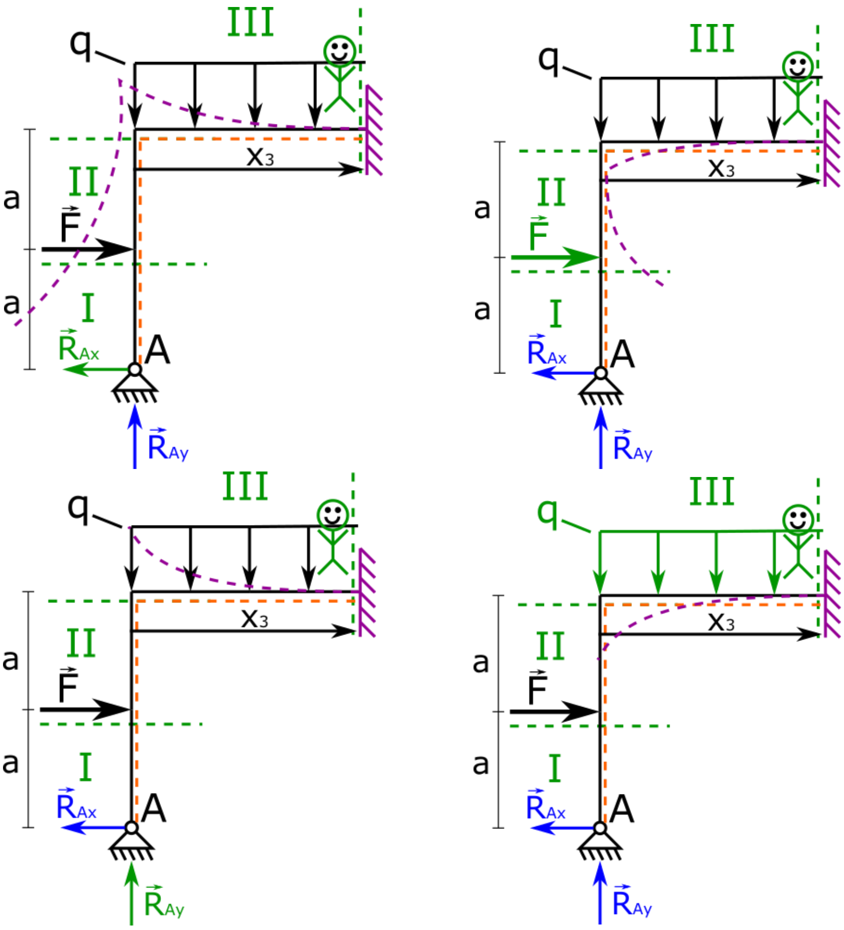
    Equations for all types of internal forces

$$M(x_2) = R_{Ax} * x_2 - F * (x_2 - a)$$
$$T(x_2) = R_{Ax} - F$$
$$N(x_2) = -R_{Ay}$$

11. III section within $0 \leq x < 2a$.

The bending moment will be from forces: R_{Ax}, F, R_{Ay} and from distributed force q. The bending moment from each of force will be shown separately (dashed purple line). Cutting forces from R_{Ay}, and distributed force q. Normal force R_{Ax} and force F.



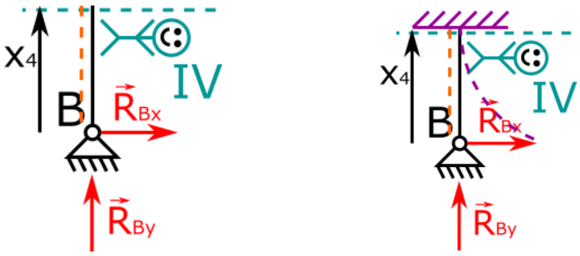


Equations for all types of internal forces

$$M(x_3) = R_{Ax} * 2a - F * a + R_{Ay} * x_3 - \frac{q * x_3^2}{2}$$
$$T(x_3) = R_{Ay} - q * x_3$$
$$N(x_3) = R_{Ax} - F$$

12. IV section within $0 \leq x < a$ from right side **it is important to mark that we are calculating sections from the right side.**

The bending moment will be only from the R_{Bx} force and will look like in the picture (dashed purple line). Cutting force only from R_{Bx}. Normal force R_{By}.

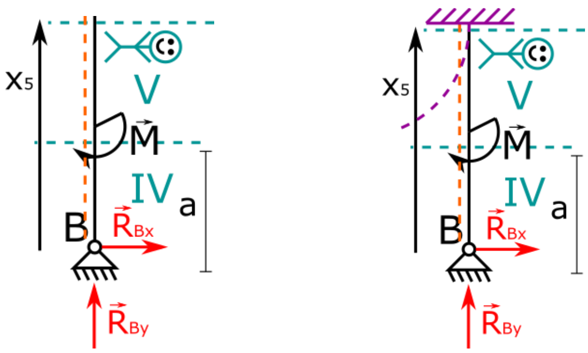


Equations for all types of internal forces

$$M(x_4) = R_{Bx} * x_4$$
$$T(x_4) = -R_{Bx}$$
$$N(x_4) = -R_{By}$$

13. V section within $a \leq x < 2a$ from right side **it is important to mark that we are calculating sections from the right side.**

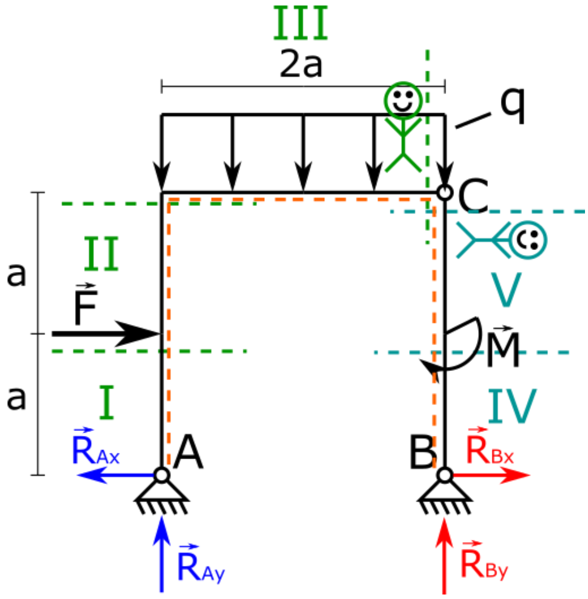
The bending moment will be from the R_{Bx} force and moment M. The bending from R_{Bx} will be the same as it was in previous section. Bending moment from moment M will look like in the picture (dashed purple line). Cutting forces from R_{Bx}. Normal force R_{By}.



Equations for all types of internal forces

$$M(x_5) = R_{Bx} * x_5 - M$$
$$T(x_2) = -R_{Bx}$$
$$N(x_2) = -R_{By}$$

14. Let's write equations for all sections in one place to clarify everything.



I Section $0 \le x_1 < a$

$$M(x_1) = R_{Ax} * x_1$$
$$T(x_1) = R_{Ax}$$
$$N(x_1) = -R_{Ax}$$

II Section $a \le x_1 < 2a$

$$M(x_2) = R_{Ax} * x_2 - F * (x_2 - a)$$
$$T(x_2) = R_{Ax} - F$$
$$N(x_2) = -R_{Ay}$$

III Section $0 \le x_1 < 2a$

$$M(x_3) = R_{Ax} * 2a - F * a + R_{Ay} * x_3 - \frac{q * x_3^2}{2}$$
$$T(x_3) = R_{Ay} - q * x_3$$
$$N(x_3) = R_{Ax} - F$$

IV Section $0 \le x_1 < a$ right side

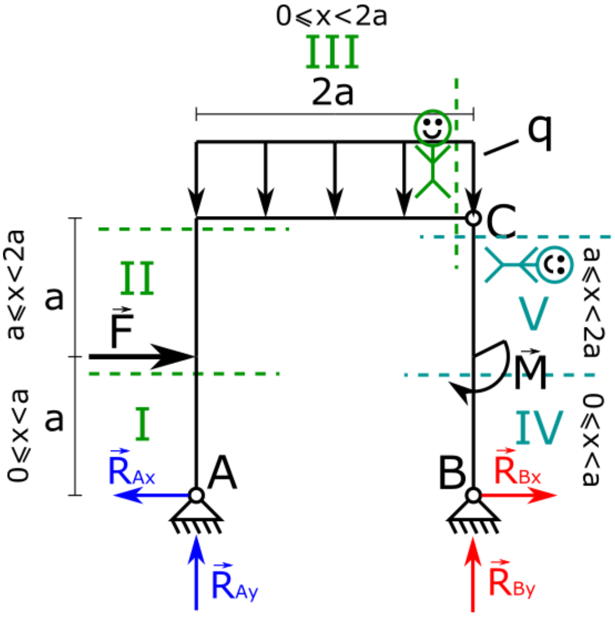
$$M(x_4) = R_{Bx} * x_4$$
$$T(x_4) = -R_{Bx}$$
$$N(x_4) = -R_{By}$$

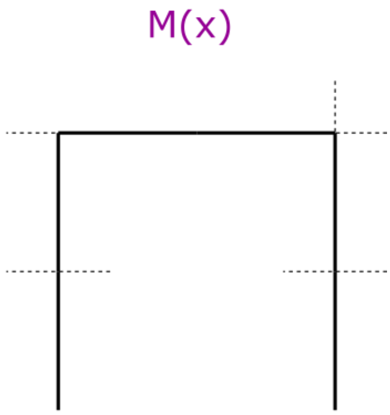
V Section $a \le x_1 < 2a$ right side

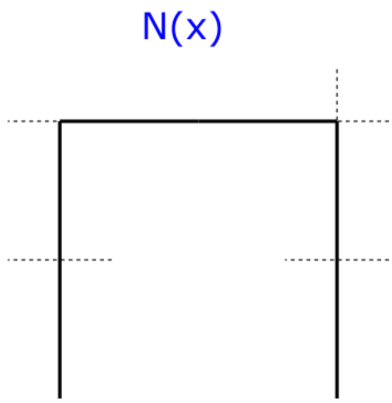
$$M(x_5) = R_{Bx} * x_5 - M$$
$$T(x_2) = -R_{Bx}$$
$$N(x_2) = -R_{By}$$

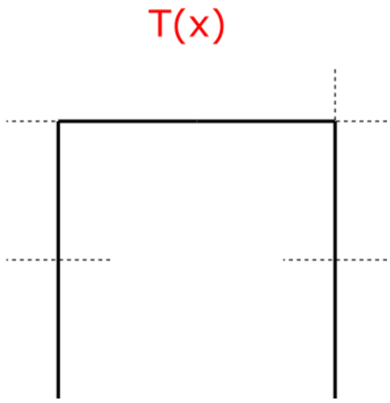
CHARTS

15. The last part related to solving frames – charts. To easily draw charts, it is best to draw them next to the frame as a frame with the same shape as given frame, as shown in the figure.

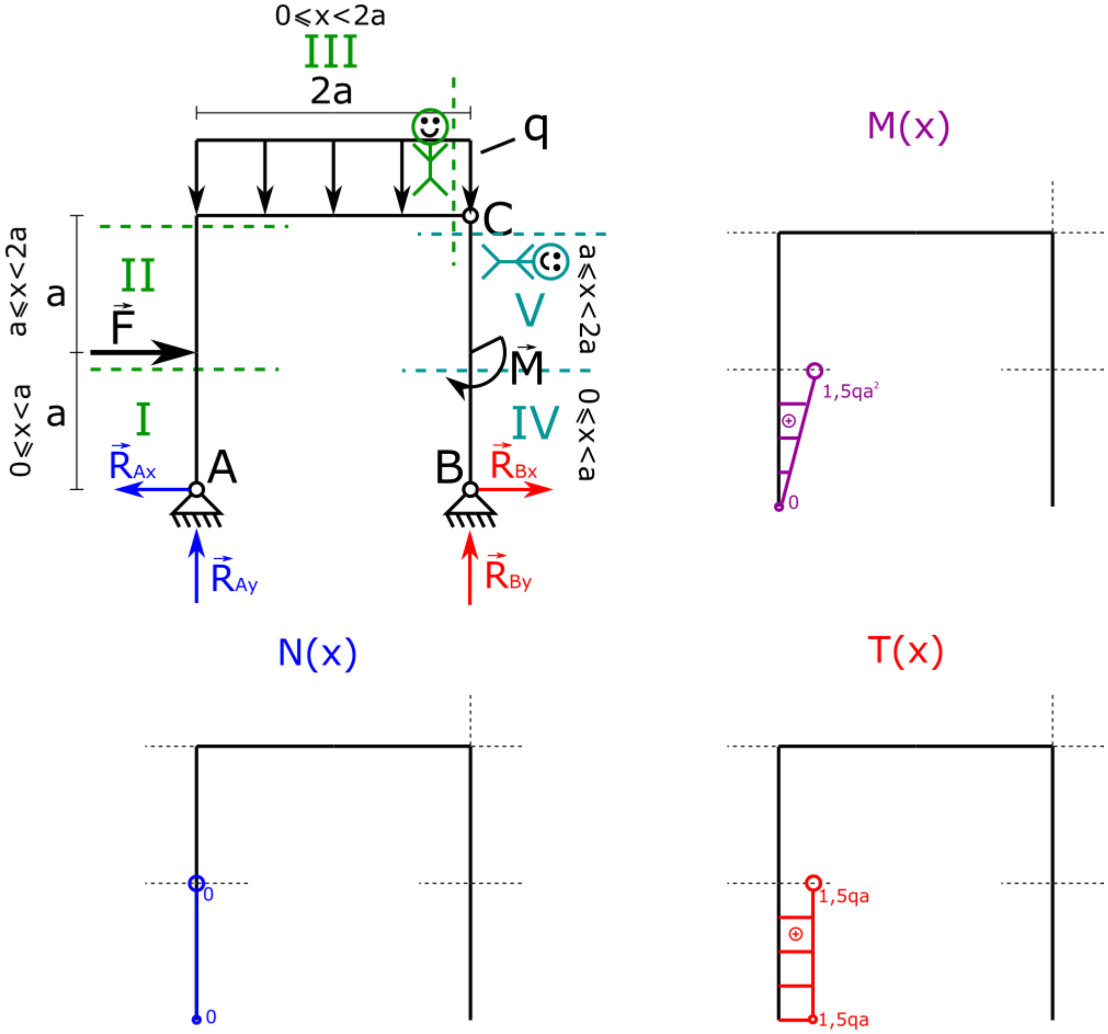








16. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the first section.

$$M(x_1) = R_{Ax} * x_1$$
$$T(x_1) = R_{Ax}$$
$$N(x_1) = -R_{Ax}$$

We know the limits of the first section.

$$0 \leq x < a$$

We substitute the boundary values into our equations (for x_1).

$$M(0) = R_{Ax} * 0 = 0$$

$$M(a) = R_{Ax} * a = 1{,}5qa * a = 1{,}5qa^2[kNm]$$

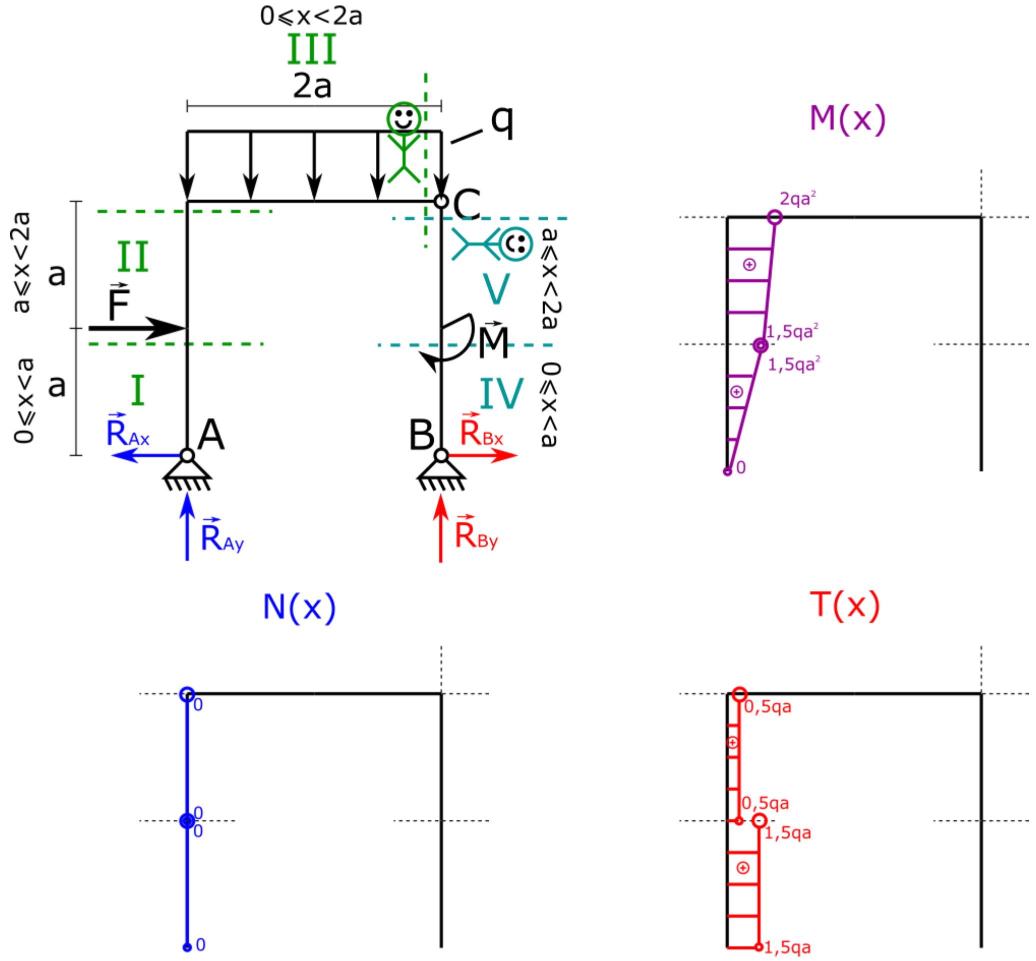
$$T(0) = R_{Ax} = 1{,}5qa[kN]$$

$$T(a) = R_{Ax} = 1{,}5qa[kN]$$

$$N(0) = -R_{Ay} = 0$$

$$N(a) = -R_{Ay} = 0$$

17. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the second section.

$$M(x_2) = R_{Ax} * x_2 - F * (x_2 - a)$$
$$T(x_2) = R_{Ax} - F$$
$$N(x_2) = -R_{Ay}$$

We know the limits of the second section.

$$a \le x < 2a$$

We substitute the boundary values into our equations (for x_2).

$$M(a) = R_{Ax} * a - F * (a - a) = 1{,}5qa^2[kNm]$$

$$M(2a) = R_{Ax} * 2a - F * (2a - a) = 1{,}5qa * 2a - 1qa * a = 2qa^2[kNm]$$

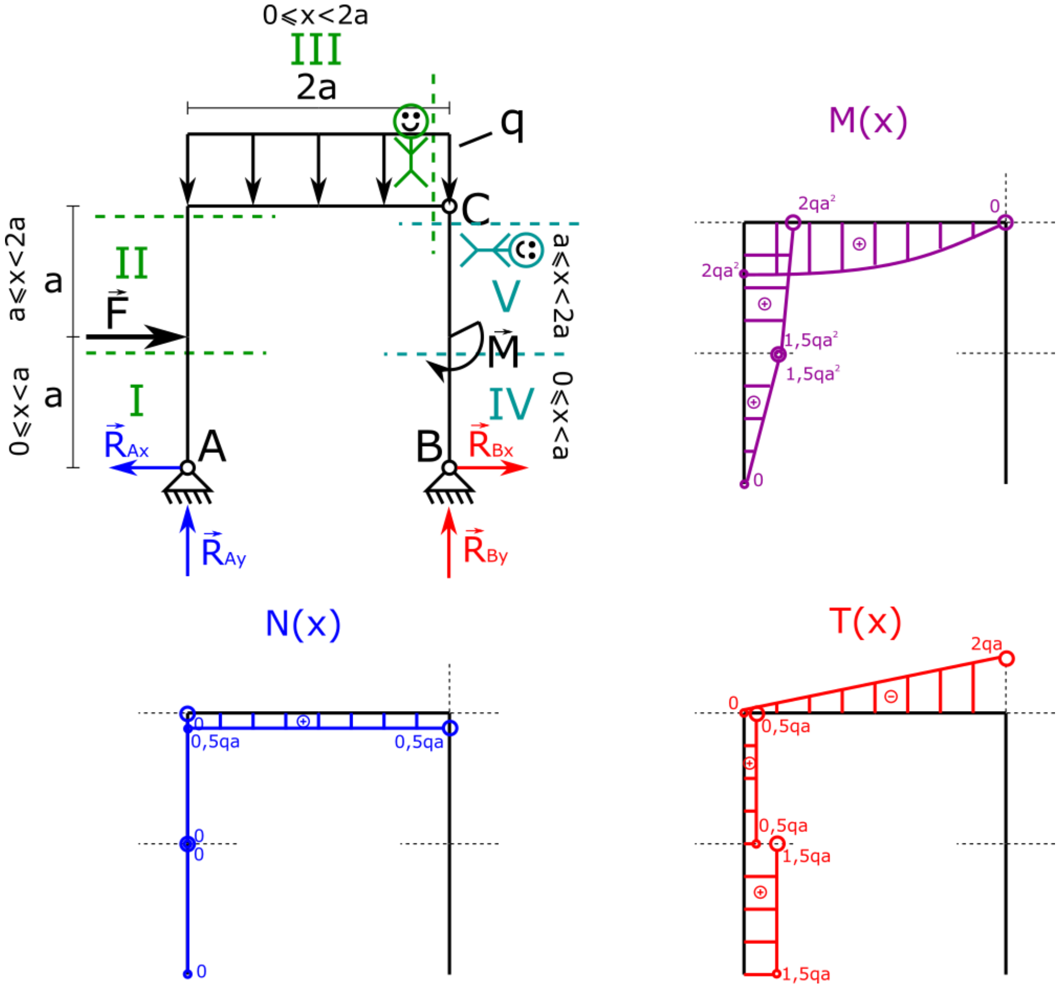
$$T(a) = R_{Ax} - F = 1{,}5qa - 1qa = 0{,}5qa[kN]$$

$$T(2a) = R_{Ax} - F = 1{,}5qa - 1qa = 0{,}5qa[kN]$$

$$N(a) = -R_{Ay} = 0$$

$$N(2a) = -R_{Ay} = 0$$

18. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the third section.

$$M(x_3) = R_{Ax} * 2a - F * a + R_{Ay} * x_3 - \frac{q * x_3^2}{2}$$
$$T(x_3) = R_{Ay} - q * x_3$$
$$N(x_3) = -R_{Ax}$$

We know the limits of the third section.

$$0 \le x < 2a$$

We substitute the boundary values into our equations (for x_3).

$$M(0) = R_{Ax} * 2a - F * a + R_{Ay} * 0 - \frac{q * 0}{2} = 1{,}5qa * 2a - qa * a = 2qa^2[kNm]$$

$$M(2a) = R_{Ax} * 2a - F * a + R_{Ay} * 2a - \frac{q * (2a)^2}{2} = 2qa^2 + 0 * 2a - 2qa^2 = 0[kNm]$$

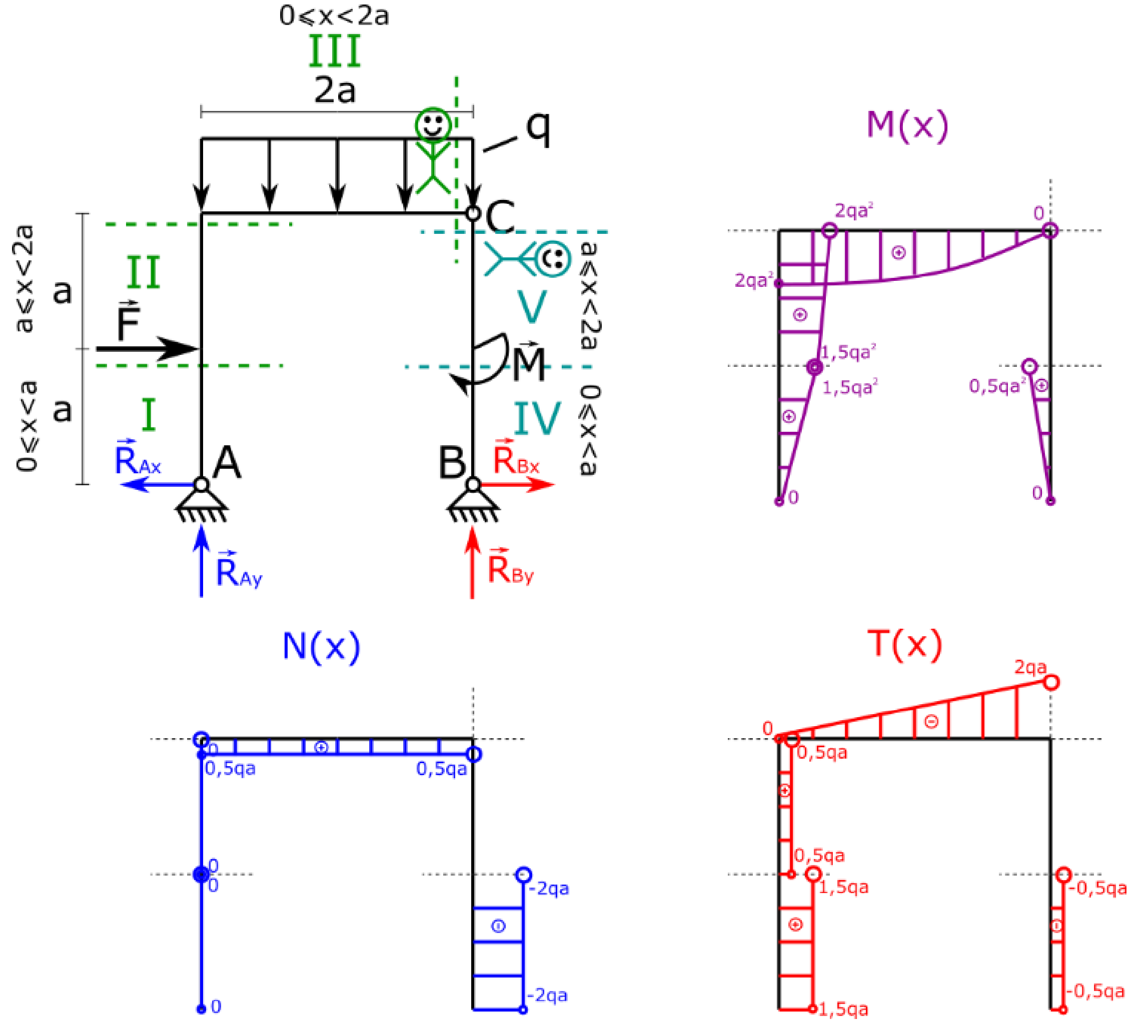
$$T(0) = R_{Ay} - q * 0 = 0 - 0 = 0[kN]$$

$$T(2a) = R_{Ay} - q * 2a = 0 - 2qa = -2qa[kN]$$

$$N(0) = 1{,}5qa - F = 1{,}5qa - qa = 0{,}5qa[kN]$$

$$N(2a) = 1{,}5qa - F = 1{,}5qa - qa = 0{,}5qa[kN]$$

19. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the fourth section.

$$M(x_4) = R_{Bx} * x_4$$
$$T(x_4) = -R_{Bx}$$
$$N(x_4) = -R_{By}$$

We know the limits of the fourth section.

$$0 \le x < a$$

We substitute the boundary values into our equations (for x_4).

$$M(0) = R_{Bx} * 0 = 0$$

$$M(a) = 0{,}5qa * a = 0{,}5qa^2[kNm]$$

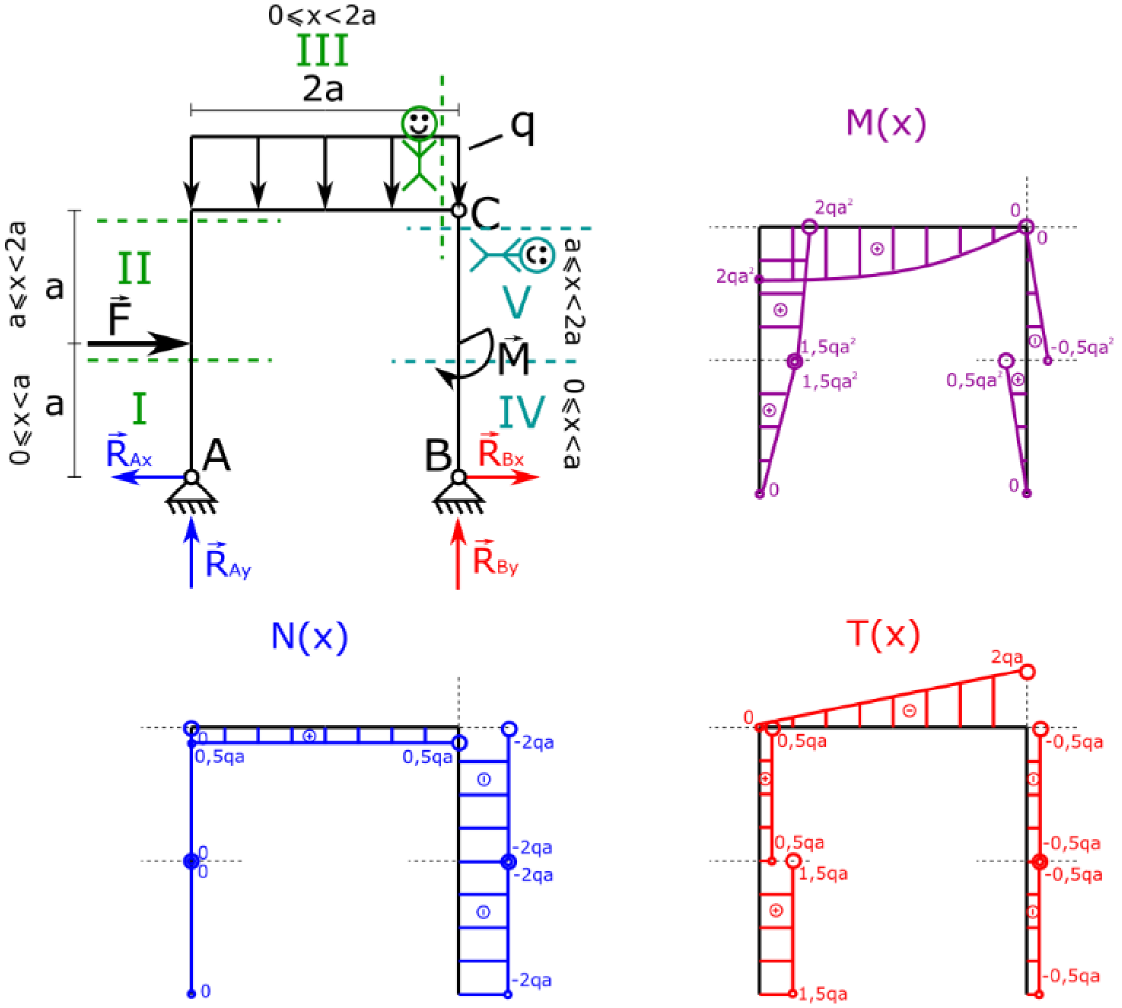
$$T(0) = -R_{Bx} = -0{,}5qa[kN]$$

$$T(a) = -R_{Bx} = -0{,}5qa[kN]$$

$$N(0) = -R_{By} = -2qa[kN]$$

$$N(a) = -R_{By} = -2qa[kN]$$

20. In this example, the charts will be drawn for all internal forces section by section.



We will need the equation for the fifth section.

$$M(x_5) = R_{Bx} * x_5 - M$$
$$T(x_2) = -R_{Bx}$$
$$N(x_2) = -R_{By}$$

We know the limits of the fifth section.

$$a \leq x < 2a$$

We substitute the boundary values into our equations (for x_5).

$$M(a) = R_{Bx} * a - M = 0{,}5qa * a - qa^2 = -0{,}5qa^2[kNm]$$

$$M(2a) = R_{Bx} * 2a - M = 0{,}5qa * 2a - qa^2 = 0[kNm]$$

$$T(a) = -R_{Bx} = -0{,}5qa[kN]$$

$$T(2a) = -R_{Bx} = -0{,}5qa[kN]$$

$$N(a) = -R_{By} = -2qa[kN]$$

$$N(2a) = -R_{By} = -2qa[kN]$$