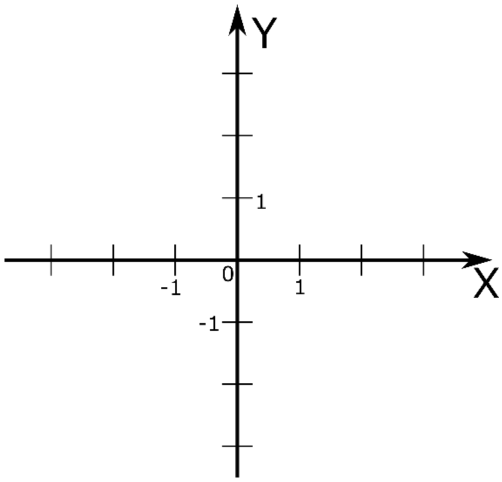
Kinematics of the particle

Example 1.

Having given the equation of motion of a material point, determine its velocities, accelerations, motion path and radius of curvature.

$$x = 4t; y = 16t^2 - 1 \; dla \; t_1 = 0{,}5s$$

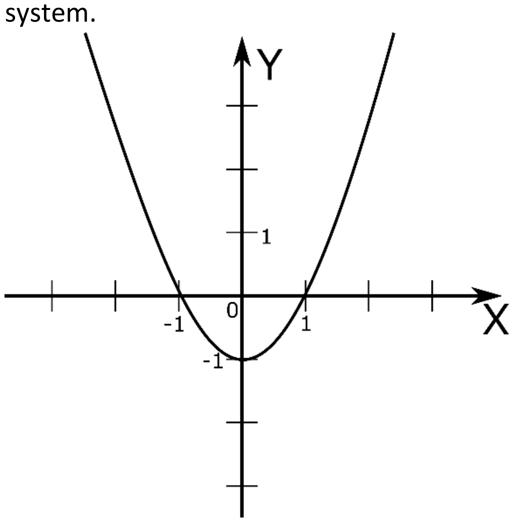
1. First, enter the coordinate system.



2. Knowing the equations of motion, one can determine the trajectory of motion, getting rid of time from them

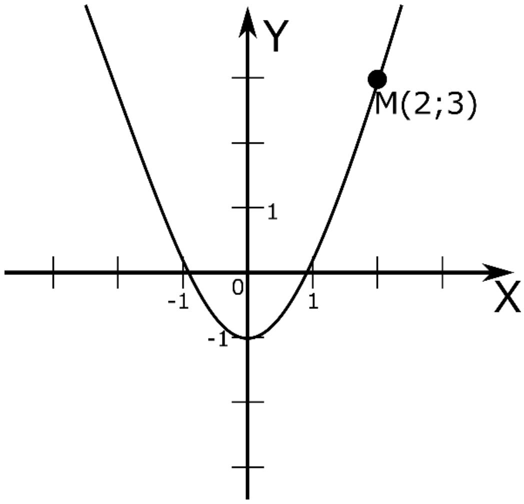
$$y = x^2 - 1$$

You can see that this is a parabola equation that we plot on the introduced coordinate system.



3. Then it is worth finding the position of the point on the path for the given time.

$$x(t_1) = 2; y(t_1) = 3$$



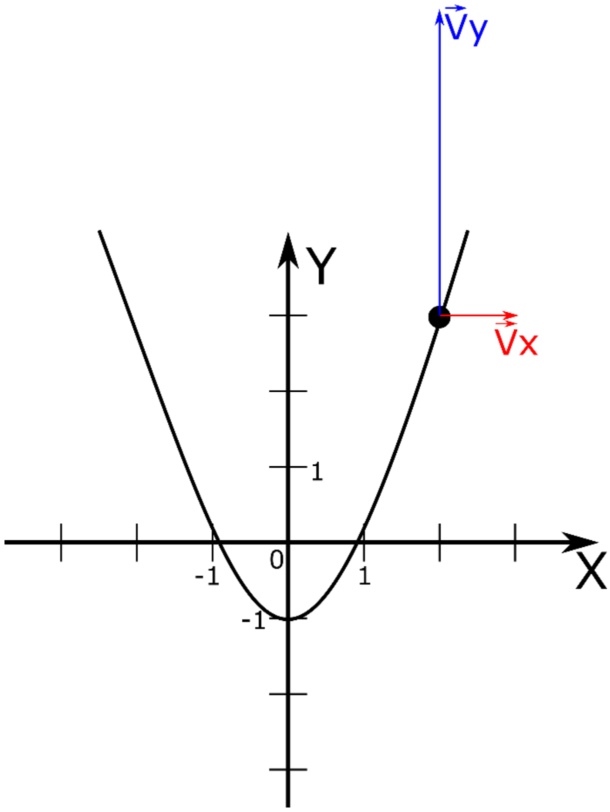
4. Then you can go to determine the velocity components. Since we know the equations of motion along the X and Y axes, it is possible to determine the velocity components along these two axes. To get the velocity, one must differentiate the path equation over time.

$$V_x = \frac{dx}{dt} = 4;\;\; V_y = \frac{dy}{dt} = 32t$$

Since we are looking for a velocity value for a specific time, we put the time into the equations above

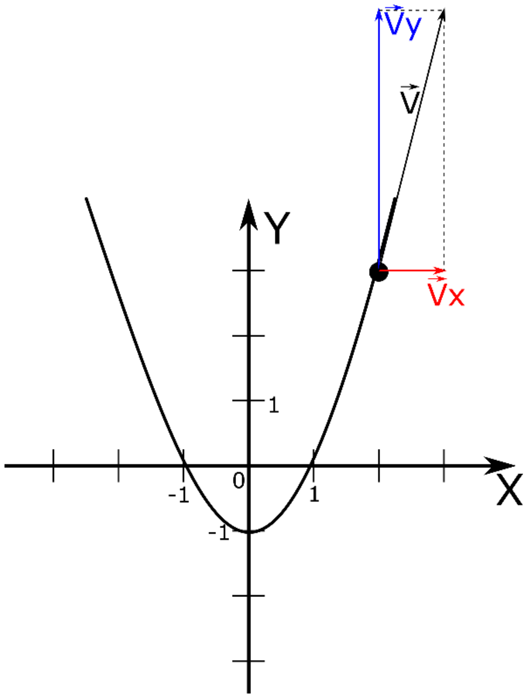
$$V_x(t_1) = 4\frac{m}{s};\;\; V_y(t_1) = 16\frac{m}{s}$$

We obtained the values of the components of the velocity vector on individual axes. Now you can plot these vectors on the coordinate system.



5. After plotting the vectors on the drawing, we can determine the modulus of the velocity vector and plot this vector.

$$|\vec{V}| = \sqrt{{V_x}^2 + {V_y}^2} = \sqrt{16+256} \cong 16{,}5\frac{m}{s}$$



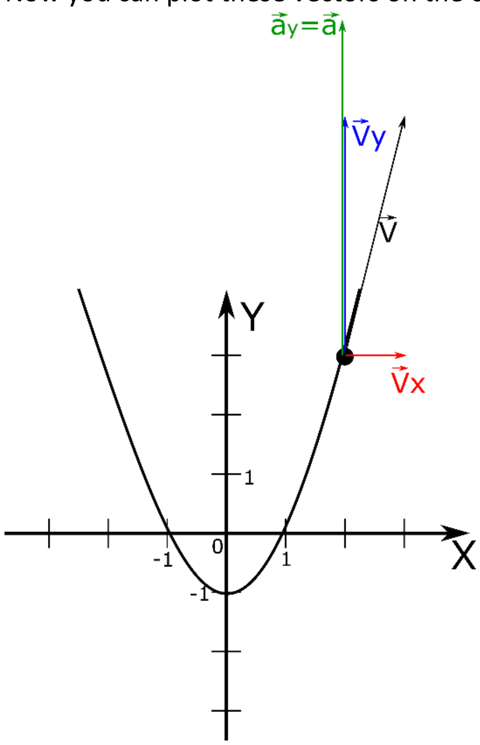
6. Knowing the velocity, you can go to accelerations. In the first step, we do the same as in the case of velocity, knowing that acceleration is a derivative of velocity.

$$a_x = \frac{dV_x}{dt} = 0;\;\; a_y = \frac{dV_y}{dt} = 32$$

Since we are looking for acceleration values for a specific time, we insert time into the above equations

$$a_x(t_1) = 0\frac{m}{s};\;\; a_y(t_1) = 32\frac{m}{s}$$

We obtained the values of the components of the acceleration vector on the individual axes. Now you can plot these vectors on the coordinate system.



7. After plotting the vectors on the drawing, we can determine the modulus of the acceleration vector and plot this vector.

$$|\vec{a}| = \sqrt{{a_x}^2 + {a_y}^2} = 32\frac{m}{s^2}$$

8. In the case of acceleration, its tangential and normal components should also be determined. We determine the tangential acceleration by differentiating the modulus of the velocity vector over time.

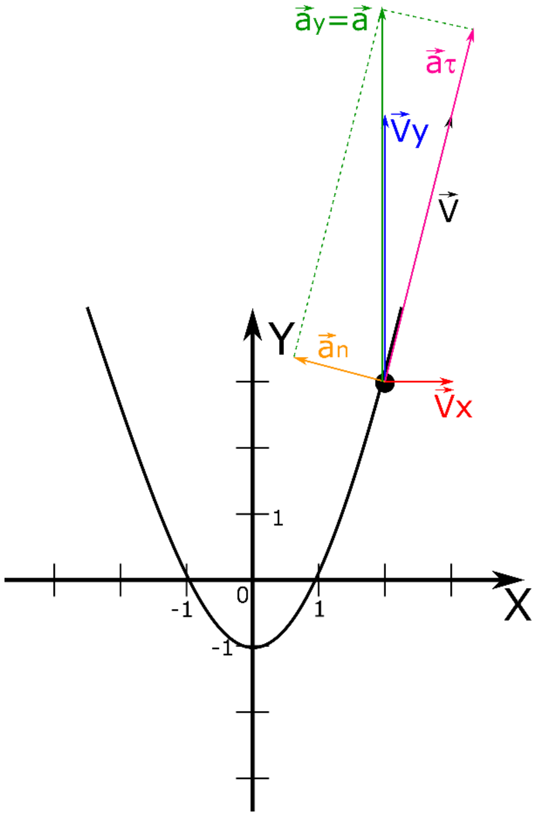
$$a_\tau = \left|\frac{d\vec{V}}{dt}\right| = \frac{2V_x\dot{V}_x + 2V_y\dot{V}_y}{2\sqrt{{V_x}^2 + {V_y}^2}} = \frac{V_x a_x + V_x a_x}{V} = \frac{512}{16{,}5} = 31\frac{m}{s^2}$$

The positive sign at the tangential acceleration value indicates that the point motion is accelerated (the senses of vectors $\vec{V}$ i $\overrightarrow{a_\tau}$ are consistent).

In the next step, we will calculate the normal acceleration

$$a_n = \sqrt{a^2 - {a_\tau}^2} = \sqrt{32^2 - 31^2} = 7{,}94\frac{m}{s^2}$$

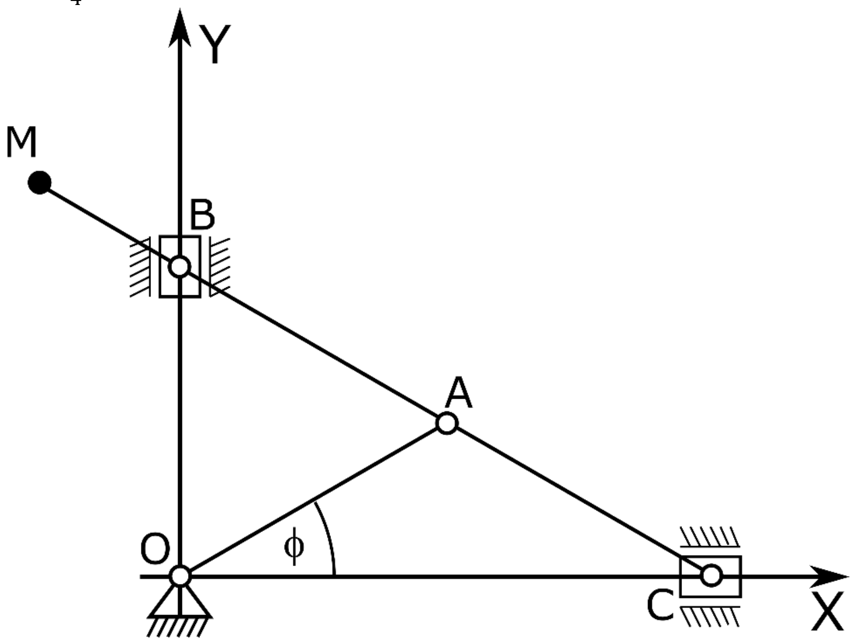
in the case of the normal component of the acceleration vector, it should be remembered that the sense of this vector is always towards the center of curvature. On this basis, these vectors can be marked on the drawing.



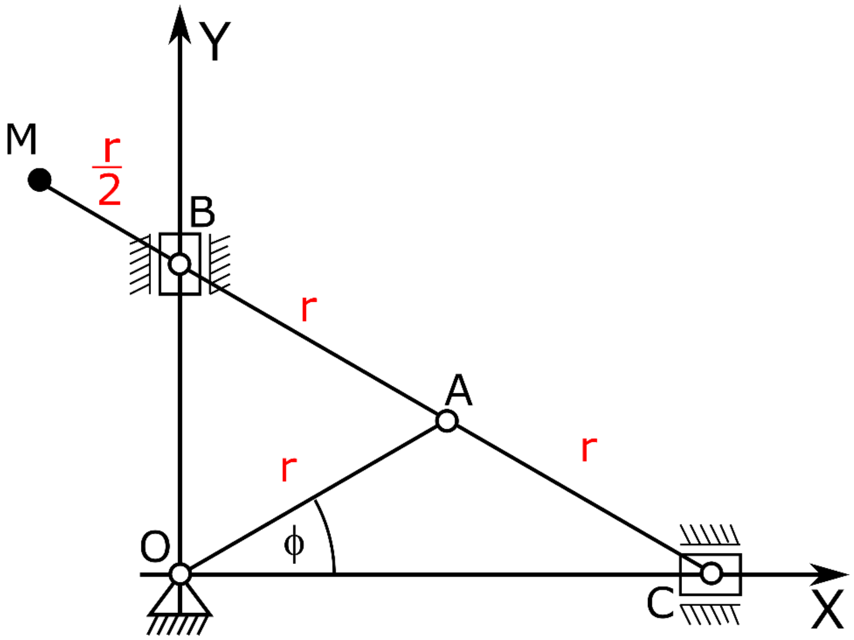
9. Finally, we determine the radius of curvature ρ of the trajectory on which the material point is located for the given time t_1.

$$\rho = \frac{V^2}{a_n} = \frac{16{,}5^2}{7{,}94} = 34{,}3$$

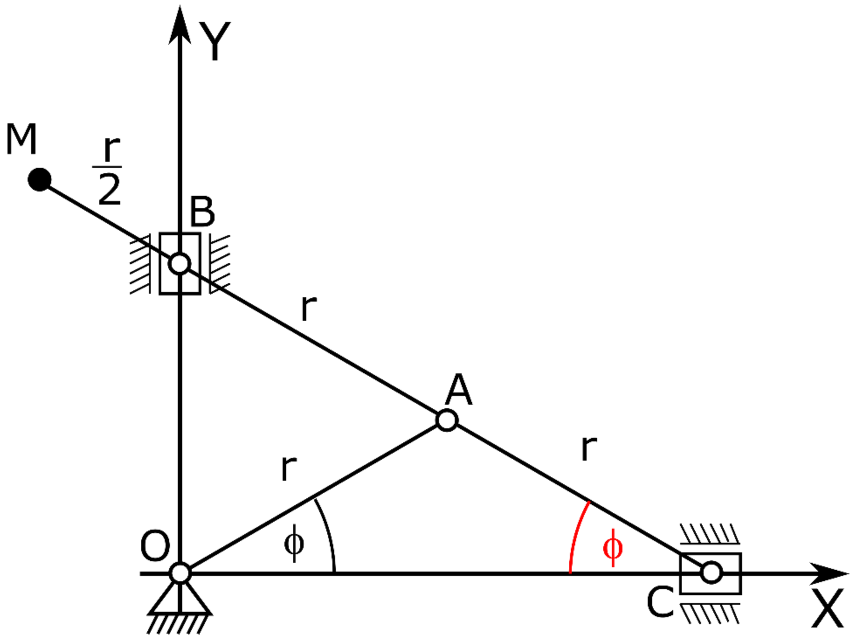
Example 2. For the M point on the mechanism below, determine the velocities, accelerations, path and radius of curvature at the moment t_1. Data: AB=AC=r, MB=$\frac{r}{2}$,r=2m, $t_1 = \frac{1}{4}s$, $\varphi = \pi t$.



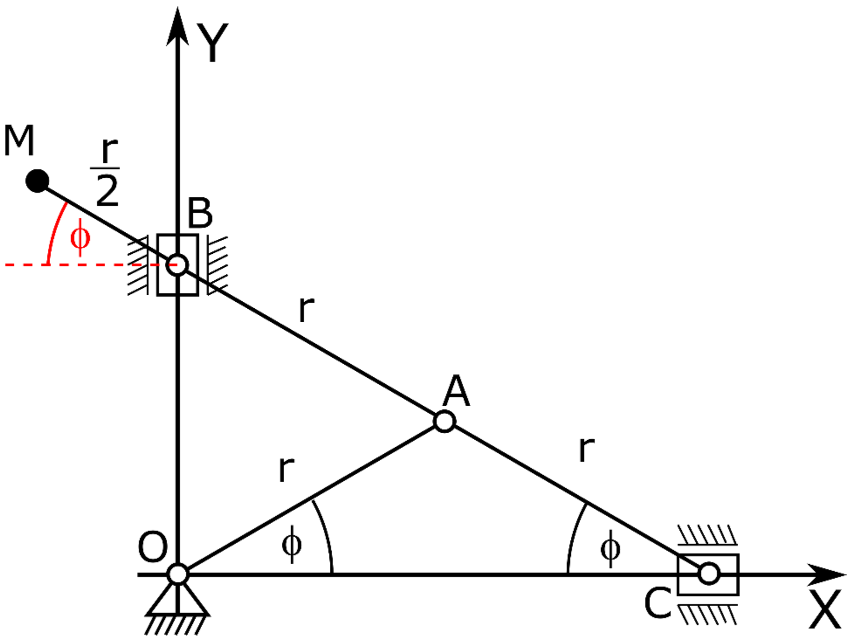
1. In the case of this type of task, the solution is a bit more difficult, compared to the previous example, due to the fact that there are no explicitly given equations of motion, and we have to determine these equations ourselves, based on the geometry of the system. In the first step, we put the given values on the drawing - we mark the segments.



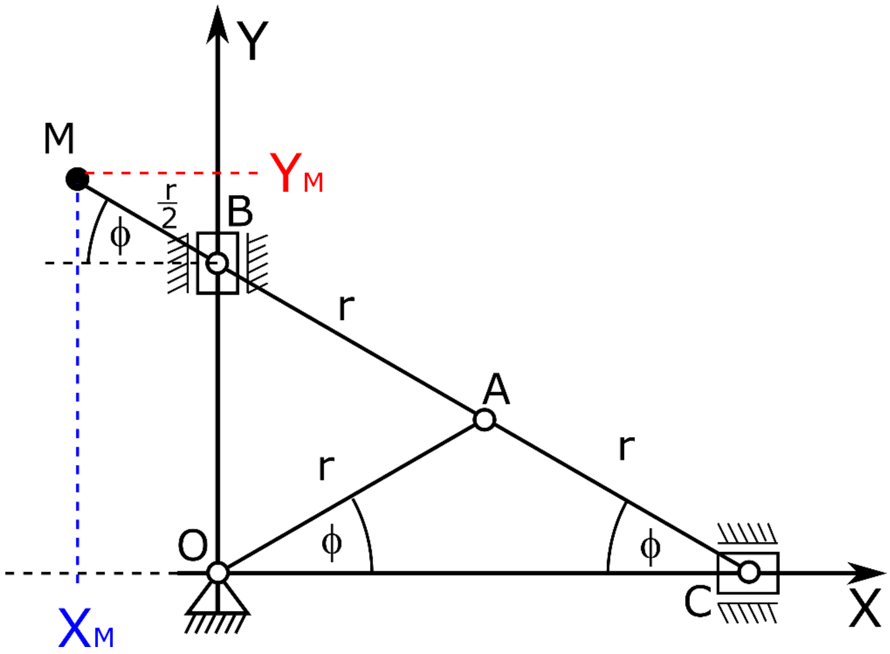
2. Then it is clearly seen that an isosceles triangle was formed, hence it can be seen that we will also have an angle at point C ϕ.



3. Analyzing the drawing further, it can be seen that the same angle will be at point B as shown.

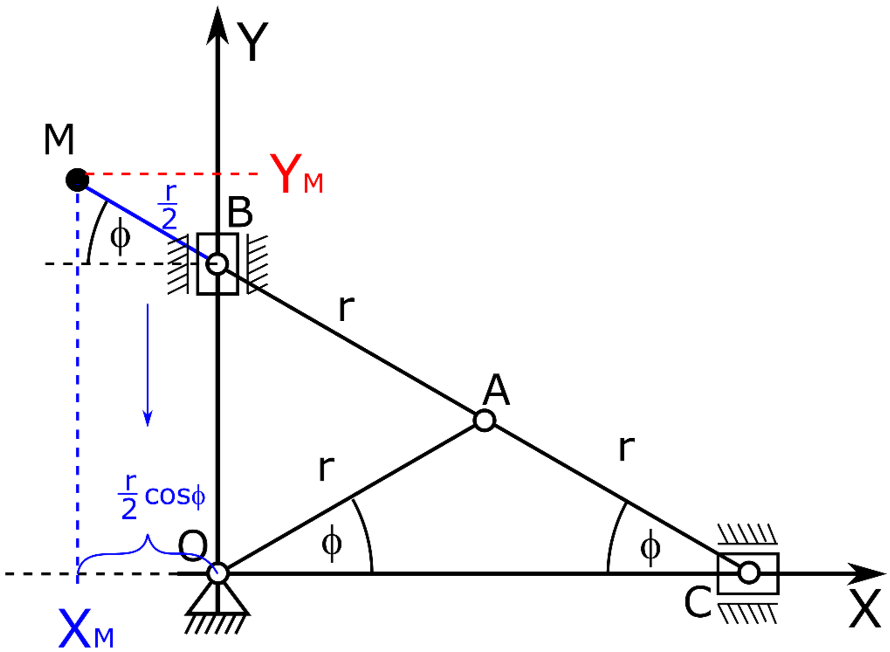


4. Once we have a well-described geometry of the mechanism, we can move on to determining the equations of motion. To do this, you need to define its X_M and Y_M coordinates, respectively, based on the given position of the M point, as shown in the figure.

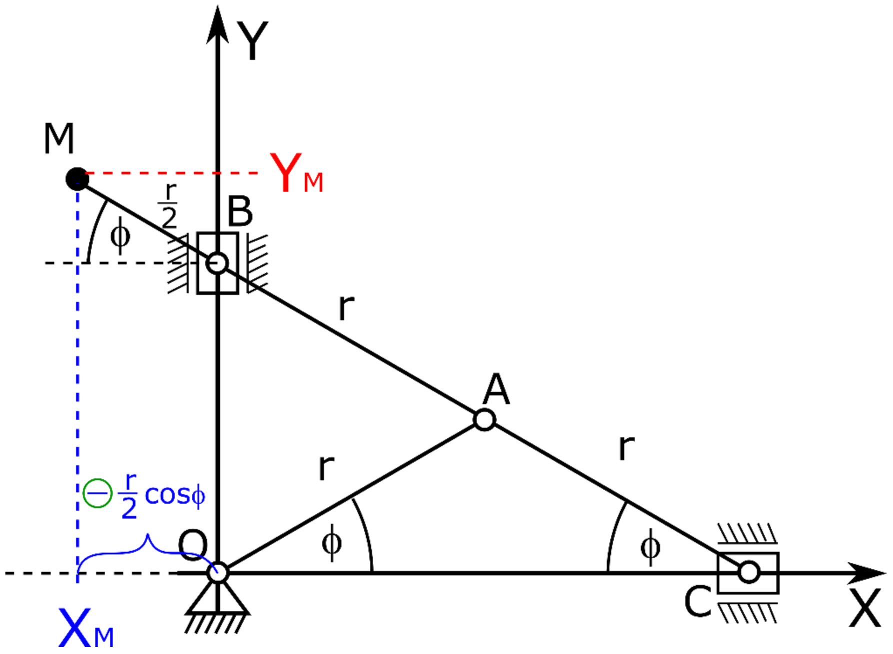


5. Let's start with determining the coordinate on the X axis. In this case, to do this, project the MB segment onto the OX axis. We will then obtain the equation of the following form,

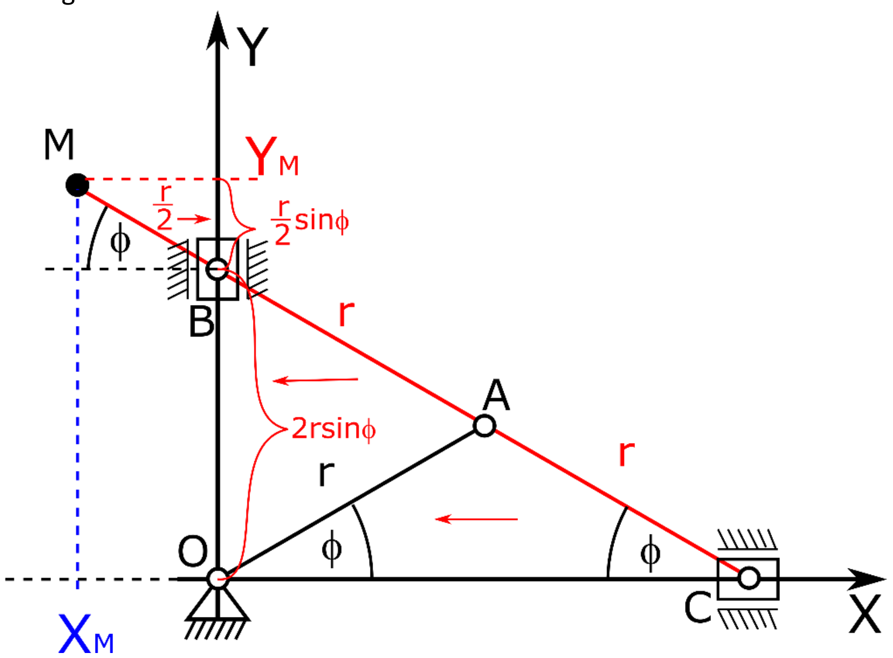
$$X_M = -0.5r\cos\phi$$



6. The negative sign in the equation results from the fact that the point M is on the negative side of the X axis coordinates, which is clearly visible in the next figure.



7. Once we have a specific value of the coordinate on the X axis, we can go to the Y axis. Similarly to the previous case, we also project the geometry to the Y axis as shown in the figure.



8. Finally, we can write the equation to the YM coordinate as follows.

$$Y_M = 2r\sin\phi + 0{,}5\sin\phi = r2{,}5\sin\phi$$

9. Finally we get the system of equations in the following form.

$$\begin{cases} X_M = -0.5r\cos\phi \\ Y_M = r2{,}5\sin\phi \end{cases}$$

10. From this point on, when we have parametric equations of motion, we proceed very much like in the first example. First, let's define the trajectory along which this point will move. Therefore, time has to be removed from the equations.

$$\begin{cases} x = -\dfrac{r}{2}\cos\phi \\ y = \dfrac{5}{2}r\sin\phi \end{cases}$$

We take the values independent of time to one side and square it on both sides.

$$\begin{cases} \left(\frac{x}{\frac{r}{2}}\right)^2 = -\cos^2\phi \\ \left(\frac{y}{\frac{5}{2}r}\right)^2 = \sin^2\phi \end{cases}$$

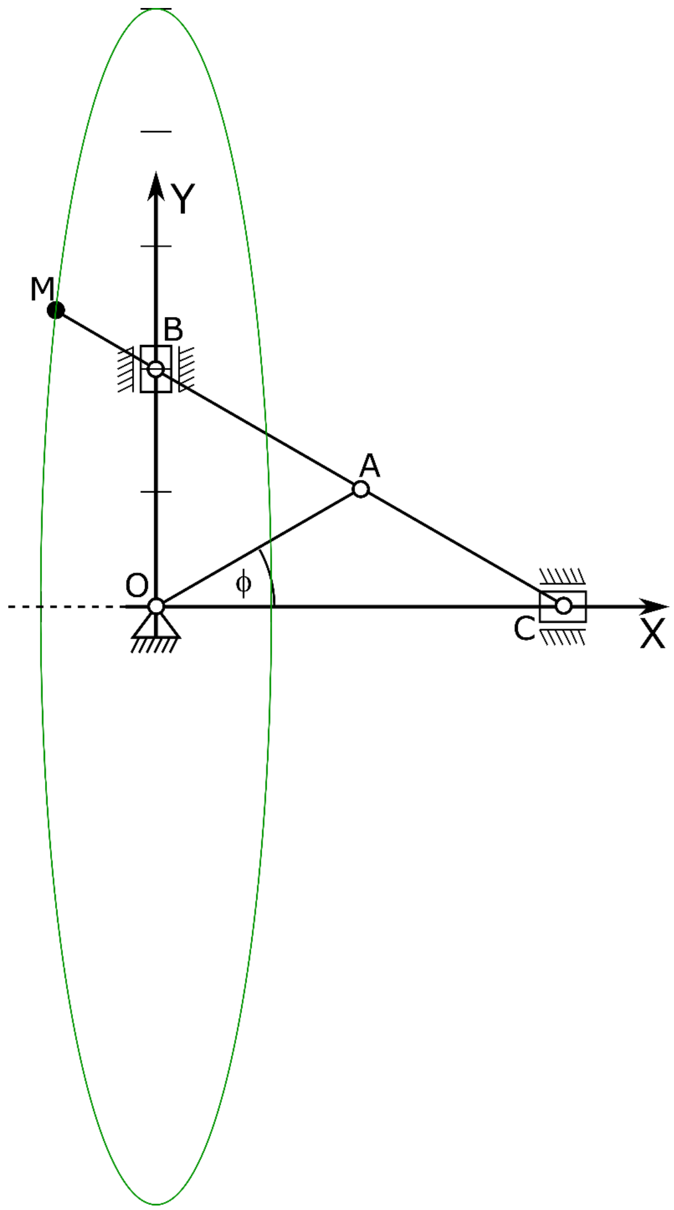
When we add the equations side by side, you can see the trigonometric one on the right

$$\left(\frac{x}{\frac{r}{2}}\right)^2 + \left(\frac{y}{\frac{5}{2}r}\right)^2 = 1$$

And transforming further, we see that we got the parabola equation.

$$\frac{x^2}{1} + \frac{y^2}{25} = 1$$

Now you can draw a trajectory along which point M follows.



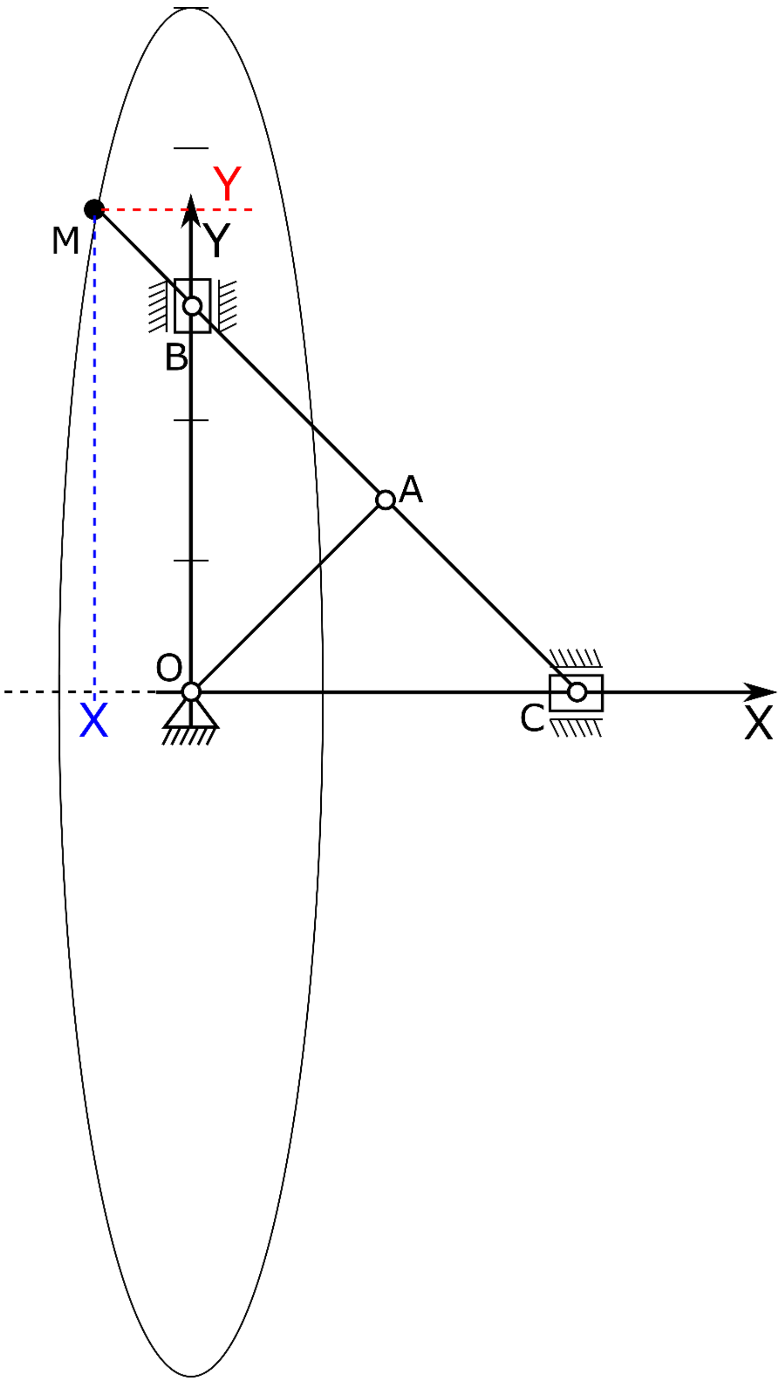
11. Next, we are looking for the coordinates of the point location for the given time.

$$\begin{cases} x = -\cos \pi t \\ y = 5 \sin \pi t \end{cases}$$

$$\begin{cases} x(t_1) = -\cos \dfrac{\pi}{4} \\ y(t_1) = 5 \sin \dfrac{\pi}{4} \end{cases}$$

$$\begin{cases} x(t_1) = -0{,}707 \\ y(t_1) = 3{,}53 \end{cases}$$

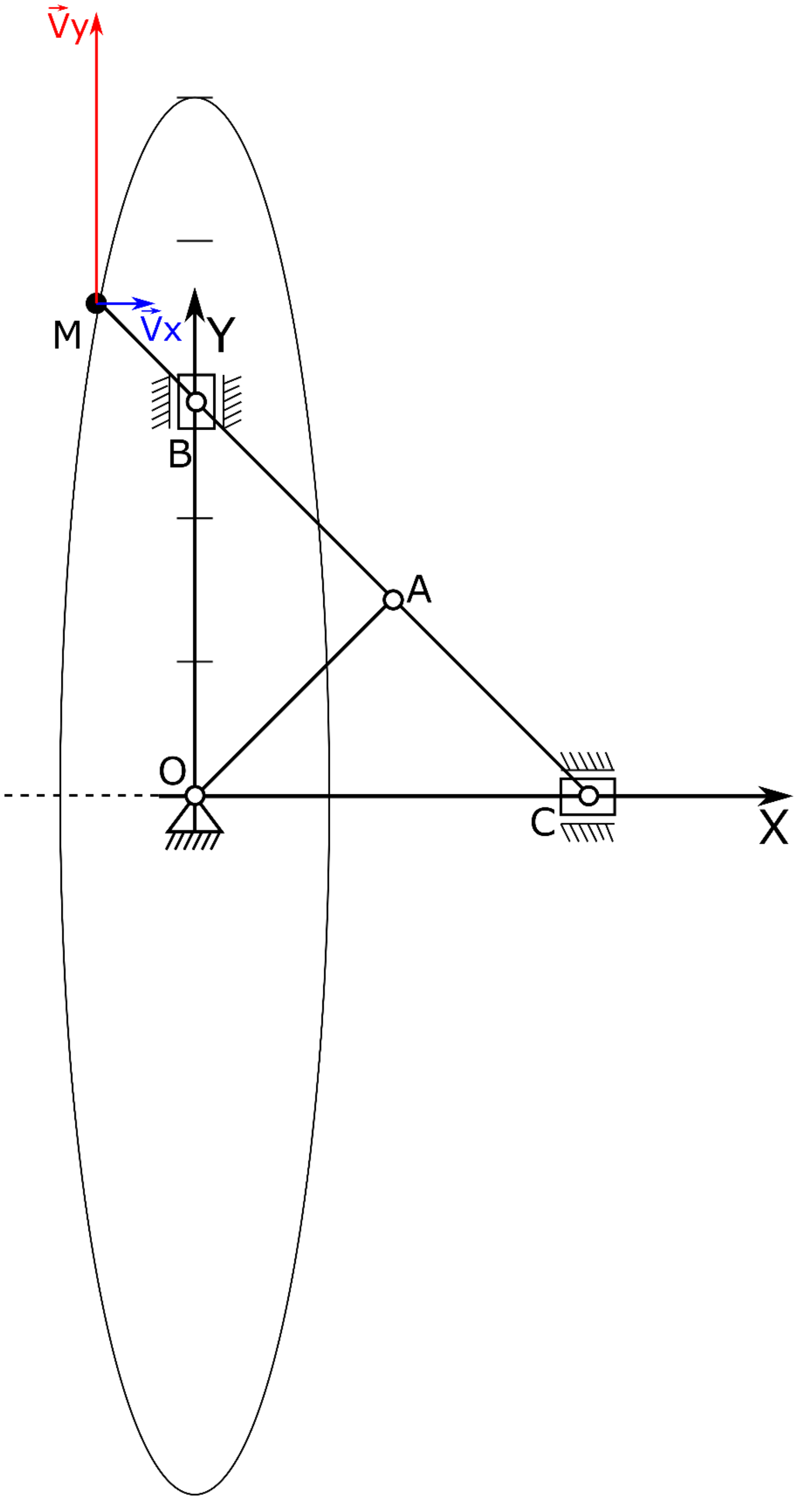
We plot the location of the point on the trajectory.



12. Next we look for speed (derivative of the road over time)

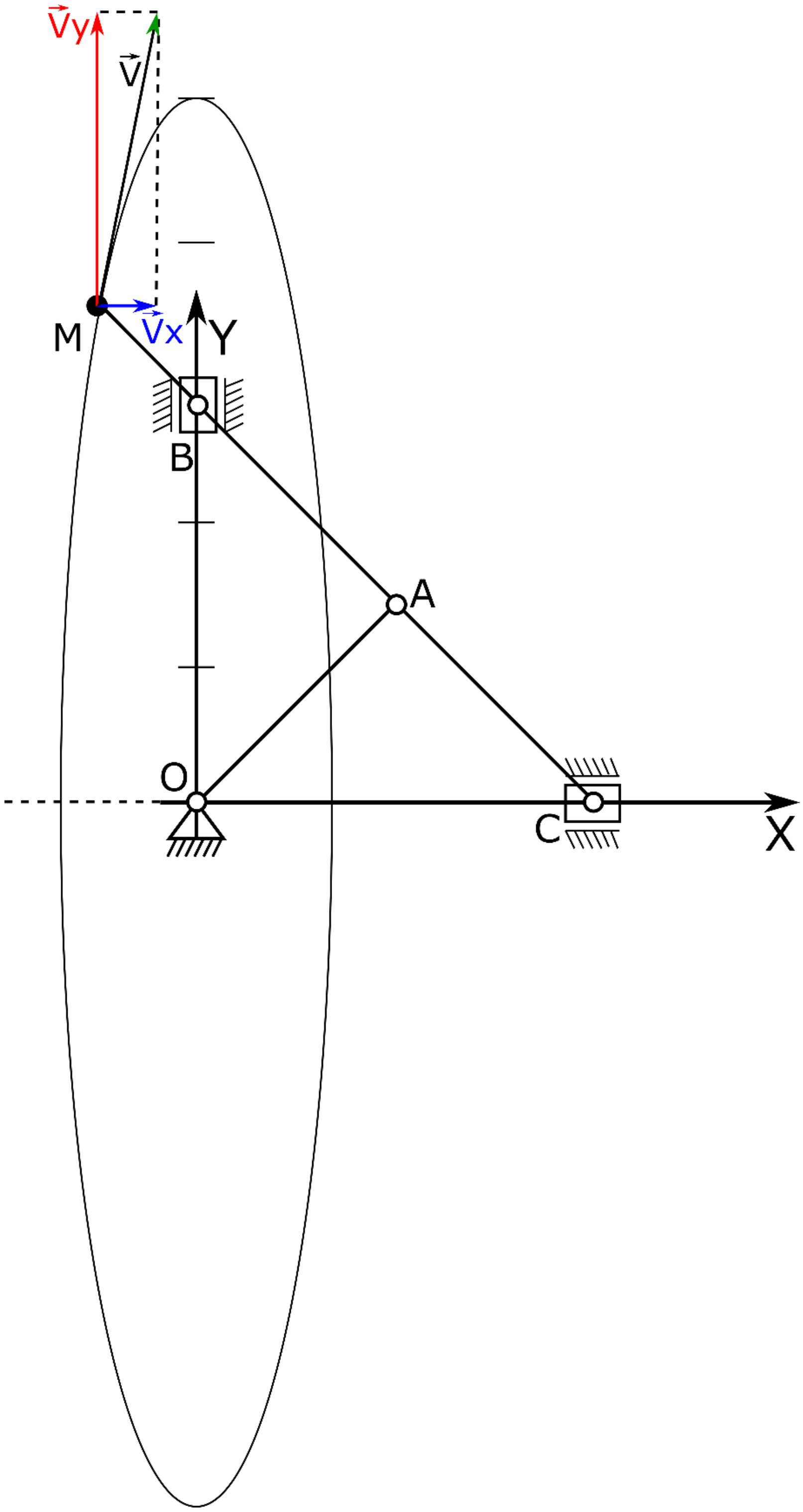
$$\begin{cases} \dot{x} = \dfrac{dx}{dt} = \pi \sin \pi t = 2{,}22 \dfrac{m}{s} \\ \dot{y} = \dfrac{dy}{dt} = 5\pi \cos \pi t = 11{,}1 \dfrac{m}{s} \end{cases}$$

After finding the value of the projections of the velocity vector on the X and y axes for a given time, we mark the vectors on the drawing.



13. After plotting the vectors on the drawing, we can determine the modulus of the velocity vector and plot this vector.

$$|\vec{V}| = \sqrt{{V_x}^2 + {V_y}^2} = 11{,}32\frac{m}{s}$$



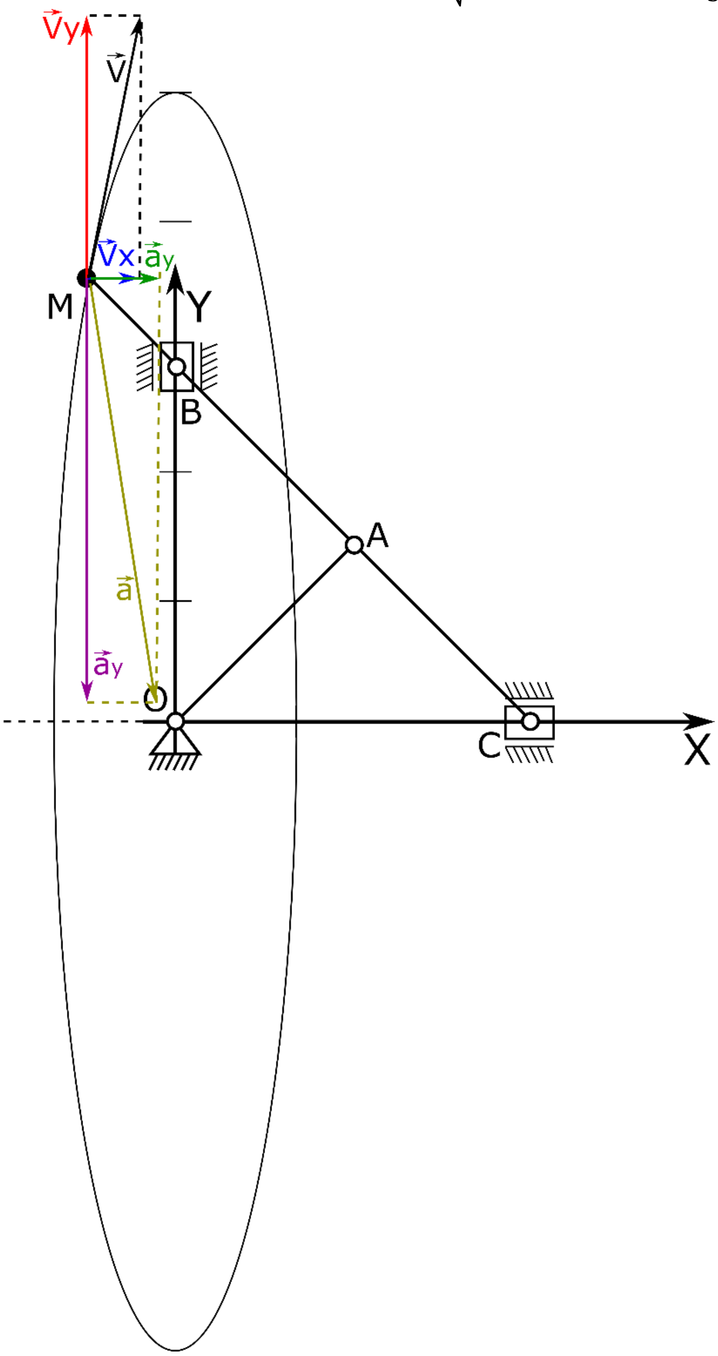
14. Next we are looking for accelerations (derivative of velocity over time)

$$\begin{cases} \dot{V}_x = \dfrac{dV_x}{dt} = \pi^2 \cos \pi t = 6{,}97 \dfrac{m}{s^2} \\ \dot{V}_y = \dfrac{dV_y}{dt} = -5\pi^2 \sin \pi t = -34{,}87 \dfrac{m}{s^2} \end{cases}$$

After finding the projections of the acceleration vector on the X and y axes for a given time, we mark the vectors on the drawing.

After plotting the vectors on the drawing, we can determine the modulus of the acceleration vector and plot this vector.

$$|\vec{a}| = \sqrt{{a_x}^2 + {a_y}^2} = 35{,}56 \frac{m}{s^2}$$



15. Finally, we calculate the values of the tangential acceleration, normal acceleration, and the radius of curvature.

$$a_\tau = \left|\frac{d\vec{V}}{dt}\right| = \frac{2V_xV_x + \dot{2}V_y\dot{V_y}}{2\sqrt{{V_x}^2 + {V_y}^2}} = \frac{V_xa_x + V_xa_x}{V} = -32{,}77\,\frac{m}{s^2}$$

The negative sign at the tangential acceleration value indicates that the point motion is delayed (the senses of vetors $\vec{V}$ i $\vec{a_\tau}$ are opposite). In the next step, we will calculate the normal acceleration

$$a_n = \sqrt{a^2 - {a_\tau}^2} = \sqrt{35{,}56^2 - 32{,}77^2} = 13{,}93\,\frac{m}{s^2}$$

And the radius of curvature

$$\rho = \frac{V^2}{a_n} = \frac{11{,}32^2}{13{,}93} = 9{,}199$$