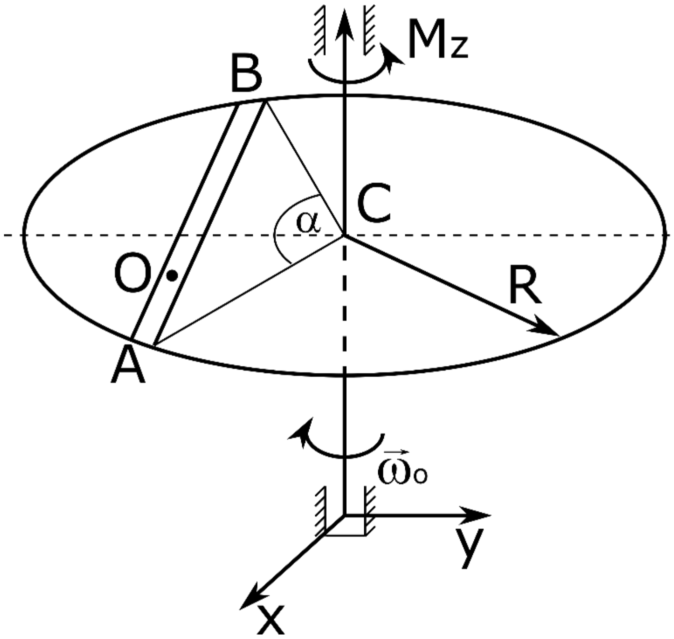
Moment of momentum

Example. A body N with mass $m_1$rotates around the vertical axis "z" with a constant angular velocity ω_o where at the point O of the groove AB of the body N, at a distance AO from point A along the groove, there is a material point of mass m_2. At a certain moment (t = 0), a torque Mz starts acting on the system. At the moment t = τ the torque stops to acting, and at the same time the point L starts relative motion from point O along the groove AB towards point B according to the formula OL. Determine the angular velocity of the body N for the times t = τ and t = T, disregarding the resistance to rotation of the body H. Show the vectors. Dane: $m_1 = 200\ kg, m_2 = 60kg, \omega_o = 2s^{-1}, R = 2m,$ $\alpha = 120°, OA = \frac{\sqrt{3}}{2}, M_z = 101\ Nm, \tau = 5s, T = 6s, OL = \sqrt{3}(t-\tau)^2$



We use principle of the moment of momentum

Stage I, $M_z$is acting

$$\frac{d\vec{K}^o}{dt} = \sum M_i^o$$

$$\frac{dK_z}{dt} = M_z$$

$$K_z = K_z{}^N + K_z{}^L$$

$$K_z{}^N = I_z * \omega$$

$$K_z{}^L = m_2 * V * OC$$

$$\vec{V} = \vec{V}_R + \vec{V}_L$$

Since the point does not move at the beginning, there is no $\vec{V}_R$, so $V = V_L$.

$$K_z{}^L = m_2 * V_L * OC$$

$$V_L = \ \omega * OC$$

$$K_z{}^L = m_2 * V_L * OC$$

$$K_z{}^L = m_2 * \omega * OC^2$$

$$K_z = \omega(I_z + m_2 * OC^2)$$

Finding the OC section

$$\frac{AF}{R} = \cos 30°$$

$$AF = R\cos 30° = \sqrt{3}$$

$$\frac{FC}{R} = \sin 30°$$

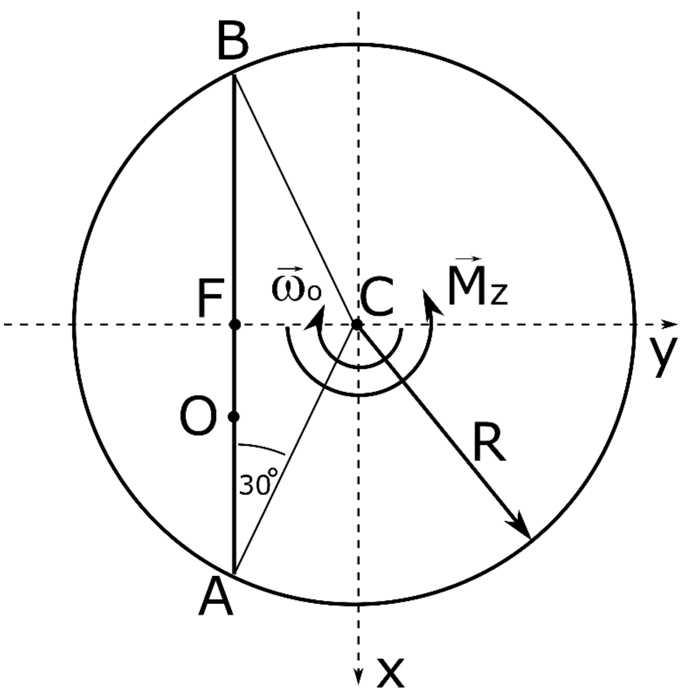
$$FC = R\sin 30° = 1$$

$$FO = AF - OA = \frac{\sqrt{3}}{2}$$

$$OC = \sqrt{FO^2 + FC^2} = 1{,}32$$

The above geometrical relationships will be used at a later stage of the task. From the law of cosines.

$$OC = \sqrt{OA^2 + R^2 - 2 * OA * R * \cos 30°} = 1{,}32$$



$$\frac{dK_z}{dt} = M_z$$

$$K_z = \int M_z dt$$

$$K_z = \int 101 dt$$

$$K_z = 101t + C_1$$

Determination of the integration constant

For $t = 0$, i.e. for the beginning of the movement, we know that the whole system was spinning with$\omega = \omega_o$

$$\omega_o(I_z + m_2 * OC^2) = C_1$$

$$\omega(I_z + m_2 * OC^2) = 101t + \omega_o(I_z + m_2 * OC^2)$$

$$\omega = \omega_o + \frac{101t}{I_z + m_2 * OC^2}$$

Now you only need to determine the moment of inertia of the circular disk with respect to the Z axis.

$$I_z = \frac{mR^2}{2} = 400$$

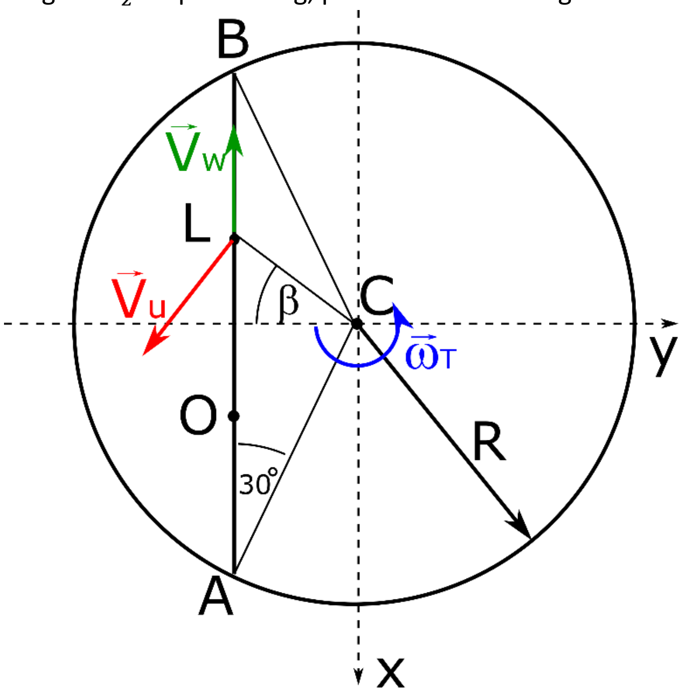
Determination of $\omega(\tau)$, that is for the 5th second of the movement.

$$\omega(\tau) = \omega_o + \frac{101\tau}{I_z + m_2 * OC^2} = 1$$

Next we will calculate the moment of momentum value for τ.

$$K_z(\tau) = \omega(\tau)(I_z + m_2 * OC^2) = 504{,}5$$

Stage II M_z stops working, point L starts moving



$$M_z = 0$$

$$\frac{dK_z}{dt} = 0 \Rightarrow K_z = const \Rightarrow K_z{}^{(\tau)} = K_z{}^{(T)}$$

$$K_z = {K_z}^N + {K_z}^L$$

$${K_z}^N = I_z * \omega_T$$

$${K_z}^L = \left(\vec{r} \times \vec{V}\right) * m_2$$

$$\vec{V} = \vec{V}_R + \vec{V}_L$$

$${K_z}^L = \left(\vec{r} \times \vec{V}_R + \vec{r} \times \vec{V}_L\right) * m_2$$

$${K_z}^L = m_2 * V_L * LC - m_2 * V_R * LC \sin(90° - \beta)$$

$$V_L = \omega_T * LC$$

$$V_R = \frac{dOL}{dt} = 2\sqrt{3}(t - \tau)$$

$$V_R(T) = 3{,}46$$

$$OL(T) = \sqrt{3}(t - \tau) = \sqrt{3}$$

$$LC = CO = 1{,}32$$

Because,

$$FO = \frac{\sqrt{3}}{2}; LO = \sqrt{3} = 2 * FO$$

$$LC \sin(90° - \beta) = LC \cos\beta = FC;\ FC = 1$$

$$K_z(T) = I_z * \omega_T + m_2 * \omega_T * LC^2 - m_2 * V_R * FC = 504{,}5$$

$$I_z * \omega_T + m_2 * \omega_T * LC^2 - m_2 * V_R * FC = 504{,}5$$

$$\omega_T = \frac{504{,}5 + m_2 * V_R * FC}{I_z + m_2 * LC^2} = 0{,}59$$