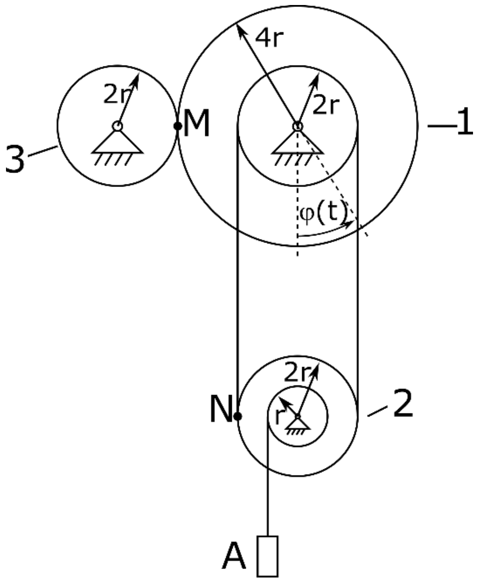
Motion of a point on the circle

Example. Having given the equation of the rotational motion of the disc 1 $\varphi(t) = 3 + 20t^2$ determine the equation of the translational motion of the solid A and $\omega_1, \omega_2, \omega_3, V_M, V_N, V_A, \varepsilon_1, \varepsilon_2, \varepsilon_3, a_M, a_N, a_A$



First we determine the angular velocity and angular acceleration for wheel 1. And then the linear velocity for point M. It can be seen that this point belongs to wheels 1 and 3, therefore the linear velocities will be the same.

$$\omega_1 = \frac{d\varphi(t)}{dt} = 40t$$

$$\varepsilon_1 = \frac{d^2\varphi(t)}{dt^2} = 40$$

$$V_M = \omega_1 * r_1 = 40t * 4r = 160tr$$

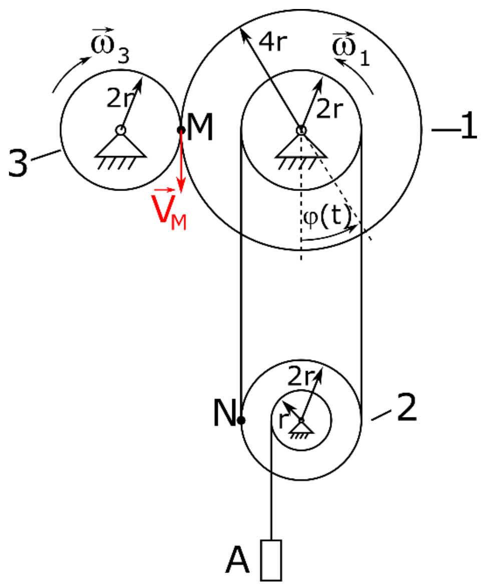
$$V_M = \omega_3 * r_3 \Rightarrow \omega_3 = \frac{V_M}{r_3} = \frac{160tr}{2r} = 80t$$

$$\varepsilon_3 = \frac{d\omega_3}{dt} = 80$$

$$a_M^n = \omega_1^2 * 4r = (40t)^2 * 4r = 6400rt^2$$

$$a_M^\tau = \varepsilon_1 * 4r = 160r$$

$$a_M = \sqrt{{a_M^n}^2 + {a_M^\tau}^2} = \sqrt{(160r)^2 + (6400rt^2)^2}$$



Then we can look at point N. The point is connected to wheel 1 by a transmission belt. Therefore the linear velocity on the circumference of the smaller wheel 1 will be the same as the velocity of point N.

$$V_N = \omega_1 * 2r = 40t * 2r = 80tr$$

$$V_N = \omega_2 * 2r \Rightarrow \omega_2 = \frac{V_N}{2r} = \frac{80tr}{2r} = 40t$$

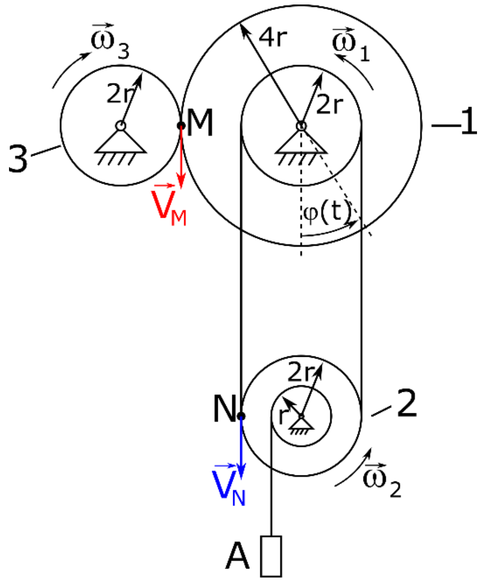
$$\varepsilon_2 = \frac{d\omega_2}{dt} = 40$$

$$\omega_1 = \omega_2\, ; \varepsilon_1 = \varepsilon_2 \Rightarrow \varphi_1 = \varphi_2$$

$$a_N^n = \omega_2^2 * 2r = (40t)^2 * 2r = 3200rt^2$$

$$a_N^\tau = \varepsilon_2 * 2r = 80r$$

$$a_N = \sqrt{{a_N^n}^2 + {a_N^\tau}^2} = \sqrt{(80r)^2 + (3200rt^2)^2}$$



Finally, we turn to point A. It is clearly visible that the linear shift of point A will be equal to the length of the rope unwound from the arc s.

$$s = \varphi * r = \varphi_2 * r = 3r + 20rt^2$$

$$V_A = \frac{ds}{dt} = \frac{d(3r + 20rt^2)}{dt} = 40tr$$

$$a_A = \frac{dV_A}{dt} = 40r$$