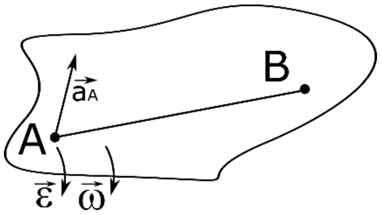
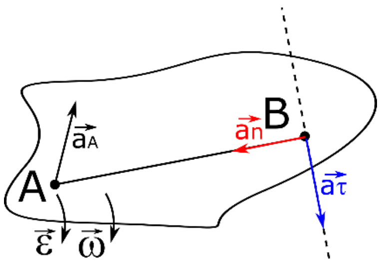
Planar motion of rigid body

If we have a rigid body and we know the acceleration of one of its points, and additionally we have information about the angular velocity and angular acceleration of this body, then we can determine the acceleration of other points of this body, using the superposition method.

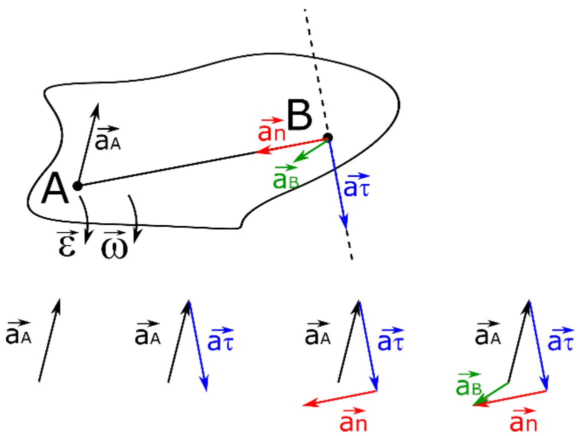
$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^{n} + \vec{a}_{BA}^{\tau}$$



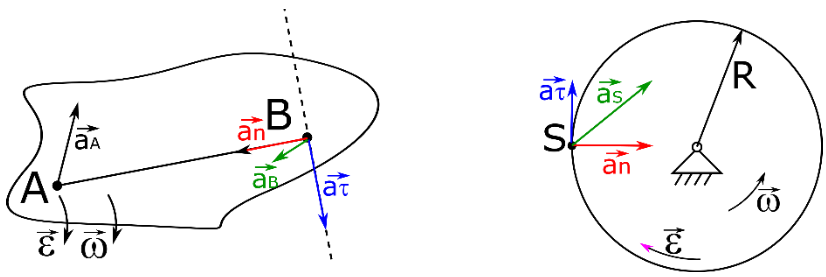
first, we take the point for which we know the acceleration as the pivot point. We plot the normal and tangential acceleration vectors to this point.



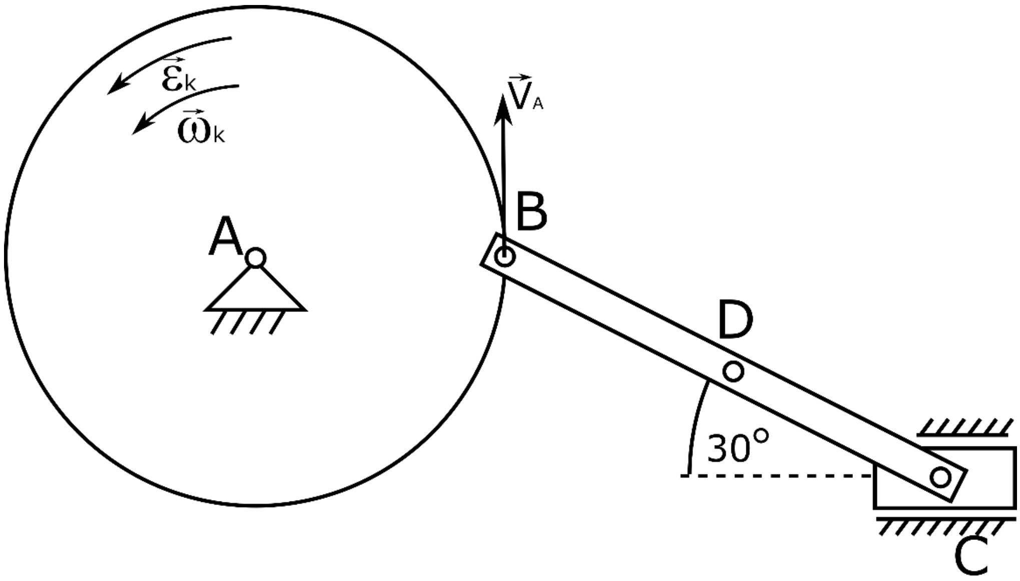
Then we construct the acceleration chain as shown in the figure below and find the acceleration of the point we are looking for.



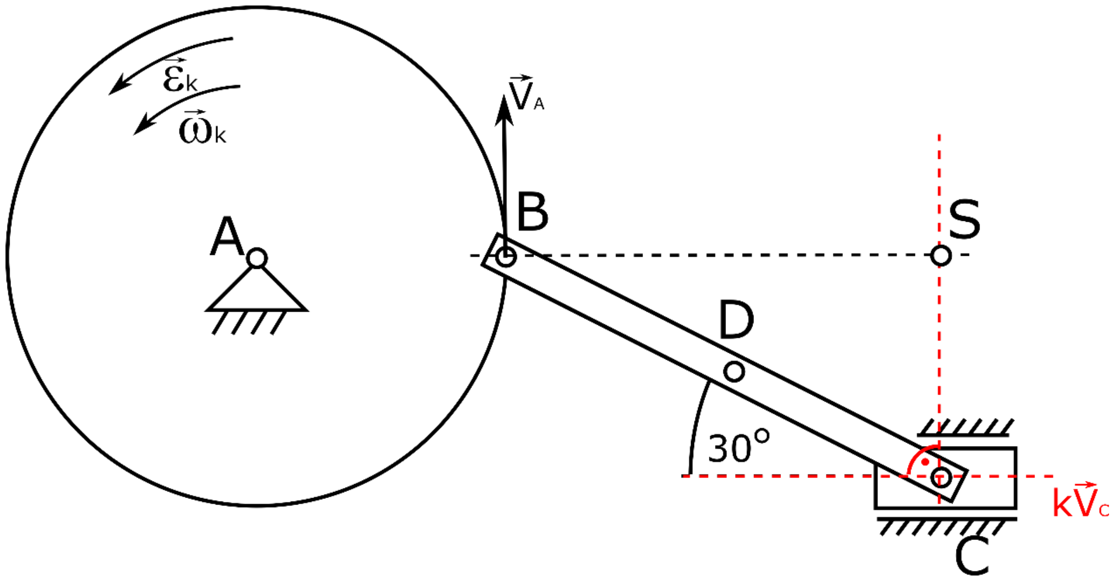
In the figure below, you can see the similarity of plane motion to circular motion.



Example. For the given mechanism, find the velocities and accelerations for points B, C and D. Data: $\omega_k = 1$, $\varepsilon_k = 1$, AB=2, BC=4, BD=2.

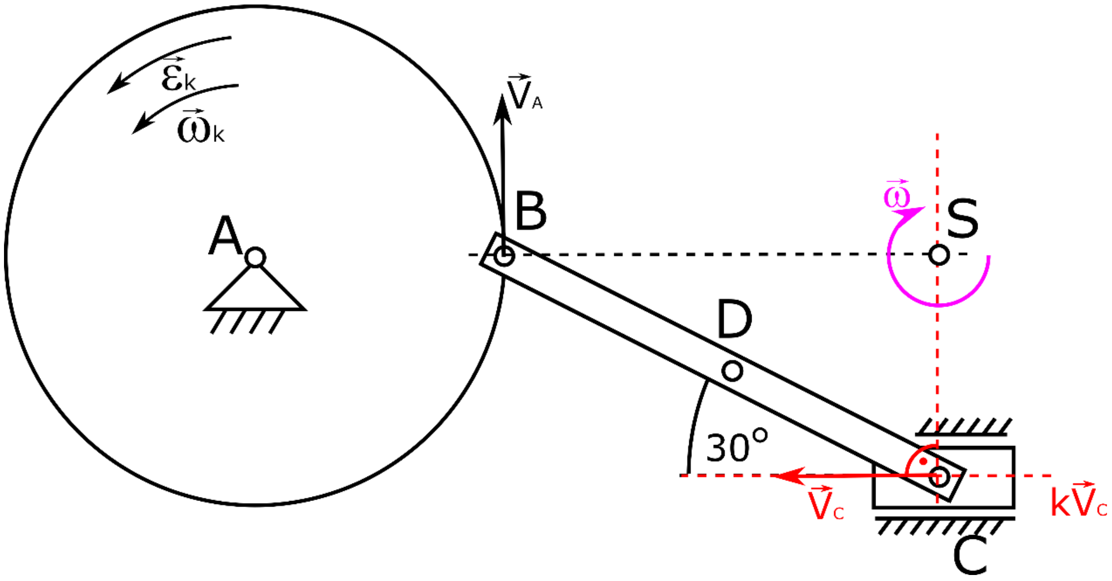


First we will deal with velocities. We calculate the speed of point B using the fact that it is located on the circumference of the circle. For the rest of the points we will use the method of the instantaneous center of rotation. The analysis of the drawing shows that we know the direction of point C's motion. It will be a horizontal direction, so the velocity vector of this point must also lie in this direction. Next we draw straight lines perpendicular to the known velocities directions. At the intersection of these lines we find a point S called the instantaneous center of rotation.



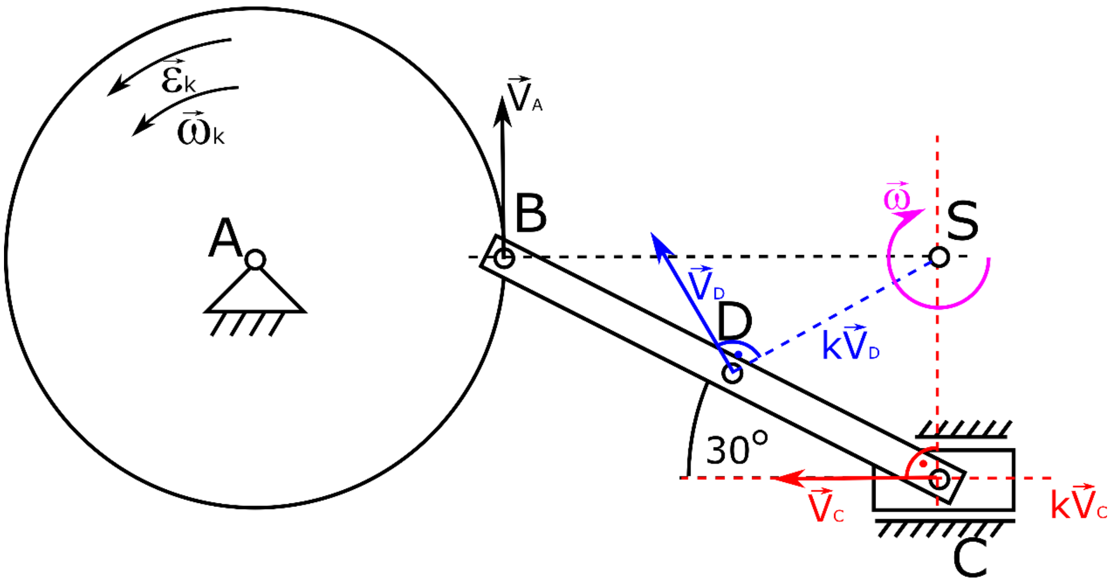
$$V_B = \omega_k * AB = 1 * 2 = 2$$

$$V_B = \omega * BS = \omega * BC * \cos 30° \Rightarrow \omega = \frac{V_B}{4 * \frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$



After determining the velocity of point B and finding the angular velocity, we can determine the velocity of point D. We run a straight line from point S to point D and the velocity vector of this point will lie on the perpendicular direction.

$$V_D = \omega * DS = \omega * \frac{1}{2}BC = \frac{2\sqrt{3}}{3}$$

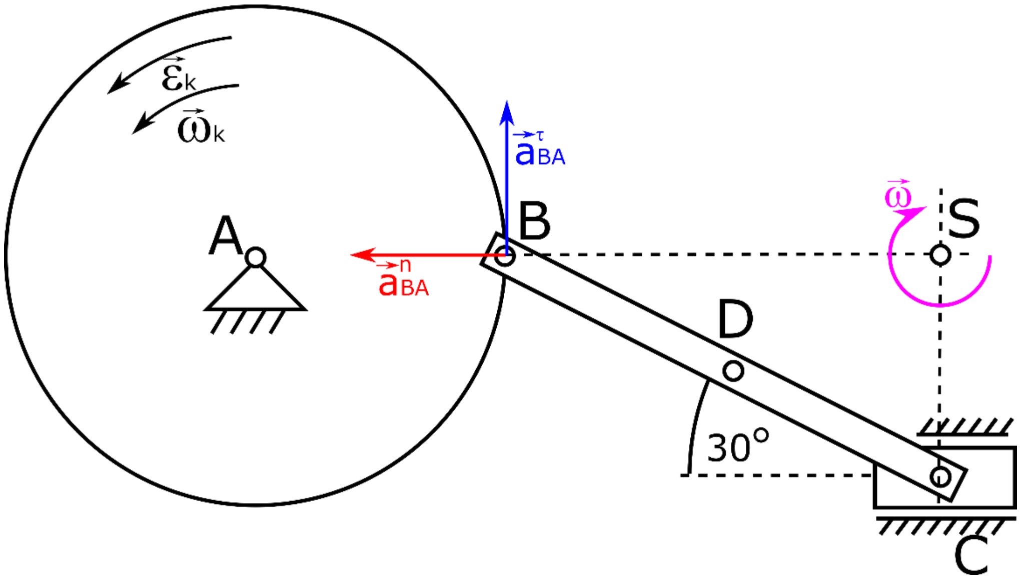


Next, we can move on to determining the accelerations. We'll start with point B. It's on a circle whose center is stationary, so we'll calculate the acceleration of point B about the center of the circle.

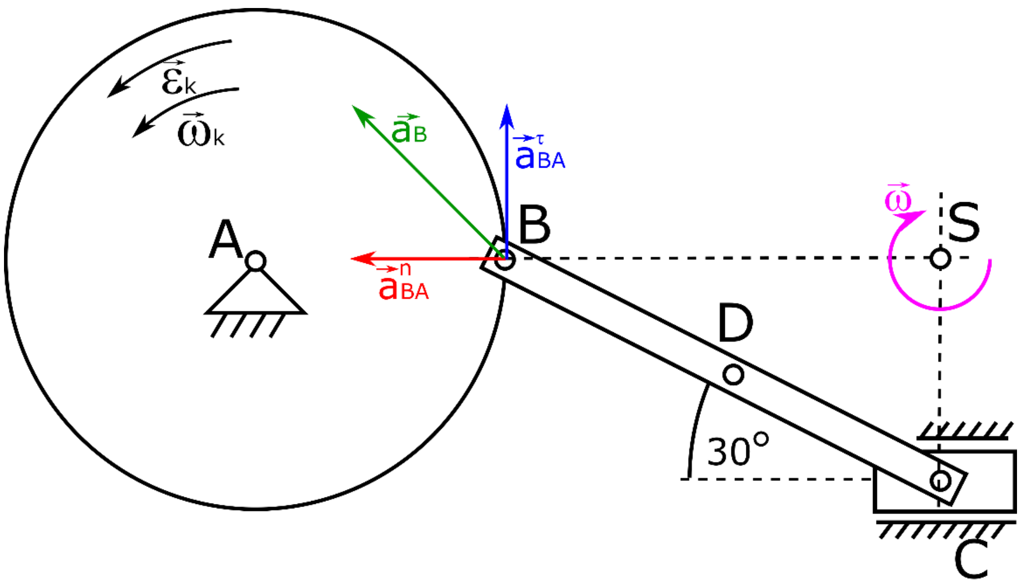
$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^n + \vec{a}_{BA}^\tau$$

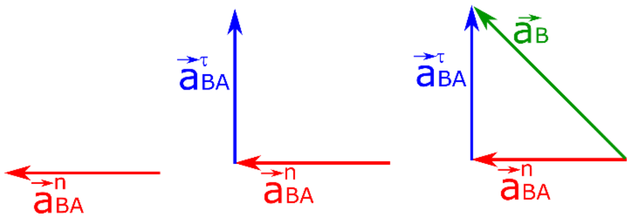
$$|\vec{a}_{BA}^n| = \omega_k^2 * AB = 2$$

$$|\vec{a}_{BA}^\tau| = \varepsilon_k * AB = 2$$



$$|\vec{a}_B| = \sqrt{{\vec{a}^{n}_{BA}}^{2} + {\vec{a}^{\tau}_{BA}}^{2}} = \sqrt{4+4} = 2\sqrt{2}$$



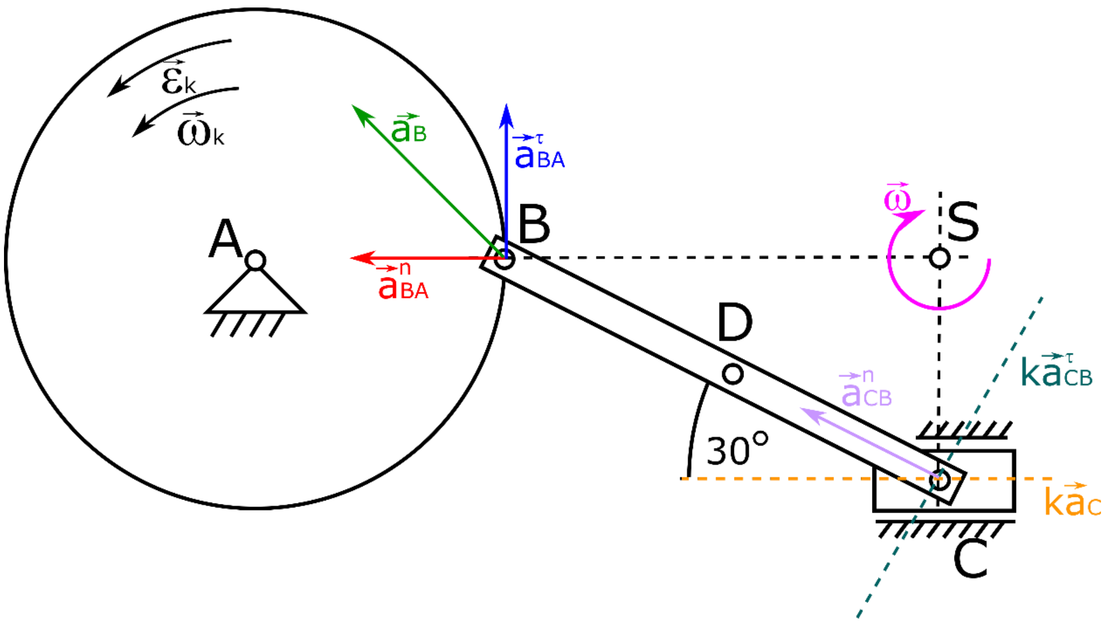


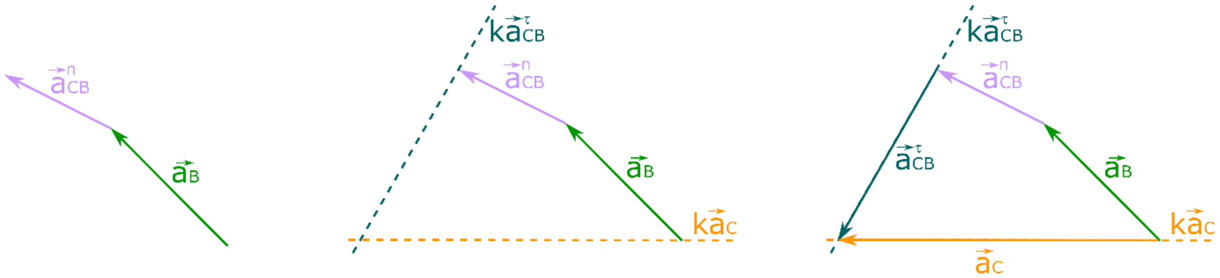
Then we calculate the acceleration for the remaining points. First choose from which point C or D we start. You can see that, as in the case of velocity, also here point C can only move horizontally, so its acceleration will be in this direction.

$$\vec{a}_C = \vec{a}_B + \vec{a}_{CB}^n + \vec{a}_{CB}^\tau$$

$$|\vec{a}_{CB}^n| = \omega^2 * BC = \left(\frac{\sqrt{3}}{3}\right)^2 * 4 = \frac{4}{3}$$

After plotting the normal acceleration of point C in relation to point B and the direction of tangential acceleration of point C in relation to point B, we can create a chain of accelerations as shown in the drawing.





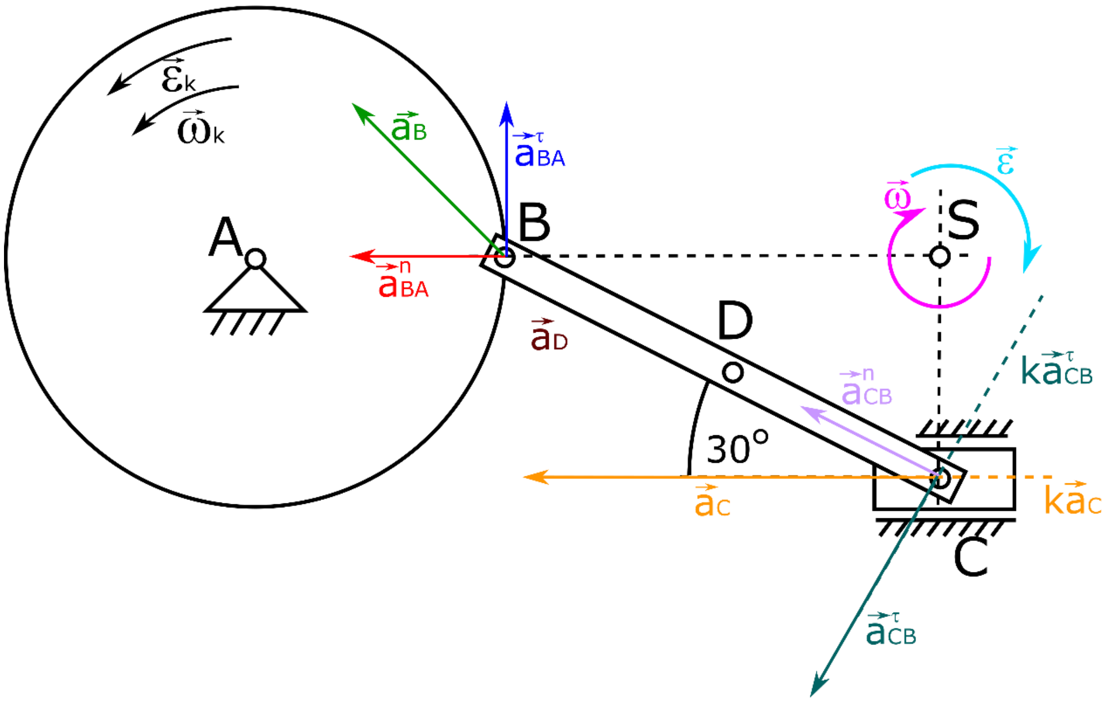
In the next step, we project the accelerations on the X and Y axes and in this way we are able to determine the acceleration of point C and the tangential acceleration of point C with respect to B. By determining the tangential acceleration, we can find the value of the angular acceleration.

$$\begin{cases} -a_C = -a_B \cos 45° - a_{CB}^n \cos 30° - a_{CB}^\tau \cos 60° \\ 0 = a_B \sin 45° + a_{CB}^n \sin 30° - a_{CB}^\tau sin60° \end{cases}$$

$$a_{CB}^\tau = \frac{a_B \sin 45° + a_{CB}^n \sin 30°}{sin60°} = \frac{2 + \frac{2}{3}}{\frac{\sqrt{3}}{2}} = \frac{16\sqrt{3}}{9}$$

$$a_{CB}^\tau = \varepsilon * CB \Rightarrow \varepsilon = \frac{a_{CB}^\tau}{CB} = \frac{\frac{16\sqrt{3}}{9}}{4} = \frac{4\sqrt{3}}{9}$$

$$a_C = \frac{18 + 14\sqrt{3}}{9}$$

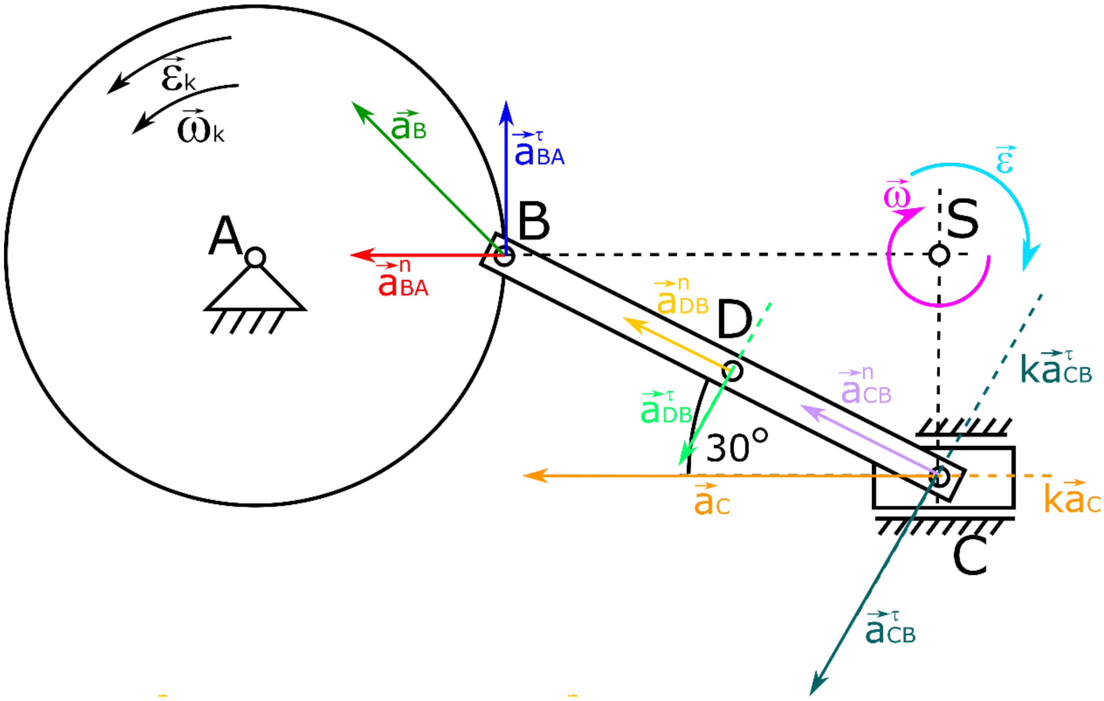


Having information about the angular velocity and the angular acceleration, we can calculate the acceleration of point D. We do not know the direction of this acceleration, but we will know the values and directions of the components of this acceleration.

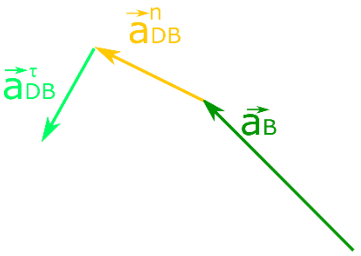
$$\vec{a}_D = \vec{a}_B + \vec{a}^n_{DB} + \vec{a}^\tau_{DB}$$

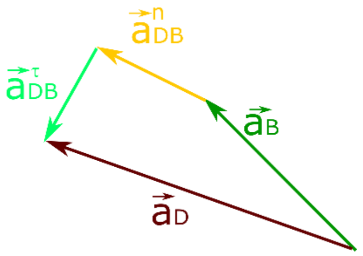
$$|\vec{a}^n_{DB}| = \omega^2 * DB = \left(\frac{\sqrt{3}}{3}\right)^2 * 2 = \frac{2}{3}$$

$$|\vec{a}^\tau_{DB}| = \varepsilon * DB = \frac{4\sqrt{3}}{9} * 2 = \frac{8\sqrt{3}}{9}$$



After plotting these accelerations on the drawing, we can create a chain of accelerations, just like in the case of point C.





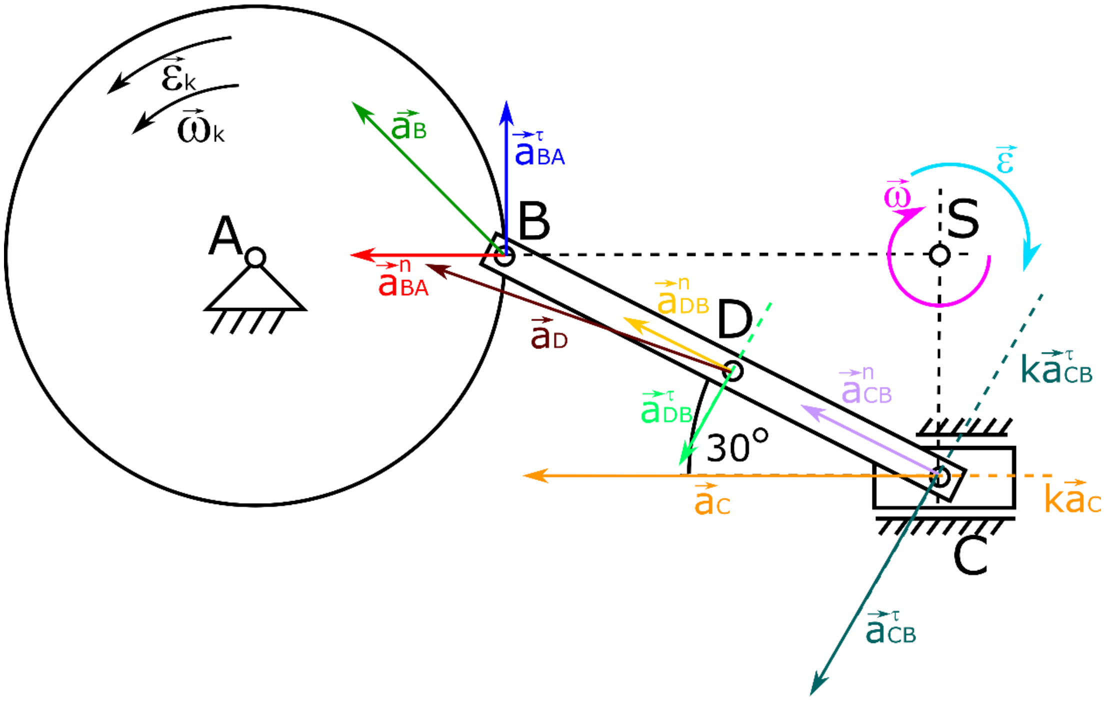
In the next step, we project the accelerations on the X and Y axes and in this way we are able to determine the acceleration of point D. It is true that we first determine the values of its components on the X and Y axes, but determining the final value of acceleration D is then a formality.

$$\begin{cases} -a_{Dx} = -a_B \cos 45° - a_{DB}^n \cos 30° - a_{DB}^\tau \cos 60° \\ a_{Dy} = a_B \sin 45° + a_{DB}^n \sin 30° - a_{DB}^\tau sin 60° \end{cases}$$

$$a_{Dx} = 2\sqrt{2} * \frac{\sqrt{2}}{2} + \frac{2}{3} * \frac{\sqrt{3}}{2} + \frac{8\sqrt{3}}{9} * \frac{1}{2} = \frac{18 + 7\sqrt{2}}{9}$$

$$a_{Dy} = 2\sqrt{2} * \frac{\sqrt{2}}{2} + \frac{2}{3} * \frac{1}{2} - \frac{8\sqrt{3}}{9} * \frac{\sqrt{3}}{2} = 1$$

$$a_D = \sqrt{{a_{Dx}}^2 + {a_{Dy}}^2} = 3{,}49$$



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