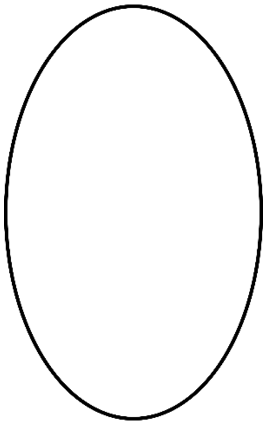
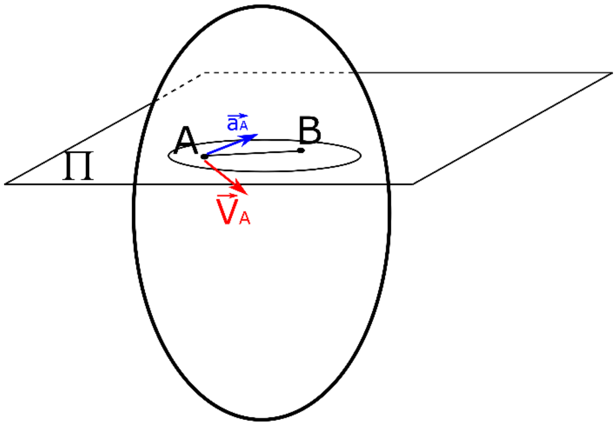
Planar movement of rigid body

A planar motion, is a motion in which all points of the body move in planes parallel to a certain plane called the plane of the planar (directing) motion.

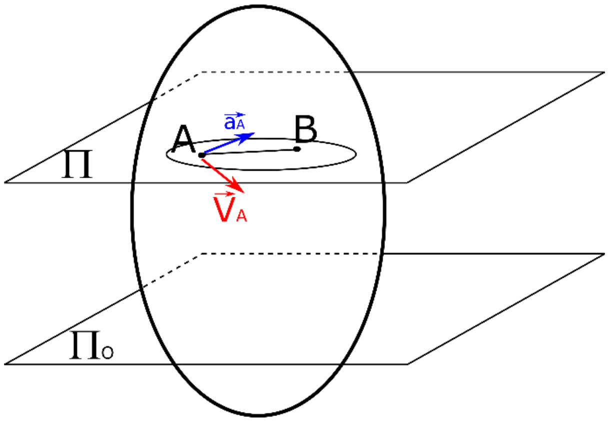
Let's imagine ourselves, a rigid body in space



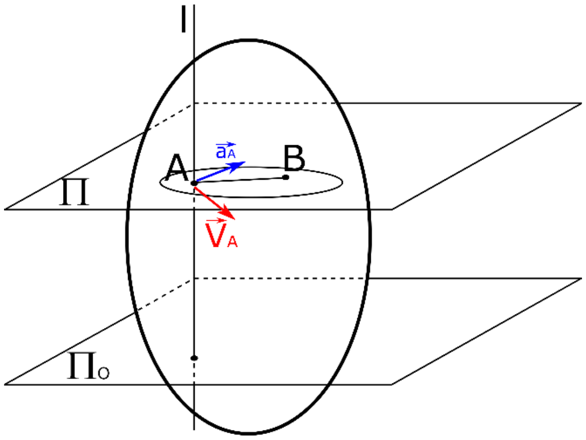
Let’s cut our body with the plane π, on which we will place points A and B. The distance between the points of this body is constant during the movement. Suppose we know the velocity and acceleration of point A, as shown in the figure.



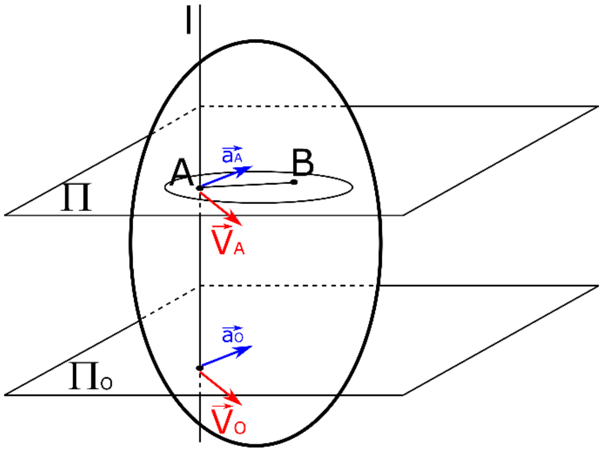
Now let's put another plane into our system, it will be the π_O plane, which will be parallel to the πplane and will pass through our body in a different place



Now, if we pass through the point A perpendicular to the plane π, we will see that the same point appears on the plane π_O.

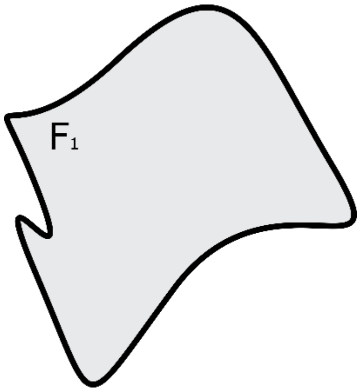


Since the body is rigid, it means that the point created on the π_O plane will have the same velocity and acceleration as point A.

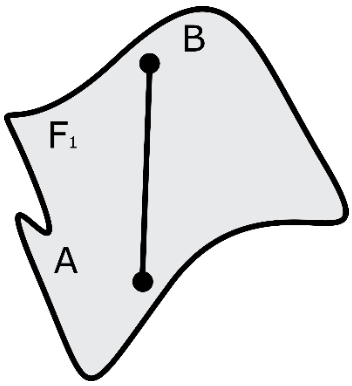


Summarizing, it can be stated that the movements in planes π and π_O are identical. In general, it can be said that the analysis of the plane motion of a given rigid body focus on studying the motion of one body section. In this way, for one plane we can define the movement of all other points of this body.

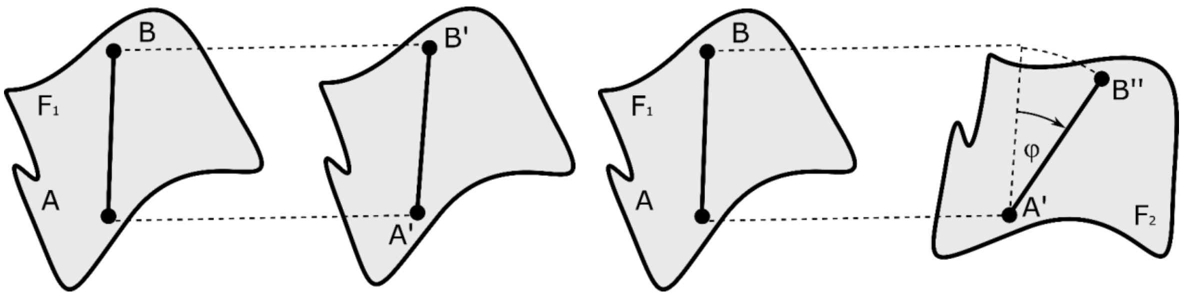
Let's consider how motion will take place on the plane. Suppose we have a flat figure F.



Now we will introduce segment AB in this figure, which will represent the movement of the entire figure.

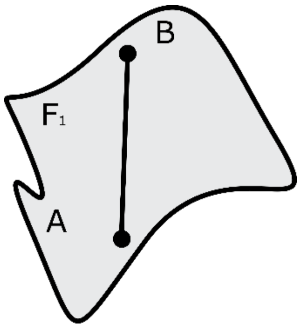


A plane figure can be moved in its plane from any position with one parallel displacement and one rotation (rotation by the angle φ).

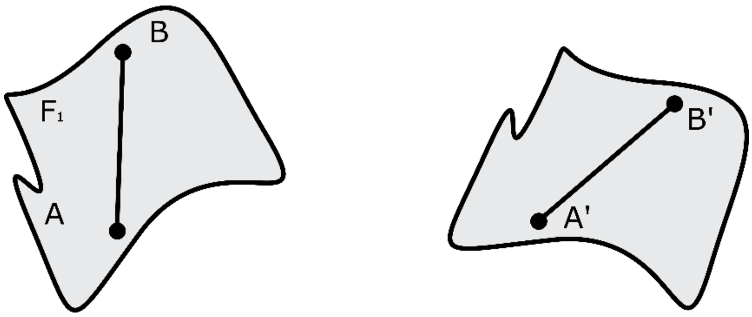


The change in the position of a plane figure in its plane can also be described by the Euclidean theorem.

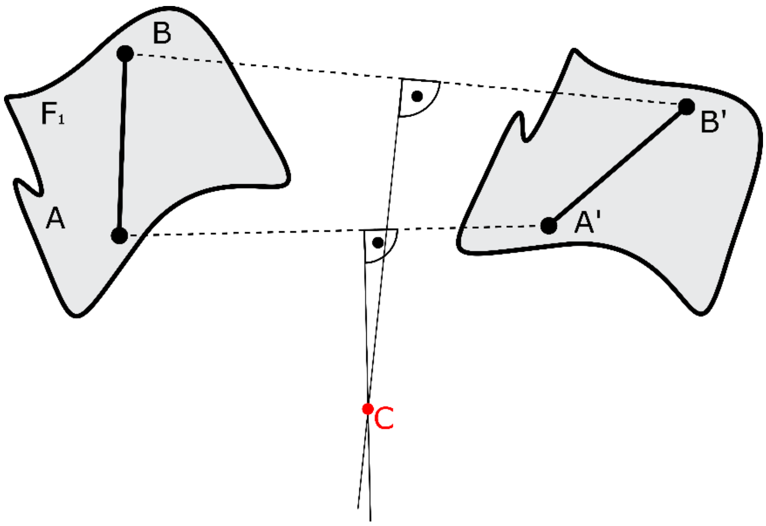
Suppose that, as before, we have a plane figure represented by the segment AB.



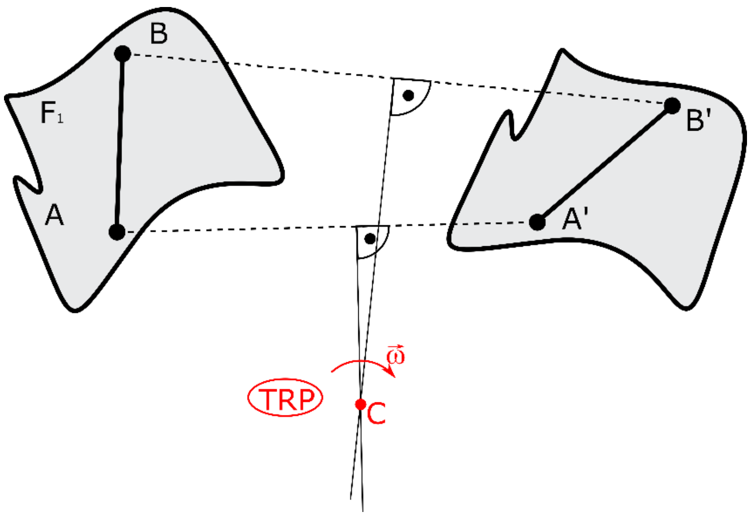
Now let's move this figure from its original position to some other position.



If we now connect the corresponding points with straight lines, and then draw the lines perpendicular to the previous segments, we will notice that the lines will intersect at one point. Let's call this point C.



The determined point C will be called the instantaneous center of rotation (or temporary rotation point (TRP)).



This point is determined by the aforementioned Euclid theorem, which says that any displacement of a plane figure in its plane can be achieved by rotation around a certain point called CENTER OF ROTATION.

Movement around each center of rotation is infinitely short, and therefore these points are called instantaneous centers of rotation.

Velocities in planar motion

In plane motion, we will describe the body using three parameters:

$$X_A = X_A(t);\ \ Y_A = Y_A(t) - parameters\ describing\ translation$$

$$\varphi = \varphi(t) - parameter\ describing\ rotation$$

Plane motion consists of instantaneous translational motion and instantaneous rotary motion

The position of the movable system in relation to the fixed system can be determined by the pole A and the angle of rotation f. The movement of the section of the body along the directing plane will be fully determined by the following equations.

$$X_A = X_A(t)$$

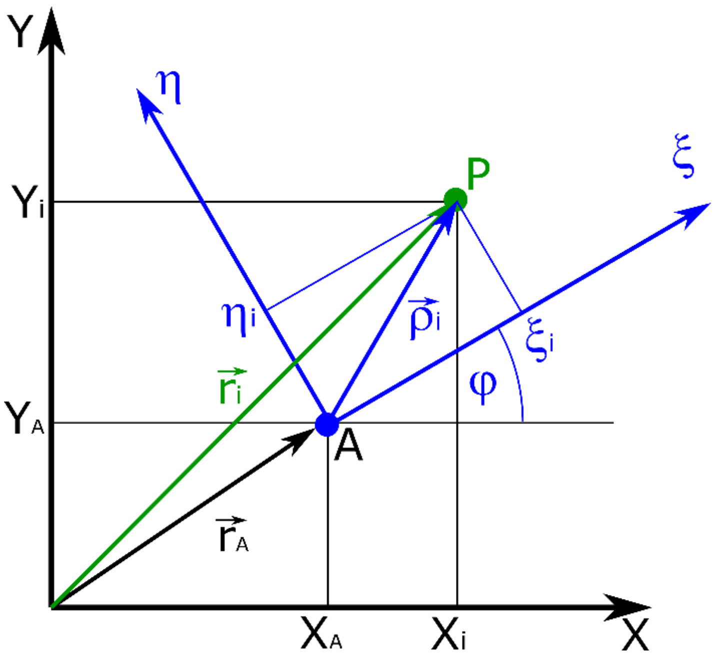
$$Y_A = Y_A(t)$$

$$\varphi = \varphi(t)$$

These are the equations of planar motion

$\vec{r}_i$- position of the P point in the X, Y stationary system

$\vec{\rho}_i$- Location of the point P in a moving system η, ξ



The position of any cross-section point can be written by the following equation.

$$\vec{r}_i = \vec{r}_A + \vec{\rho}_i$$

By transforming the equation, we can obtain information about the coordinates of the position of any point

$$\begin{cases} x_i = x_A + \xi_i \cos\varphi - \eta_i \sin\varphi \\ y_i = Y_A + \xi_i \sin\varphi + \eta_i \cos\varphi \end{cases}$$

By differentiating the displacement vector $\vec{r}_i$ over time, we get

$$\frac{\overrightarrow{dr_i}}{dt} = \frac{\overrightarrow{dr_A}}{dt} + \frac{\overrightarrow{d\rho_i}}{dt}$$

where

$\frac{\overrightarrow{dr_i}}{dt} = \vec{V}_i$- velocity of any section point,

$\frac{\overrightarrow{dr_A}}{dt} = \vec{V}_A$ - velocity of pole A (origin of the moving coordinate system),

While moving, the vector ri changes only direction , and its length remains constant. Its derivative thus represents a rotation around the A pole

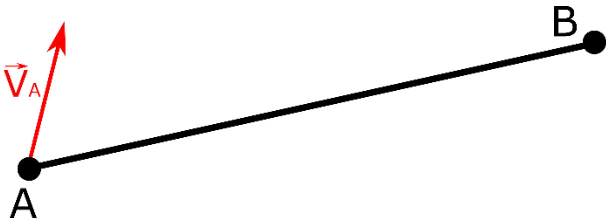
$$\frac{\overrightarrow{d\rho_i}}{dt} = \dot{\vec{\rho}}_i = \overrightarrow{\omega} \times \vec{\rho}_i$$

Ultimately, the velocity of any point in a plane motion can be written as follows.

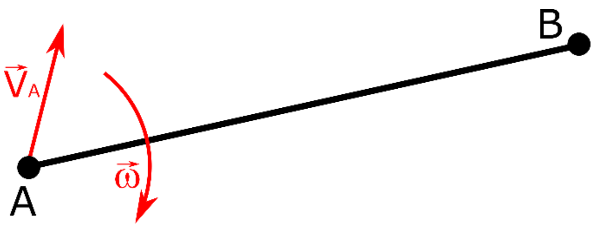
$$\vec{V}_i = \vec{V}_A + \overrightarrow{\omega} \times \vec{\rho}_i$$

As can be seen, this velocity is the sum of translational and rotary motion.

Assume we have a rigid body as shown in the figure. We know the velocity at point A and we want to find the velocity at point B.

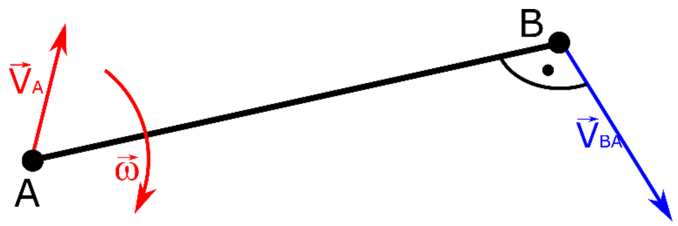


In addition, we know the angular velocity ω at which the body rotates around point A.



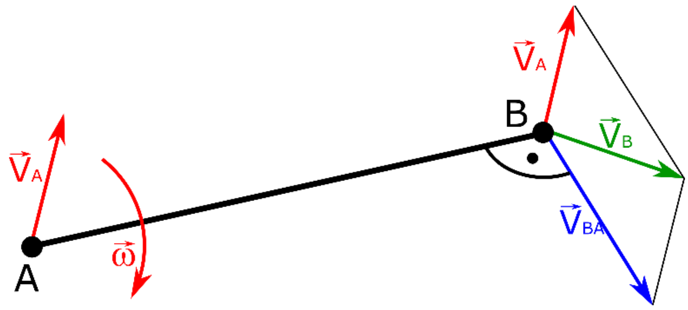
$$\vec{V}_B = \vec{V}_A + \overrightarrow{\omega} \times \overline{AB}$$

In order to find the velocity of point B, according to the above equation, we need to know the velocity of a point on this body (in our case A) and we need to find the component from the rotation of our point B relative to the pole for which we know the velocity, i.e. point A. So the velocity $\vec{V}_{BA}$.



We will refer to this method of calculating plane velocity as the superposition method

$$\vec{V}_B = \vec{V}_A + \vec{V}_{BA}$$



starting from the equation

$$\vec{V}_i = \vec{V}_A + \overrightarrow{\omega} \times \vec{\rho}_i$$

$$\vec{V}_i = \vec{V}_A + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ (x_i - x_A) & (y_i - y_A) & (z_i - z_A) \end{vmatrix} = \vec{V}_A + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega_z \\ (x_i - x_A) & (y_i - y_A) & 0 \end{vmatrix}$$
$$= \vec{V}_A + \vec{\omega}_j(x_i - x_A) + \vec{\omega}_i(y_i - y_A)$$

$$\vec{V}_i = V_{Ax}\hat{\imath} + V_{Ay}\hat{\jmath} + \hat{\imath}\big(-\omega(y_i - y_A)\big) + \hat{\jmath}\big(\omega(x_i - x_A)\big)$$

$$\begin{cases} V_{ix} = \dot{x}_A - \omega(y_i - y_A) \\ V_{iy} = \dot{y}_A - \omega(y_i - y_A) \end{cases}$$

the above equations are the velocity components in the x and y directions of a stationary system

The next step is to determine the velocity components for the η, ξ axises of moving system

$$\omega_\xi = 0;\ \omega_\xi = 0;\ \omega_\zeta = \omega$$

$$\rho_i = \xi_i\vec{\xi}_0 + \eta_i\vec{\eta}_0$$

$$\vec{V}_i = \vec{V}_A + \begin{vmatrix} \vec{\xi}_0 & \vec{\eta}_0 & \vec{\zeta}_0 \\ 0 & 0 & \omega \\ \xi_i & \eta_i & 0 \end{vmatrix} = V_{A\xi}\vec{\xi}_0 + V_{A\eta}\vec{\eta}_0 + \vec{\eta}_0\omega\xi_i - \vec{\xi}_0\omega\eta_i$$

$$\begin{cases} V_{i\xi} = V_{A\xi} - \ \omega\eta_i \\ V_{i\eta} = V_{A\eta} + \ \omega\xi_i \end{cases}$$

$$\begin{cases} V_{i\xi} = \dot{x}_A \cos\varphi + \dot{y}_A \sin\varphi - \omega\eta_i \\ V_{i\eta} = -\dot{x}_A \sin\varphi + \dot{y}_A \cos\varphi + \omega\xi_i \end{cases}$$

The above equations are the velocity components along the axis of the movable system

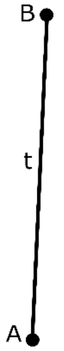
$$V_i = \sqrt{{V_{i\xi}}^2 + {V_{i\eta}}^2} = \sqrt{{V_{ix}}^2 + {V_{iy}}^2}$$

<u>Instantaneous center of rotation method (temporary rotation point method)</u>

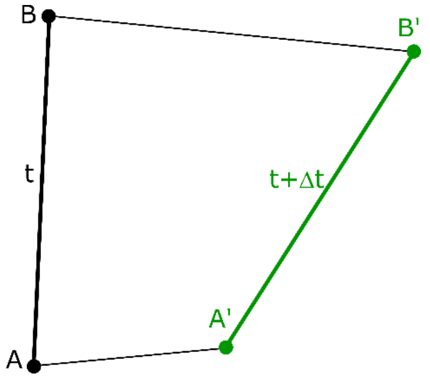
The second method, beside superposition method, allowing to determine the velocity of points in plane motion is the method of the temporary rotation point.

In the temporary rotation point method, we assume that the movement around this point is infinitely short, with a certain instantaneous angular velocity.

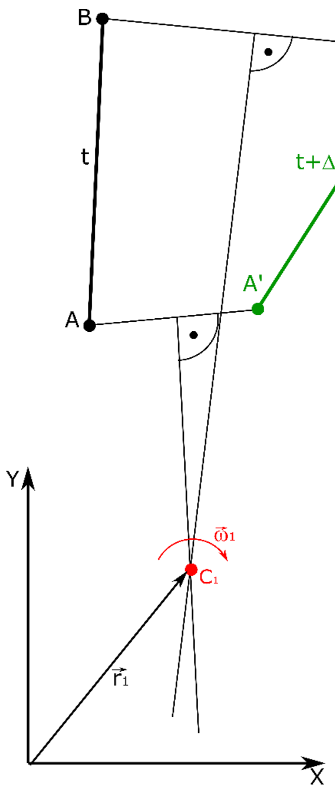
Suppose that, as before, we have a segment AB.



Now let's move this figure from its original position to some other position.

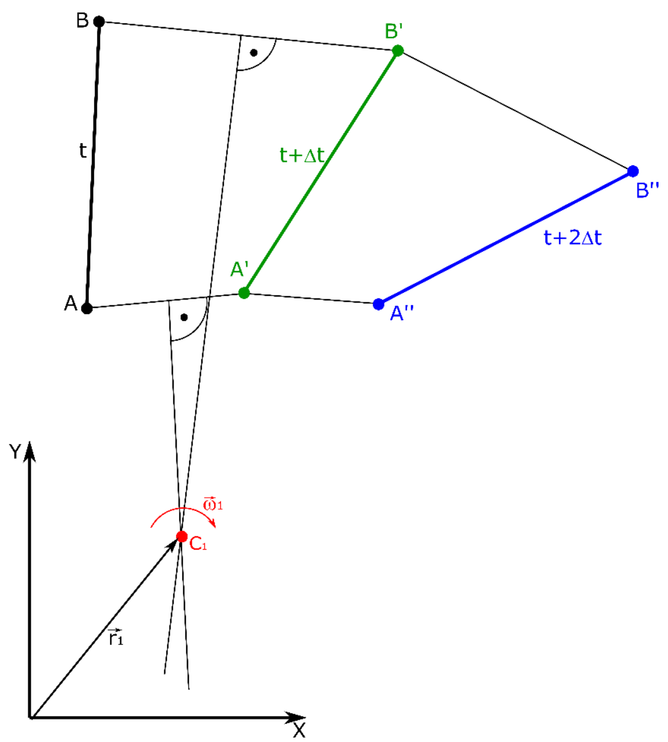


If we now connect the corresponding points with straight lines, and then draw the lines perpendicular to the previous segments, we will notice that the lines will intersect at one point. Let's call this point C_1.

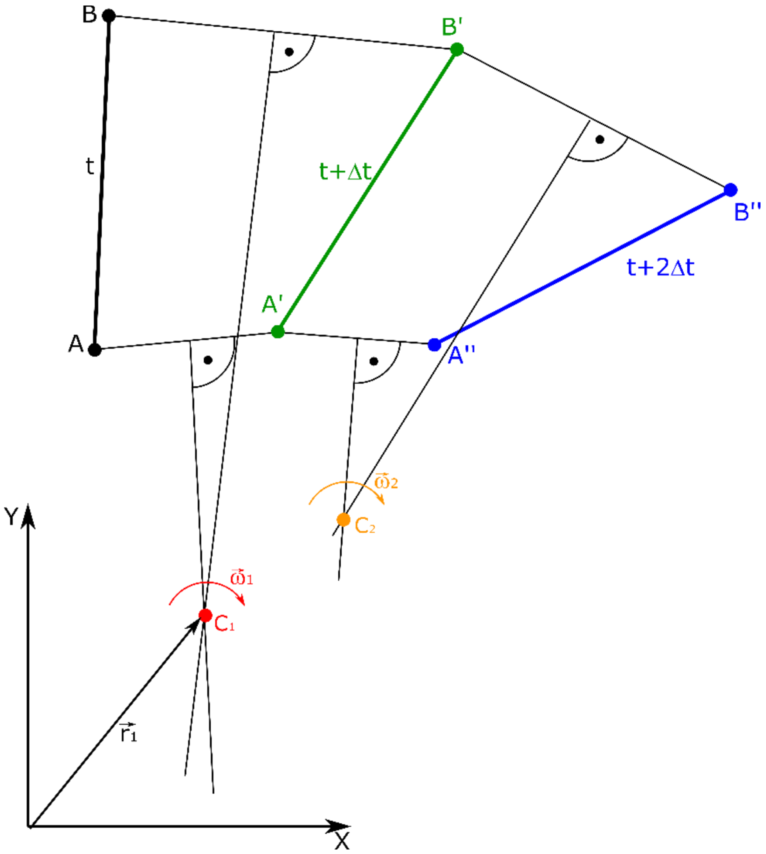


The determined point C_1 will be called the instantaneous center of rotation (or temporary rotation point (TRP)).

Let's move our object even further

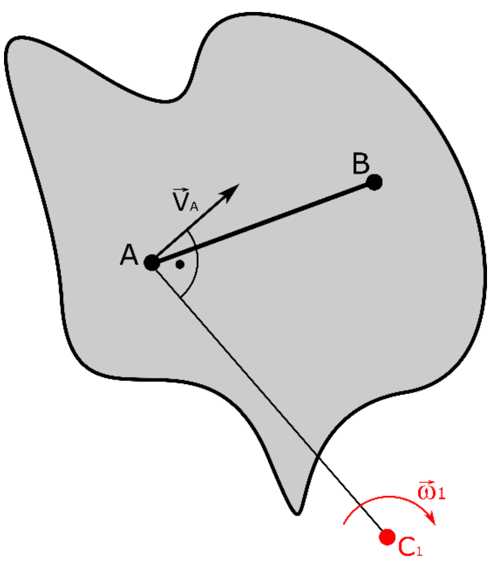


If we now connect once again the corresponding points with straight lines, and then draw the lines perpendicular, we will notice that the lines will intersect at one point. Let's call this point C_2.

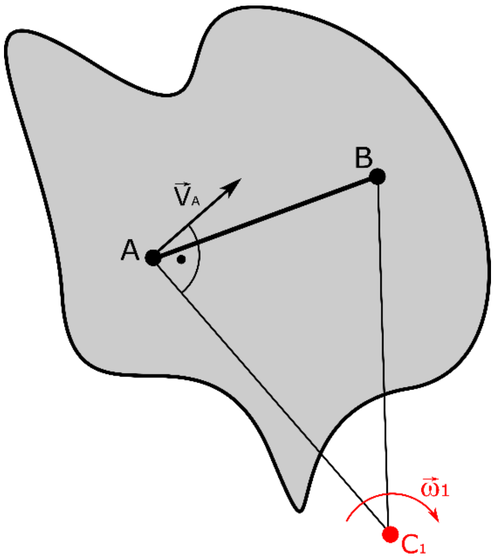


One can easily notice that as our figure moves, the position of the instantaneous center of rotation also changes. That's why we call this point instantaneous.

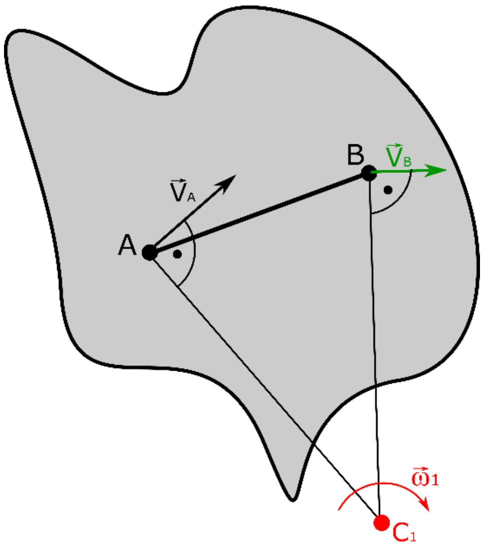
We can also reverse the situation. Suppose we know the location of the instantaneous center of rotation.



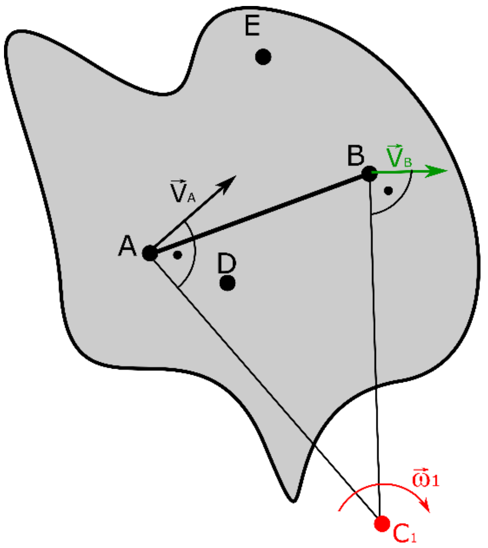
Knowing the location of this point, we can easily determine the velocities for other points of interest to us. For this purpose, a straight line should be drawn from the temporary rotation point to the point of interest.

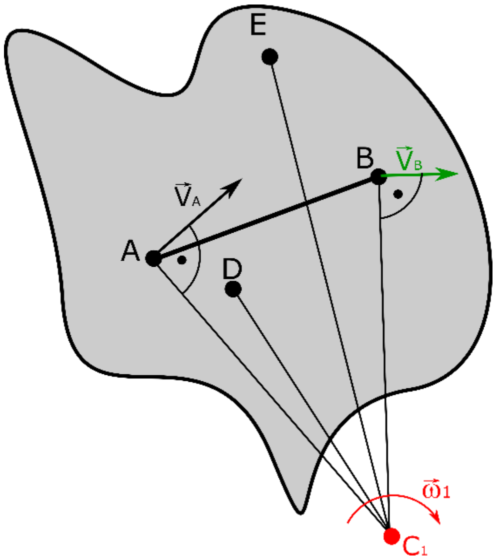


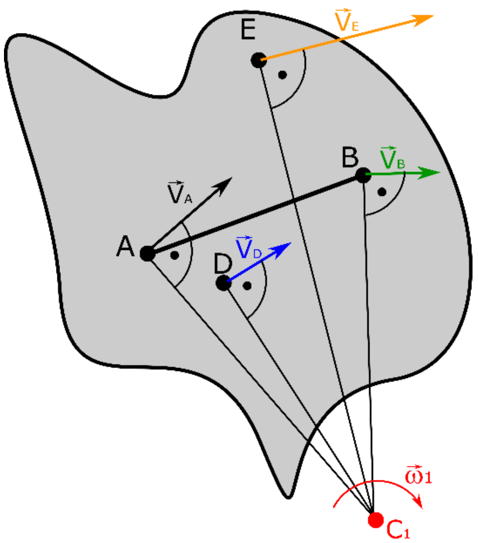
Then, in the direction perpendicular to the previously drawn section, we find the velocity of interest. The sense of velocity will be in the direction of rotation of the angular velocity.



We can do the same for other points of interest that belong to the same body.

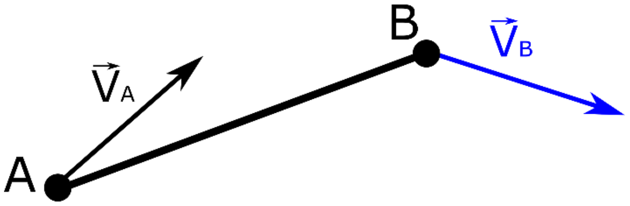




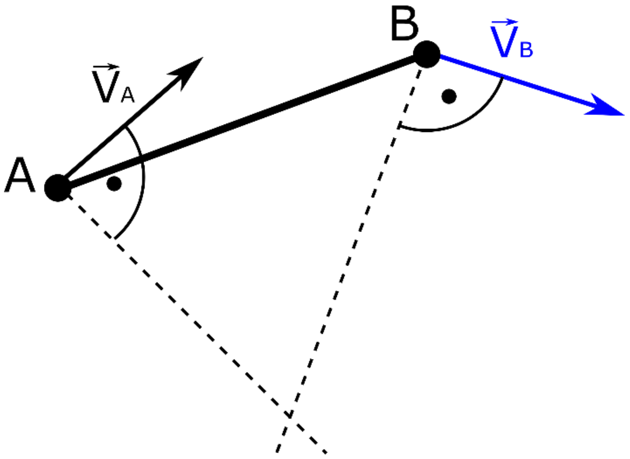


Ways of searching for the temporary rotation point, various cases.

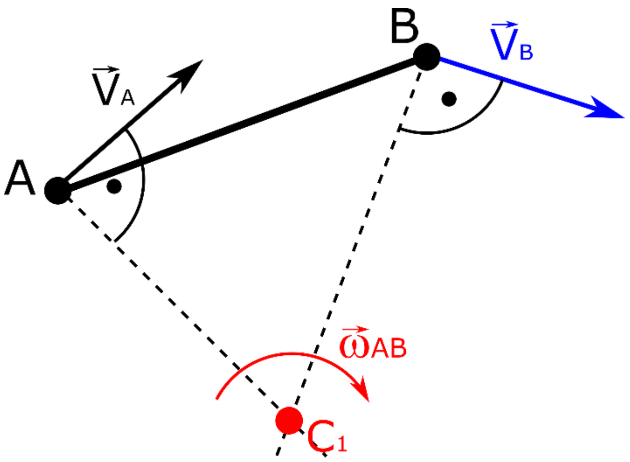
When we have two different velocities directions.



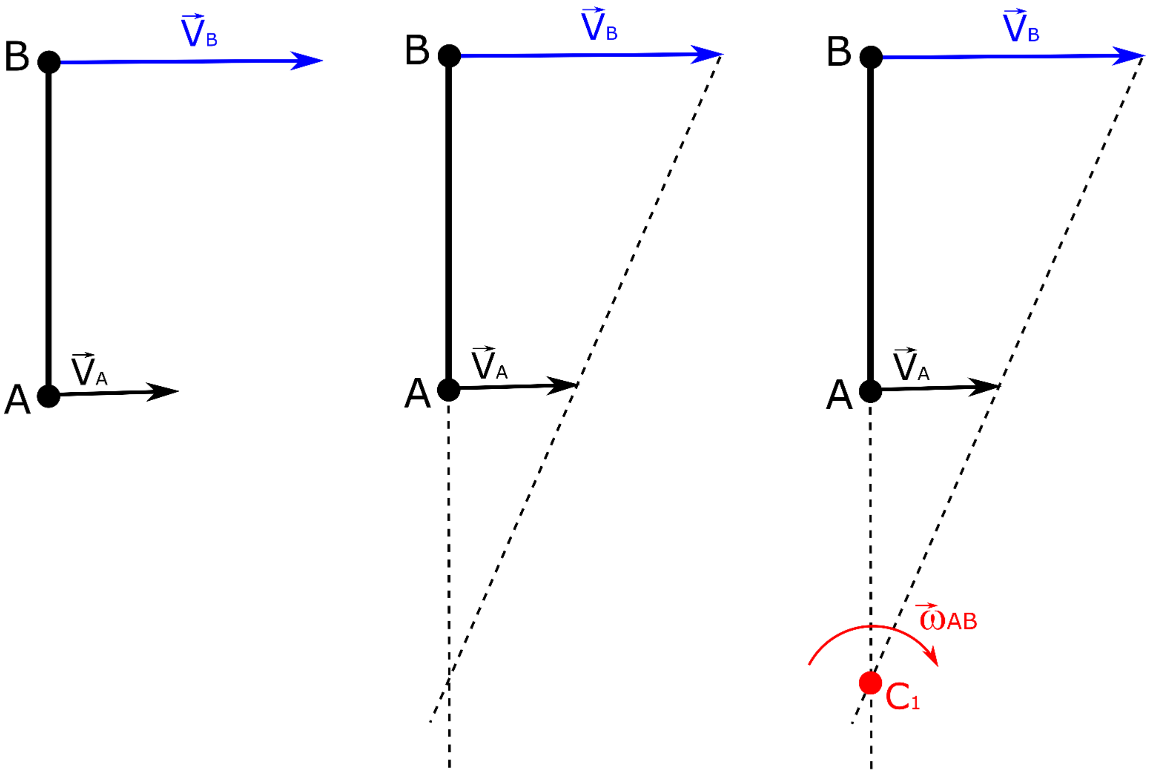
We draw lines perpendicular to the directions of velocities



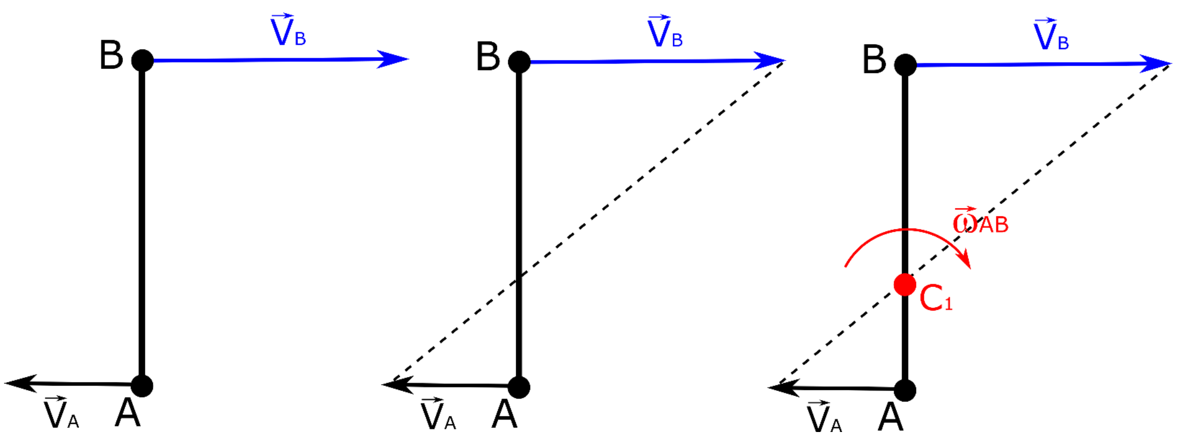
The instantaneous center of rotation will be at the intersection of the drawn lines



Velocities directions parallel to each other. The velocities have the same sense but different value.
We draw two lines connecting the beginnings of velocity vectors and their ends.
The instantaneous center of rotation will lie at the intersection of these two lines.



Velocities directions parallel to each other. The velocities have the different sense, might have same or different value. We draw two lines connecting the beginnings of velocity vectors and their ends.
The instantaneous center of rotation will lie at the intersection of these two lines.



Velocities directions parallel to each other. The velocities have the same sense and same value. We draw two lines connecting the beginnings of velocity vectors and their ends. As you can see, the drawn lines will not cross. This means that the instantaneous center of rotation is infinity and the angular velocity is zero. The body only makes a translational movement.