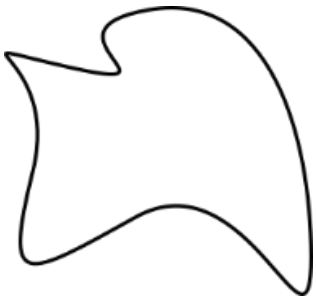
Theorem of three forces

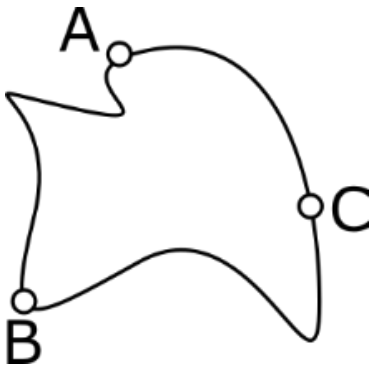
In this example, at first I will remind the theorem itself which is:

If rigid body is in balance (equilibrium) under influence of three forces on the same plane, which are unparallel to each other, then directions of these forces must crossing in one point and forces must create closed triangle.

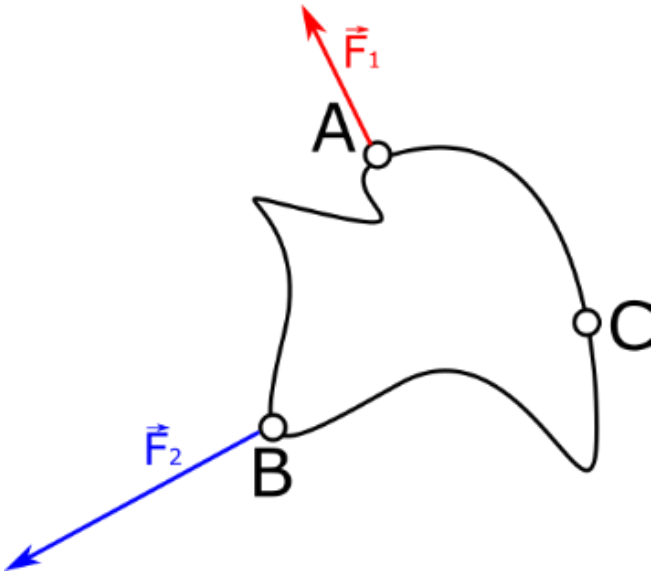
We have rigid body



We know that at point ABC some forces are acting.

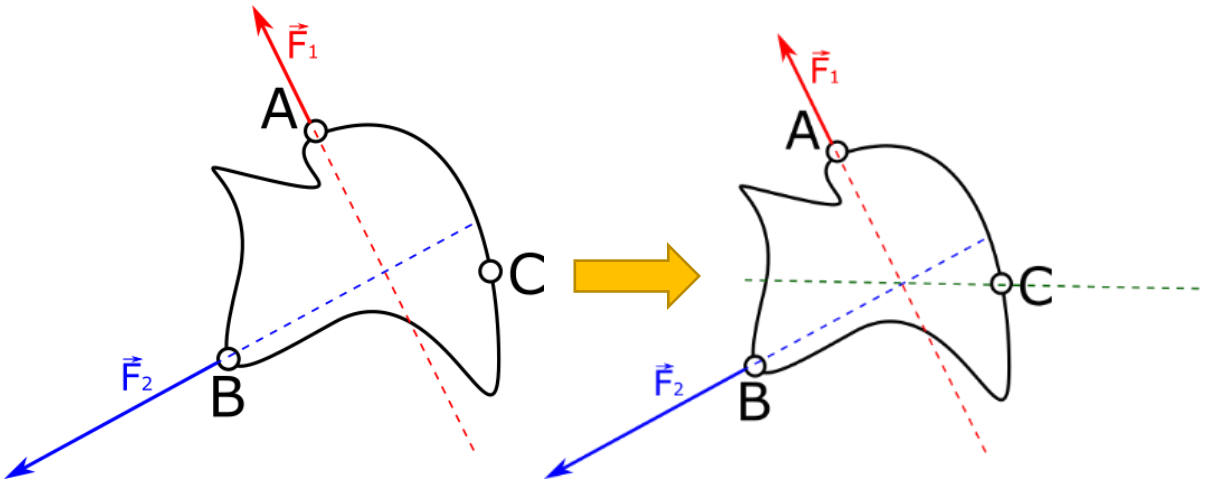


Let's assume that we know forces at points A and B.

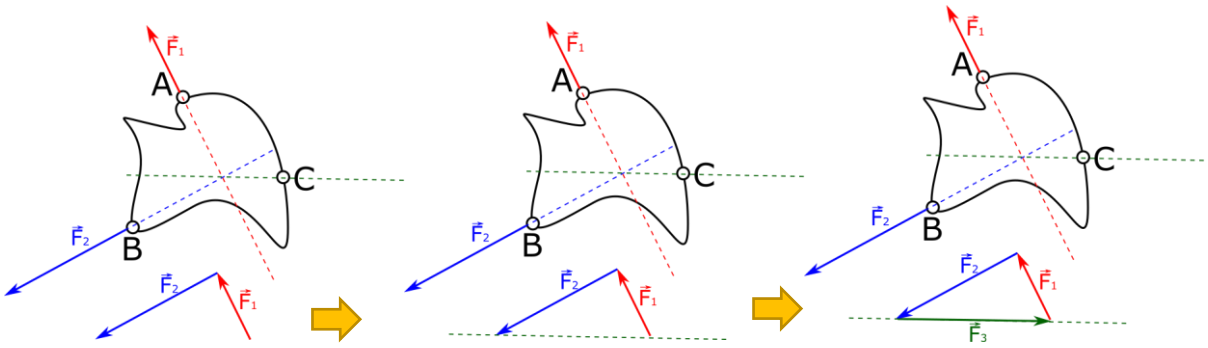


Based on above theorem we can determine force at point C.

First we will determine direction. The direction of force at point C must go through the crossing points of directions of forces F_1 and F_2.

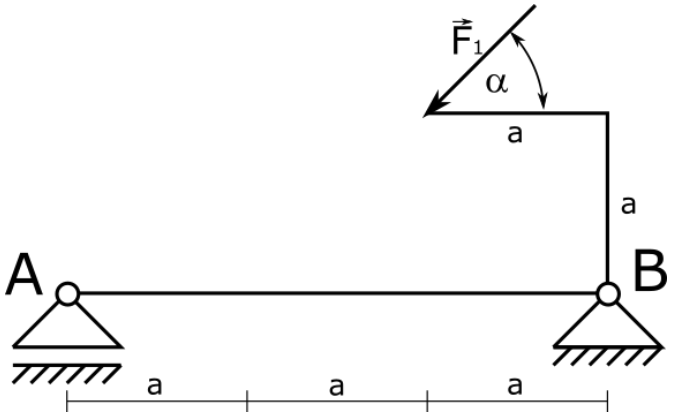


And next based one the triangle of forces we can determine value. In order to construct triangle of forces we need to draw two forces that we know, and then add the direction of third force, or if we know only one force we are adding direction of other two forces, in a way, that these directions are going through the beginning and end of known force.



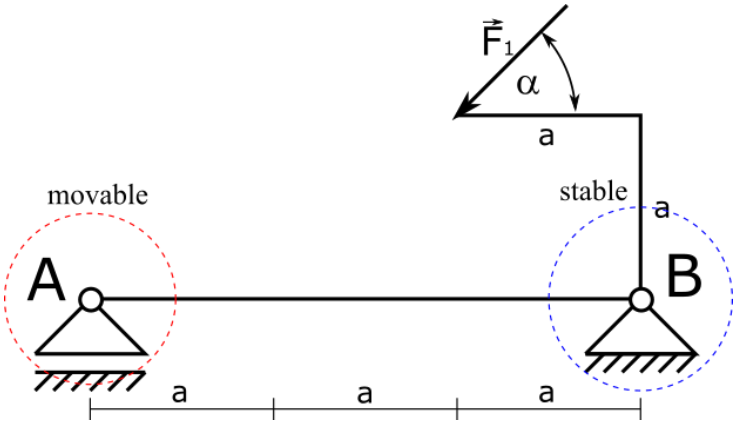
Due to the fact that our rigid body is in balance the triangle of forces must be closed. And in this way we know value and the sense of each force.

Ex1. Find the reactions in supports of frame given in the picture if force F_1= 10[kN], α=45° and a=a[m]. Use theorem of three forces

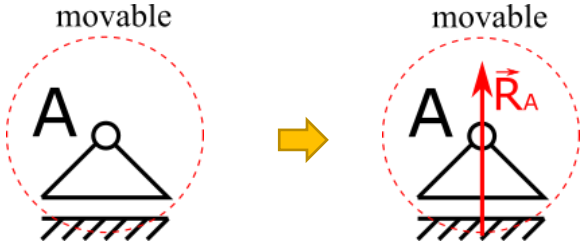


Solution

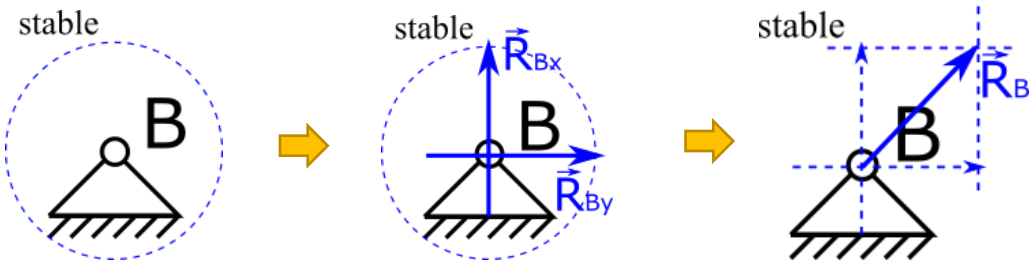
1. First of we need to think about what type of supports we have in this example. One can see that at point A there is movable support, and at point B there is stable support.



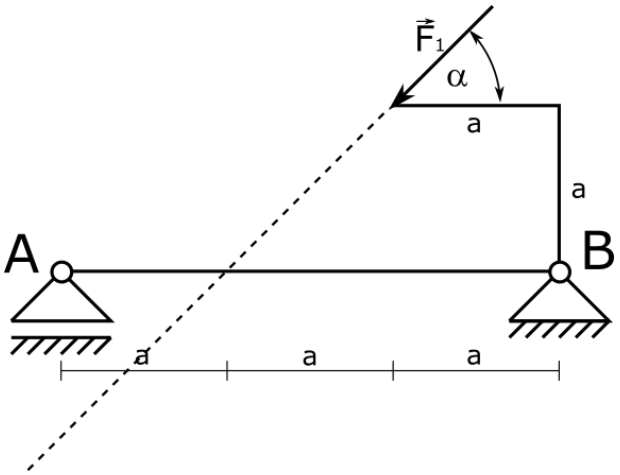
2. Now we need to remind ourselves how many reactions we can determine in each of these supports.
3. In movable support we have always one reaction which is perpendicular to the ground.



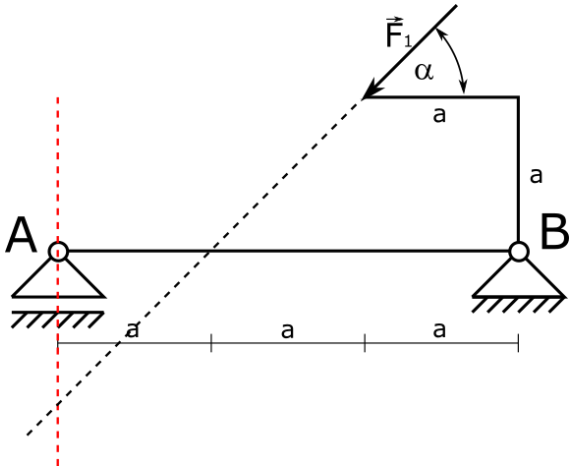
4. In stable support we can distinguish two components of reaction. One is on axis X and second on axis Y. But actually we have here also only one reaction which can be built from these two components.



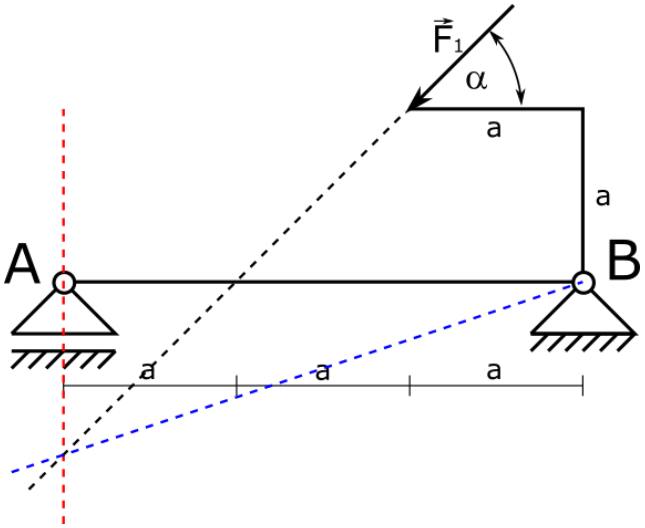
5. Based on the information above we can say that on this frame are acting three forces. F_1, reaction at point A R_A, and reaction at point B R_B.
6. We know everything only about force F_1. First let's draw the direction of this force.



7. Now going back to our reactions. We can only say something about reaction at point A. We know the direction of this force. Let's draw it.

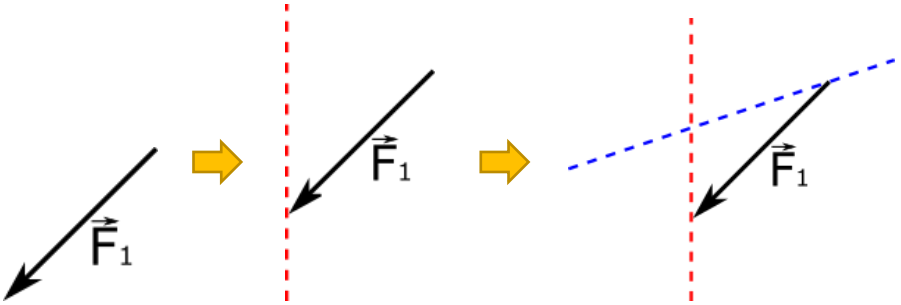


8. We just found the point where directions of two forces are crossing. Due to the fact that this body is in a balance it means that the direction of third force (reaction at point B), must going through this point as well. Let's draw it.

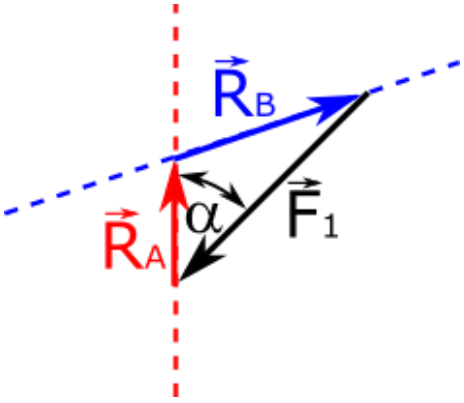


9. Now we know directions of all forces but we still need to find values. It can be done in two ways graphically and analytically. We will start with graphic way.

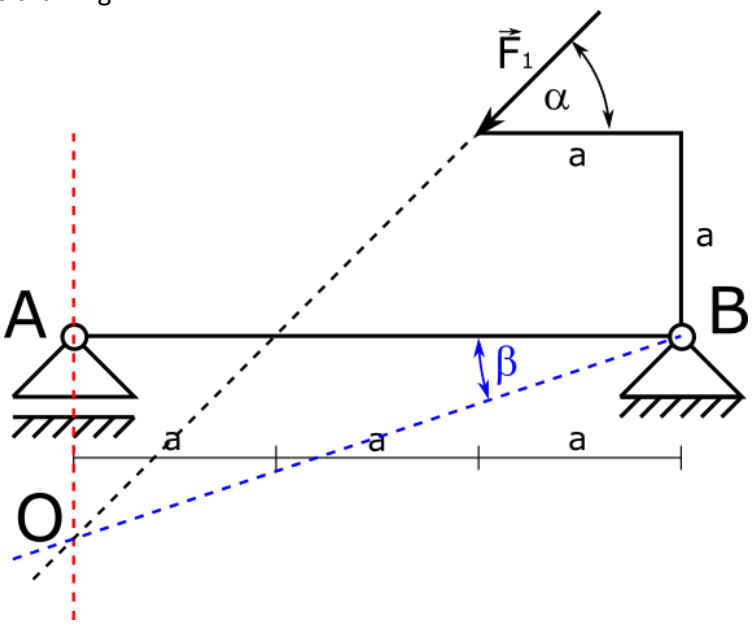
10. Because we know everything about force F1 we will start drawing triangle of forces from this force, and then we will add directions of reactions we are looking for.



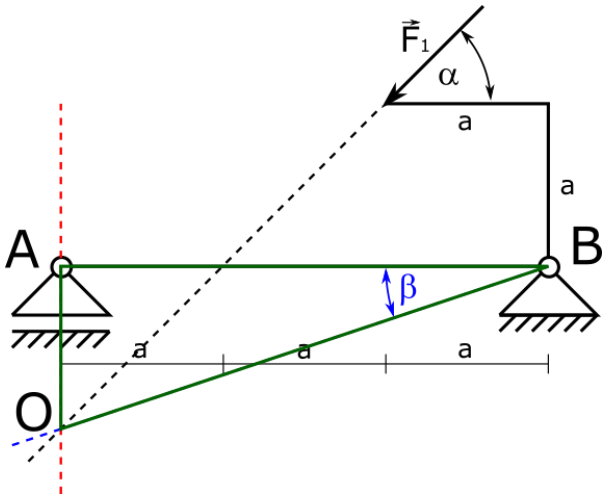
11. Based on obtained triangle we can establish values and senses of reactions. Because all forces must creating closed triangle of forces. It will look as follows.



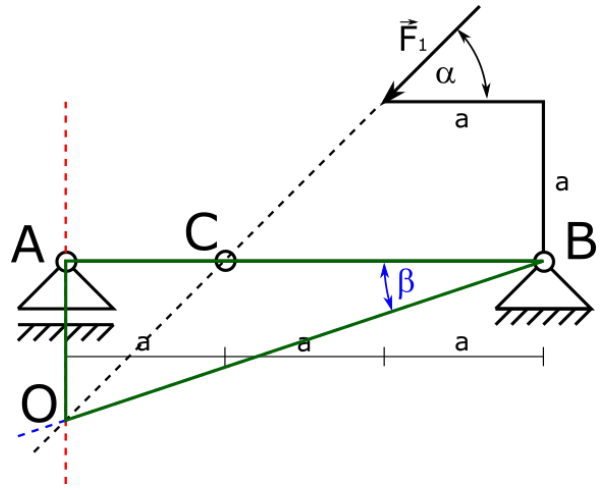
12. First we can "read" values of forces straight from the drawing, however, in order to do this it is necessary to introduce some scale.
13. Second we can calculate values based on this triangle, but unfortunately we have lack of information, because, we have only angle and one side of this triangle. We will need to find something more.
14. Let's introduce angle β between direction of reaction at point B and horizontal direction. As it is done in this drawing.



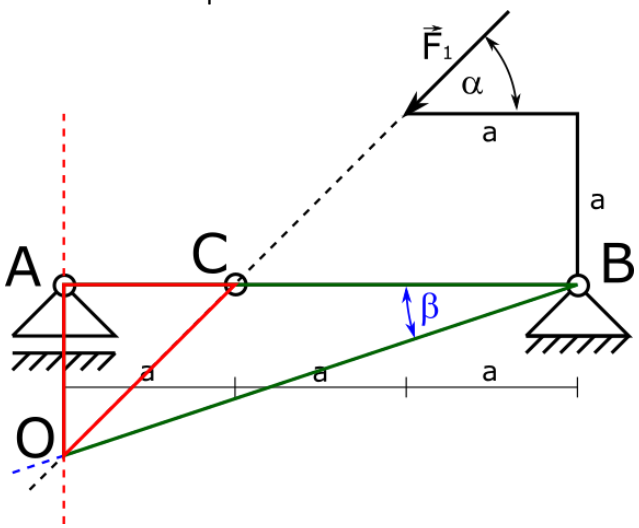
15. Now it will be useful to find this angle. One can find that we just create new triangle ABO.



16. We know that the distance AB is equal to 3a, it is hard to say something about distance OB, but we have some information about distance OA. We should remember that angle α is 45° it means that direction of force F_1 is going through the distance AB exactly between a and 2a, at point C.



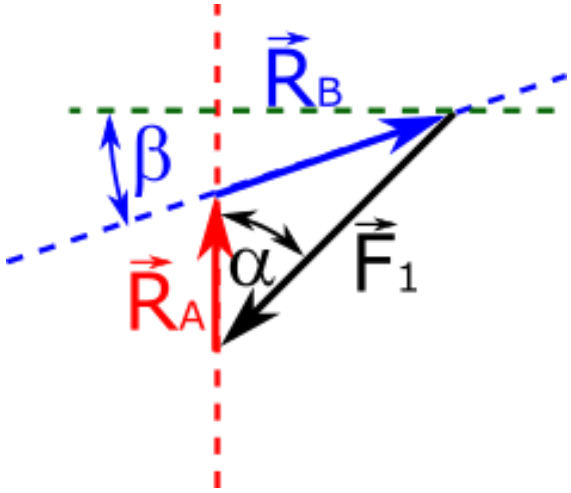
17. In this way we obtained a new triangle CAO, where sides AC and AO must be equal to each other. It means that distance AO is equal to a.



18. Based on information above we can back to the triangle ABO. We can see that:

$$\tan(\beta) = \frac{|AO|}{|AB|} = \frac{a}{3a} = \frac{1}{3}$$

19. When we know value of tan(β) we can calculate β, which is equal 18°26'. Based on this information we introduce this angle to our triangle.



20. Now we now three angles in our triangles and we can calculate values of reactions using theorem of sines.

$$\frac{F_1}{\sin(180^o - (\alpha + (45^o - \beta)))} = \frac{R_A}{\sin(45^o - \beta)} = \frac{R_B}{\sin\alpha}$$

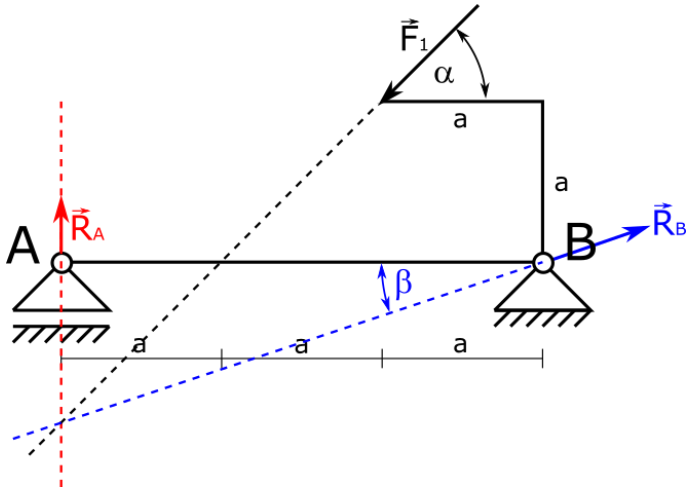
21. Based on this equation we can find that R_A = 4,71kN, and R_B = 7,47kN.

Now analytical way.

22. We can calculate values of reactions, based on equilibrium equations for planar convergent system.

$$\Sigma F_{ix} = 0$$
$$\Sigma F_{iy} = 0$$

23. Let's draw forces on our system, based on already obtained triangle of forces.



24. Now we can make projections of forces over both axis X and Y, as follows.

$\Sigma F_{ix} = 0 = -F_1\cos\alpha + R_B\cos\beta$

$\Sigma F_{iy} = 0 = -F_1\sin\alpha + R_A + R_B\sin\beta$

25. Now we just need to sole these equations.

$$R_B = F_1 \frac{\cos\alpha}{\cos\beta}$$
$$R_A = F_1 \sin\alpha - R_B \sin\beta$$

R_A = 4,71 kN

R_B = 7,47 kN