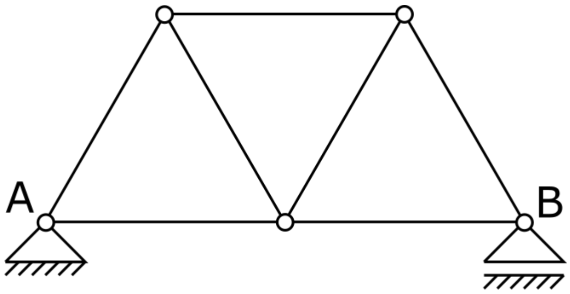
Trusses.

First of all it is necessary to define what we will describe as a truss.

For our purpose we will describe truss as: a system of rods, which ends are connected to each other with joints, and which have unchangeable geometrical form.

We can distinguish planar and spatial trusses, however, we will focus on planar trusses, where all rods are placed in one plane.

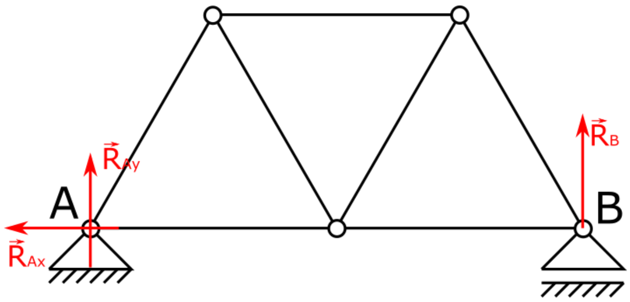
1. In the figure you can find example of a truss. Usually rods will be marked as a black lines and joints will be marked as a small circles.



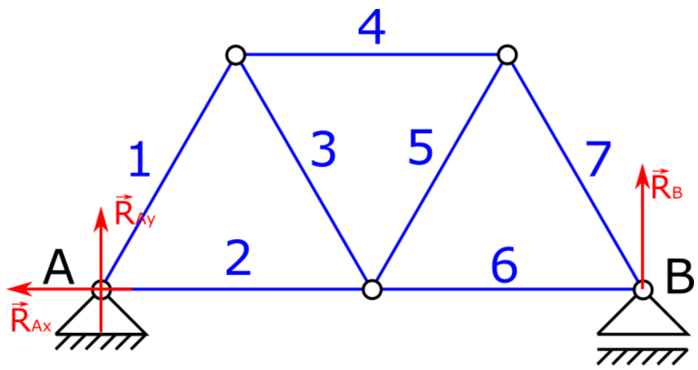
2. In this topic your task will be to solve the truss. What does it mean? It means that you need to find reactions in the supports, and forces which are in each of rods under the given load.
3. First of all we need to check if the truss is externally statically possible to solve. It means if the number of unknown forces in the supports (n) is lower or equal to the number of equilibrium equations (r) that you can write for the divergent planar system.

$$n = r \text{ , or } n \leq r$$

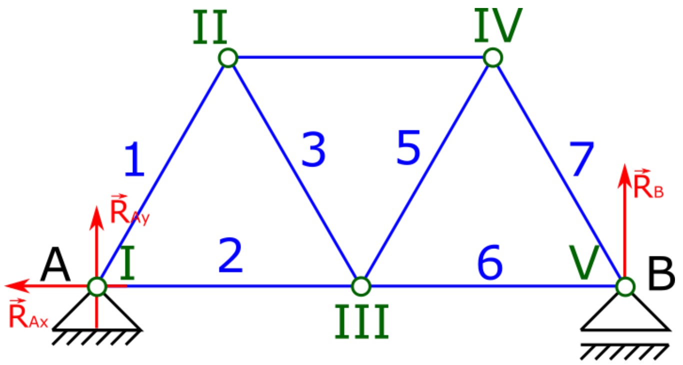
4. In this example you can see that there are 3 unknown reactions (n) in the supports, it means that it is externally, statically possible to solve.



5. Second thing that one need to check before solving truss is, if it is internally, statically possible to solve. How to do this?
   - first, you need to determine number of unknown reactions in the supports (we already know it),
   - second, you need to count number of rods in the truss (R) – in this truss we have 7 rods.



- third, you need to count number of joints (J) – in this truss we have 5 joints.



After these calculations you need to check if below equation is true (left side is equal to the right side):

$$R = 2 * J - n$$

R – number of rods,

J – number of joints,

n – number of unknown reactions in supports.

7=2*5-3

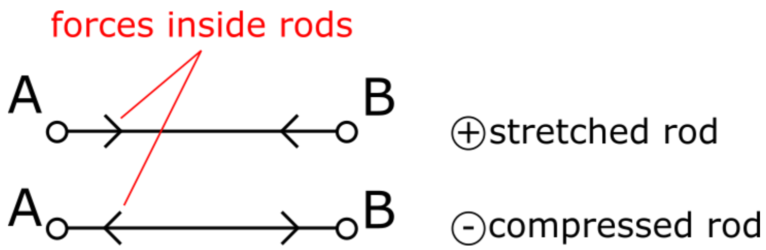
7=10-3

7=7

L=R

In this case we can see that this truss is internally, statically possible to solve.

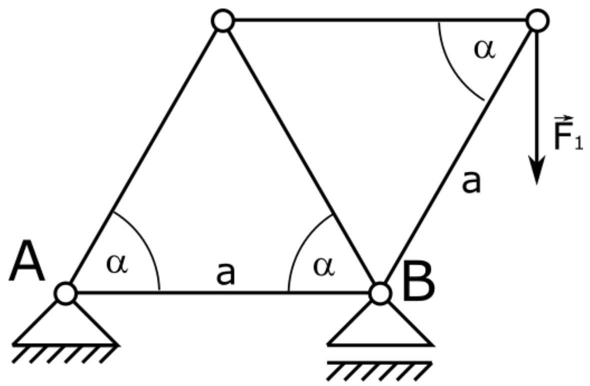
6. Next important thing which is connected with trusses is an assumption that forces in stretched rods we will be assuming as positive and in compressed rods as negative. Below you can see how we will mark these forces in rods.



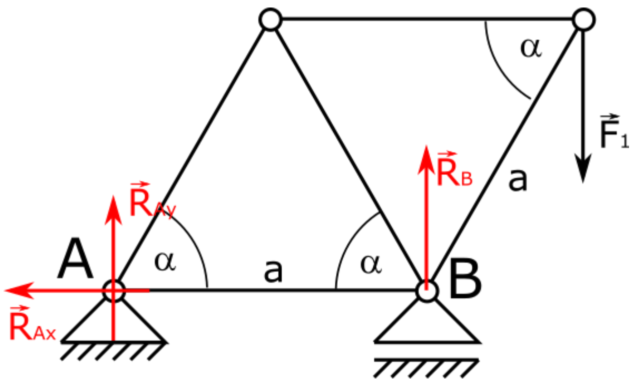
7. In this topic I will show you how you can solve trusses in three way. First two ways will analytical and third one will be graphical. Each method will be explained based on example.

Method of JOINTS

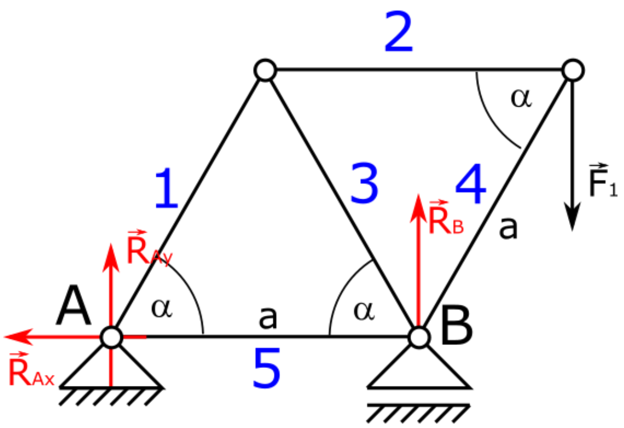
Example. Solve this truss with given data: F_1, α=60°, a.



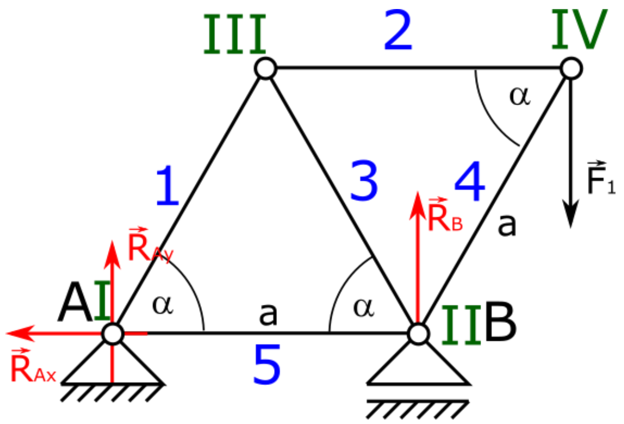
1. In order to start solving truss we will mark and count number of unknown reactions. In this example, we have 3 unknown reactions.



2. Second we will mark, and count number of rods. Here we have 5 rods.



3. Third we will mark, and count number of joints. This truss has 4 joints.



Now we can check if we are able to solve this truss

n=r

R=2*J-n

5=2*4-3

5=8-3

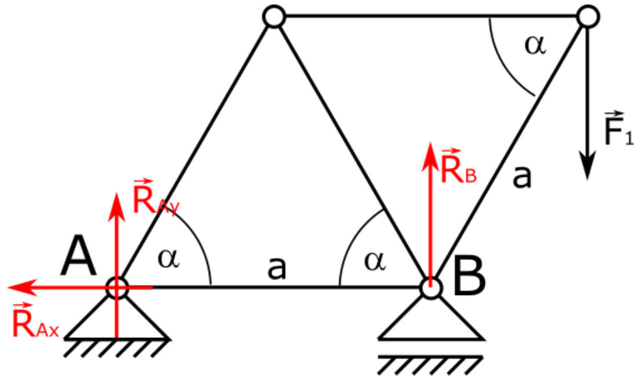
5=5

L=R

This truss is statically possible to solve.

First of all we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$

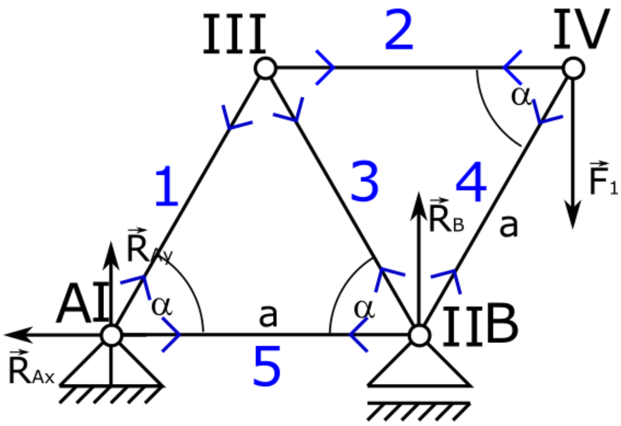


$$\sum_{i=1}^{n} F_{xi} = 0 = -R_{Ax} \rightarrow R_{Ax} = 0$$

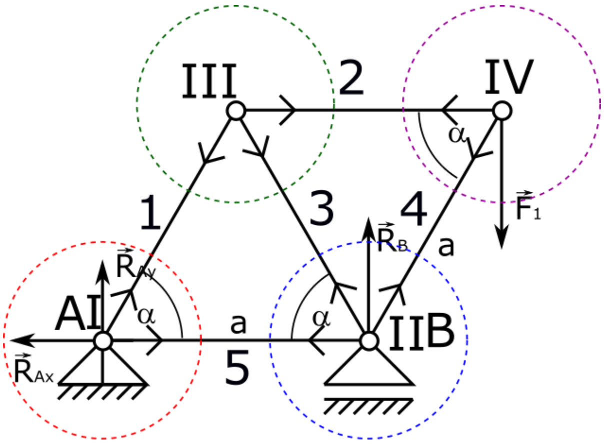
$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - F_1 + R_B \rightarrow R_{Ay} = F_1 - R_B = -\frac{1}{2}F_1$$

$$\sum_{i=1}^{n} M_A = 0 = -R_B * a + F_1 * 1{,}5a \rightarrow R_B = \frac{3}{2}F_1$$

4. All things done till this point are actually common to each of mentioned above methods.
5. Now we will make an assumption that all rods in this truss are stretched. It will help us in later calculations, because, if after calculation, we will obtain positive value, it will mean that our assumption was correct, and rod that we calculated, in real is actually stretched. However, if the result will be with negative sign, it will mean that in real this rod is compressed.



6. Now let's dig into the method of joints. First of all lets mark each of joints with the circle in order to make it more clear.

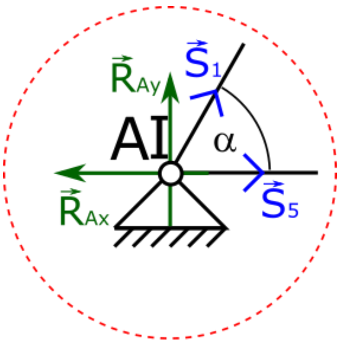


If you look closely to each of these circle you can notice that in each them we have separated convergent system of forces. Having in mind that for planar convergent system we can write only two equilibrium equations (projections over axis X and projections over axis Y), we need to choose where we can start solution.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0;$$

If you look closely to the above figure you can clearly see, that only in joints I and IV we have two unknown forces, and in joints II and III we have three unknown. It means that we can start our calculation taking into account joint I or IV. Let's start from joint I.

7. In this method we are assuming that actually this joint was cutted out from the whole system. However, in order to keep this joint in balance we need to introduce forces in cutted rods (S_1 and S_5). These blue forces are unknown, and we need to find them. Green forces (reactions), are already known, because we calculated them above.



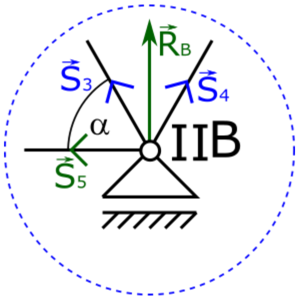
Let's write equations of equilibrium

$$\sum_{i=1}^{n} F_{xi} = 0 = -R_{Ax} + S_5 + S_1 \cos 60° \rightarrow S_5 = R_{Ax} - S_1 \cos 60° = 0 - \frac{\sqrt{3}}{3} F_1 * \frac{1}{2} = -\frac{\sqrt{3}}{6} F_1$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} + S_1 \sin 60° \rightarrow S_1 = -\frac{R_{Ay}}{\sin 60°} = -\frac{-\frac{1}{2} F_1}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3} F_1$$

In this way we have found forces in two rods 1 and 5. What is more due to the fact that at the beginning we took assumption that all rods are stretched, we can tell that in real rod 1 is stretched (positive value of force) and rod 5 is actually compressed (negative value of force).

8. Now we know a little bit more (force in rod 5), and this is why, we can now take under consideration joint II.



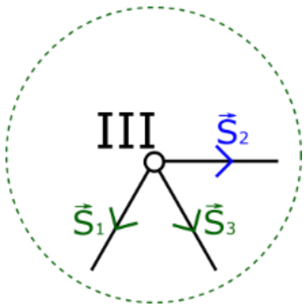
Once again: green forces, are already known, and blue are unknown. At this stage your attention should be paid on force S_5. One can noticed that at joint II this force has opposite sense in comparison to joint I. This change of sense is connected with the forces that are acting over the rod and which are inside the rod. If we are trying to stretch or compress rod, we are acting on it from both sides. At the same moment inner forces are acting on both sides of rod as well in order to prevent of stretching or compressing. This is why senses of inner forces, are drawn on both sides of each rod, and on one side of rod if it is cutted. Now, after this explanation we can write equations of balance for this joint

$$\sum_{i=1}^{n} F_{xi} = 0 = -S_5 - S_3 \cos 60° + S_4 \cos 60° \rightarrow S_4 = -\frac{2\sqrt{3}}{3} F_1$$

$$\sum_{i=1}^{n} F_{yi} = 0 = S_3 \sin 60° + R_B + S_4 \sin 60° \rightarrow S_3 = -\frac{\sqrt{3}}{3} F_1$$

In this way we have found forces in two rods 3 and 4. What is more due to the fact that at the beginning we took assumption that all rods are stretched, we can tell that in real rods 3 and 4 are actually compressed (negative value of force).

9. Finally, we can see that there only one rod left to be calculated (rod 2). Let's consider joint III in order to find it. Again: green forces, are already known, and blue are unknown. One can notice that in this joint it is enough to write only one equation of balance in order to find force in rod 2.



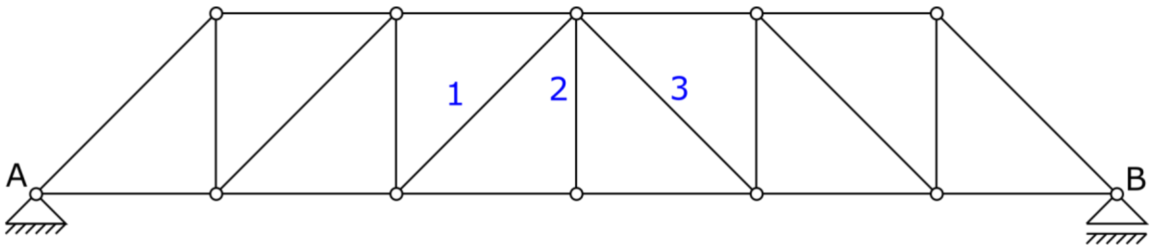
$$\sum_{i=1}^{n} F_{xi} = 0 = -S_1 \cos 60° + S_3 \cos 60° + S_2 \rightarrow S_2 = \frac{\sqrt{3}}{3} F_1$$

Ritter's method. Cutting method

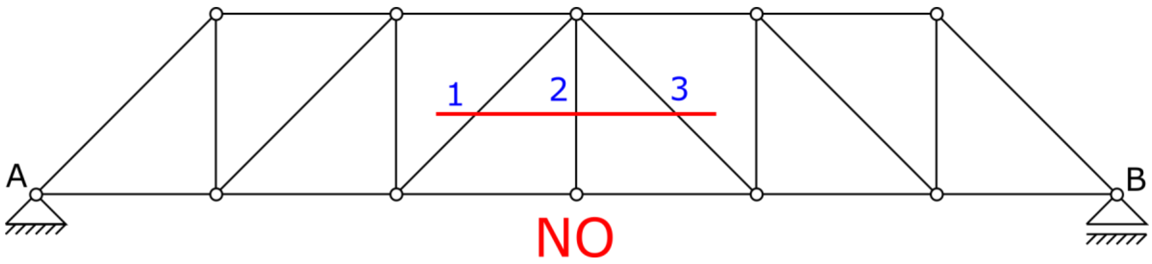
Joint method is useful when you need to calculate forces in all rods, or if the truss is small. However, sometimes there is need to calculate forces only in few specific rods, and then, it is better to use other methods, e.g. Ritter's method.

Ritter method is sometimes called as Cutting method, because in this method we are using cuts. Below I will show how this method works and when you can use it.

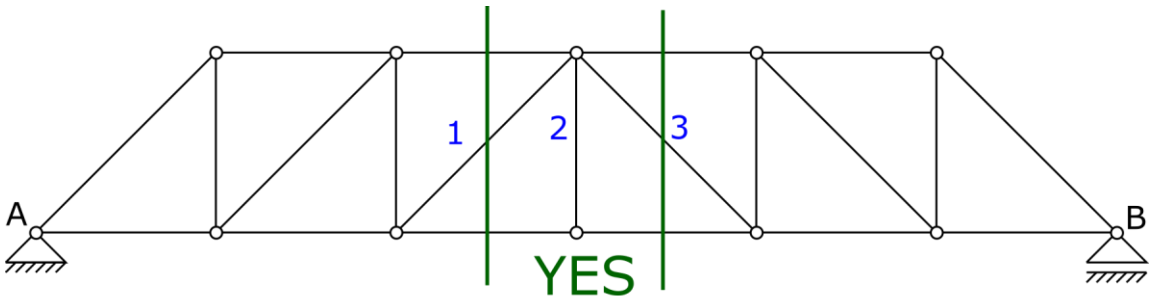
1. First thing first. Due to the fact that this is cutting method it means that you need to do a cut.



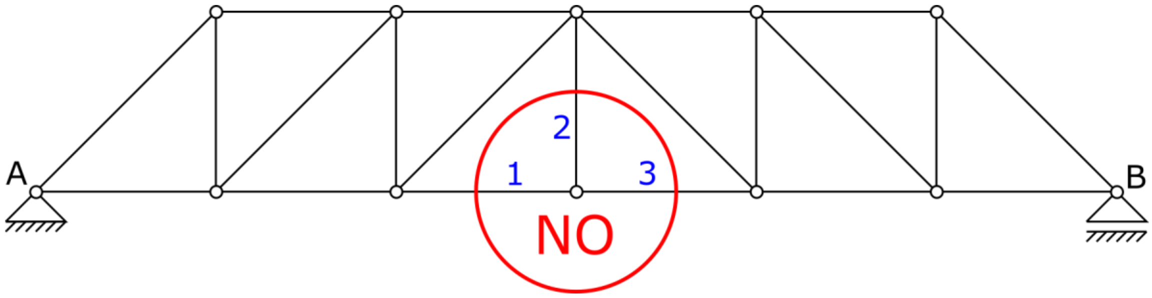
2. This cut should be done through 3 rods. But it must be done in a specific way. The cut that you will done must divide truss to two parts. As you can see below there are two examples. First is done badly, because as you can see cut is done through 3 rods, however, it does not dividing truss to two parts. This is why you cannot cut like that.



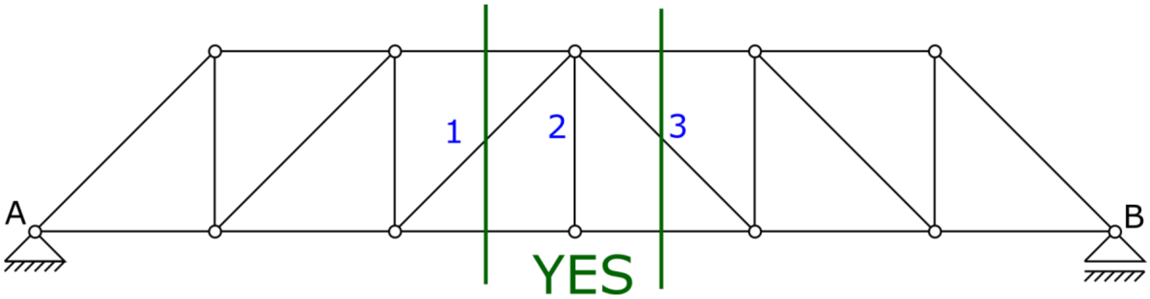
In second example we have two cuts. As you can see each of these cuts will divide truss to two parts, so they are done correctly.



3. Next rule is that three rods through which you are cutting must not starting in one joint. This is why cut done in first example is incorrect.



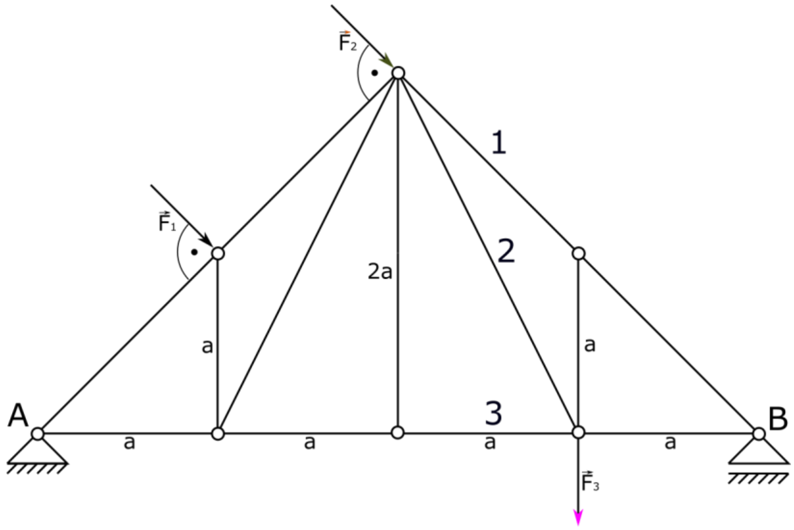
Second cut is done correct.



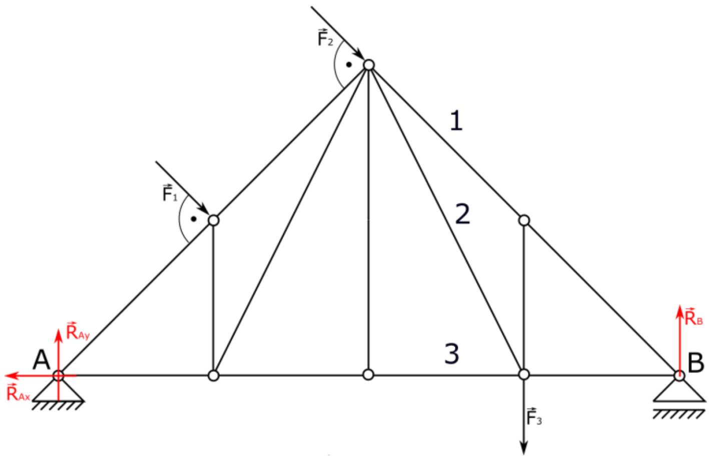
4. All rules which are connected with Ritter's method:

- **You must cut through 3 rods,**
- **Cut must divide truss to two separate parts,**
- **Rods through which you are cutting must not starting in one joint,**
- **Rods through which you are cutting cannot be parallel to each other – only two of them might be parallel.**

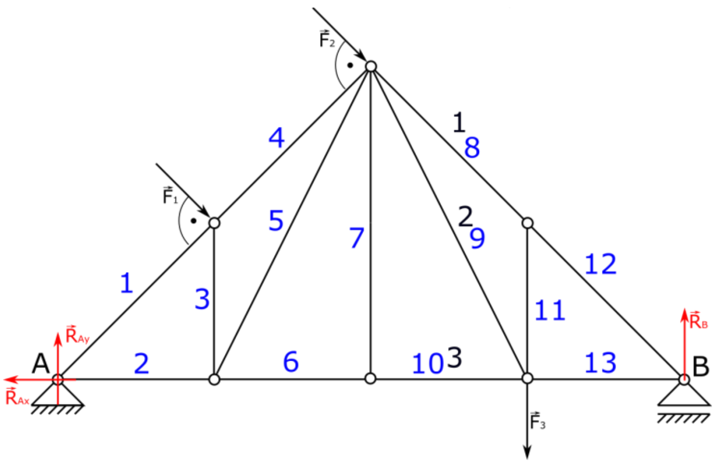
Example. Find forces in marked rods of this truss (1, 2, 3) with given data: F_1=10kN, F_2=20kN, F_3=30kN, a=1m.



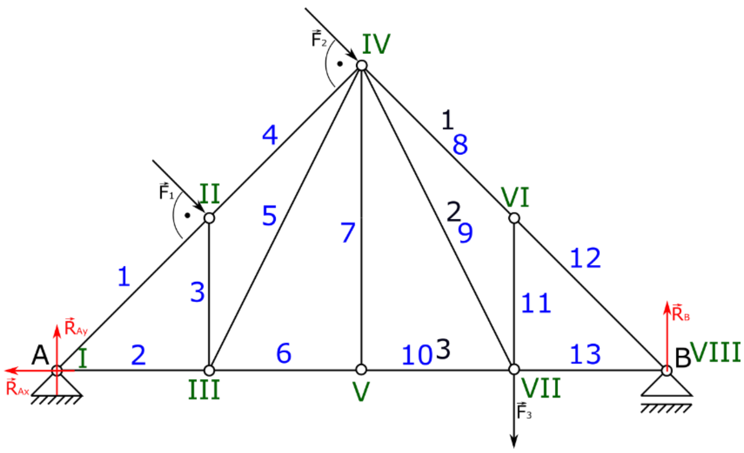
1. For each truss beginning is the same (check if we can solve it in static way). In order to start solving truss we will mark and count number of unknown reactions. In this example, we have 3 unknown reactions.



2. Second we will mark, and count number of rods. Here we have 13 rods.



3. Third we will mark, and count number of joints. This truss has 8 joints.



Now we can check if we are able to solve this truss

$$n=r$$

R=2*J-n

13=2*8-3

13=16-3

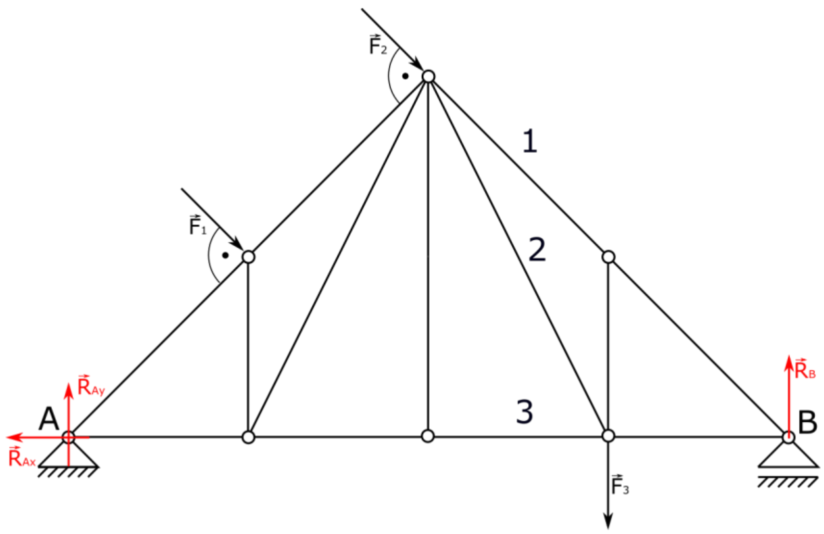
13=13

L=R

This truss is statically possible to solve.

4. Now, we can back to our basic drawing.
First of all we will find the unknown reactions in the supports. In order to do this we will use our three equations of equilibrium. Of course we cannot forget about assumption that moments rotating anticlockwise will be with positive sign.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} F_{yi} = 0; \quad \sum_{i=1}^{n} M_O = 0$$



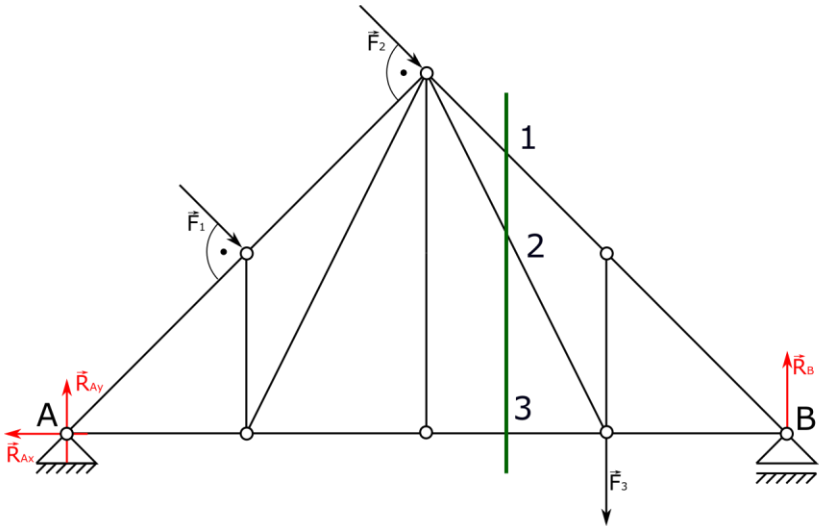
$$\sum_{i=1}^{n} F_{xi} = 0 = -R_{Ax} + F_1 \cos 45° + F_2 \cos 45° \rightarrow R_{Ax} = 21{,}15kN$$

$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - F_1 \sin 45° - F_2 \sin 45° - F_3 + R_B \rightarrow R_{Ay} = 11{,}025kN$$

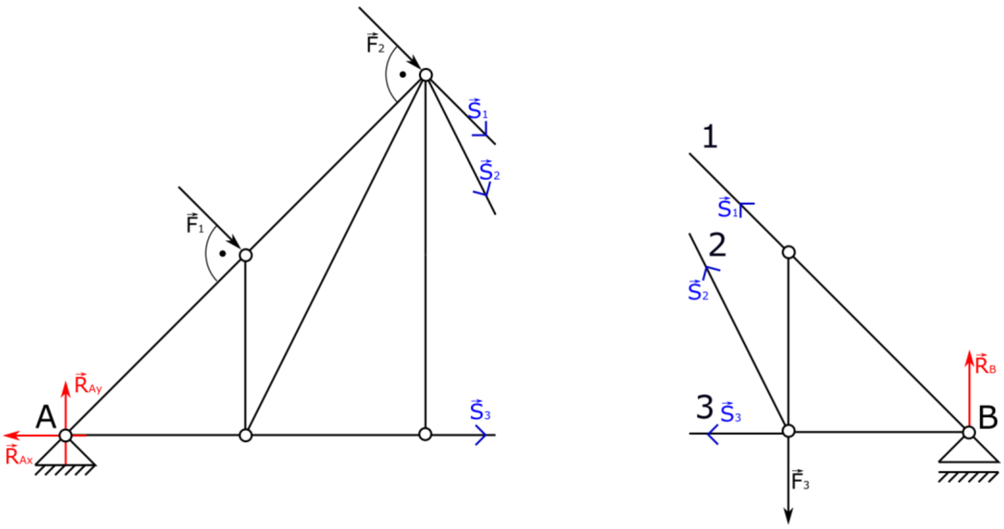
$$\sum_{i=1}^{n} M_A = 0 = R_B * 4a - F_1 * a\sqrt{2} - F_2 * 2a\sqrt{2} - F_3 * 3a \rightarrow R_B = 40{,}125kN$$

5. Next, after finding unknown reactions we can start dealing with cutting method. First of all, we need to answer ourselves questions to all these 4 rules given above:
   - Is this cut going through 3 rods? YES,
   - Is this cut dividing truss to two parts? YES,
   - Are rods through which cut is done parallel to each other? NO,
   - Are rods through which cut is done starting from one joint? NO.

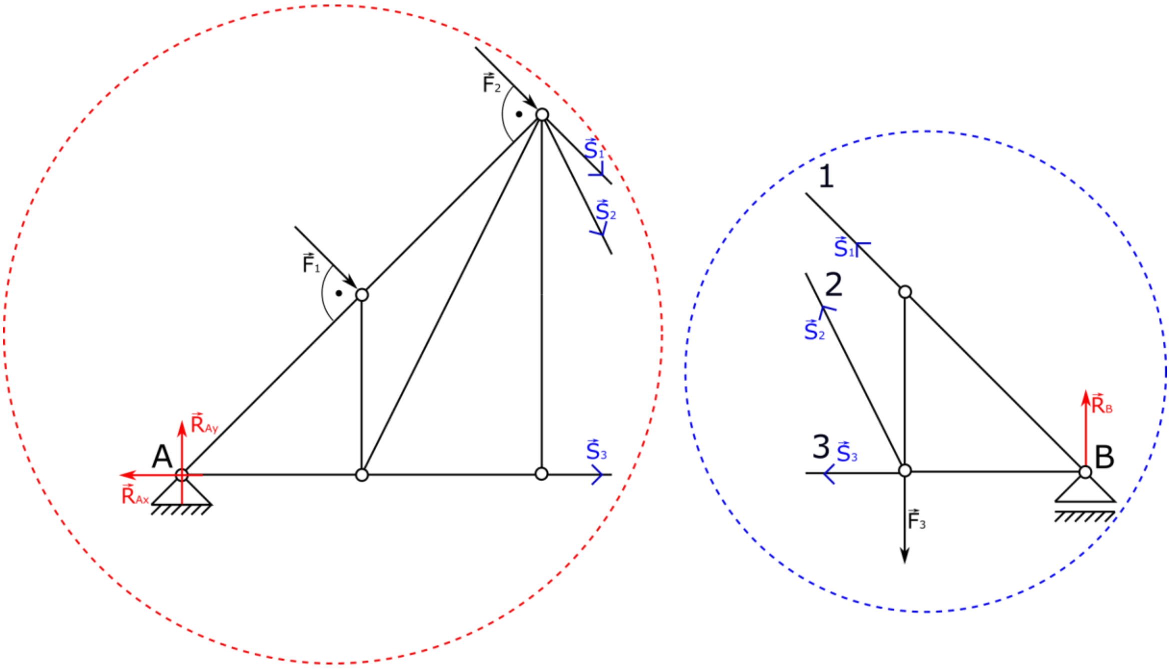
   if the answers to these questions are as above, then we can continue.



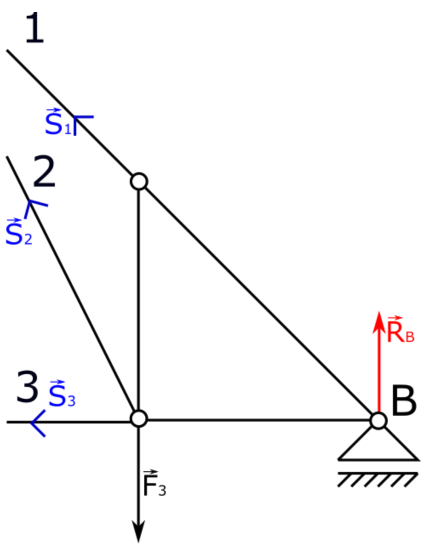
6. Let's divide truss to two separate parts. At this point we need to introduce forces to the rods through which cutting was done, in order to keep system in a balance. These introduced forces are our unknowns. WE ARE NOT INTRODUCING FORCES TO OTHER RODS.



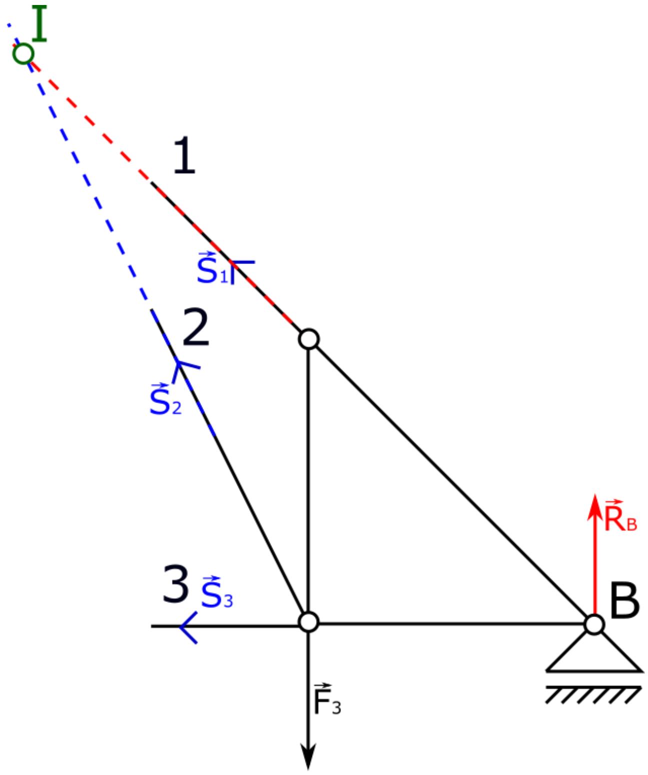
7. Now when we have two parts with introduced unknown forces we need to choose one: left or right. I suggest to choose part where is the smallest amount of forces. In order to make calculations easier. In this case we will take left part of truss (in blue circle).



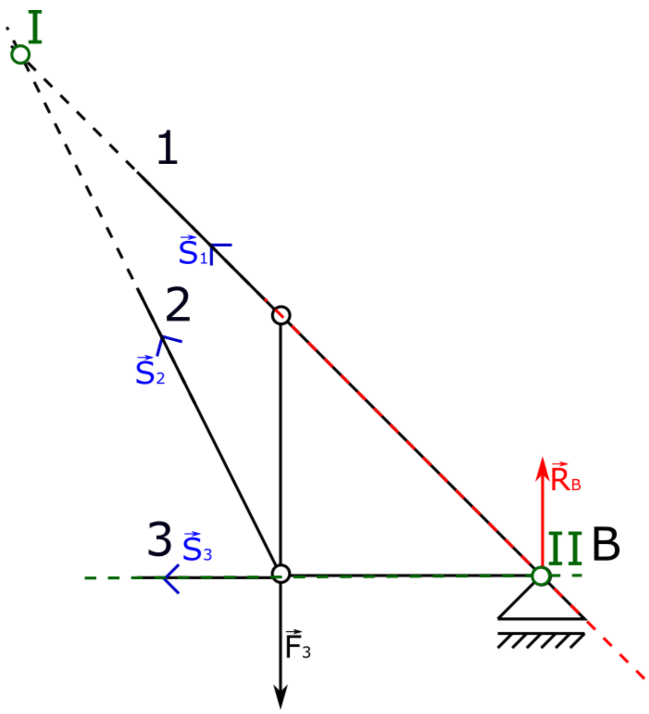
8. After choosing a part, we do not bother the other one. At this stage Ritter's method is introducing something new, so called Ritter's poles. How to find them? We need to extend directions of each of our unknown forces, and find where two directions are crossing.



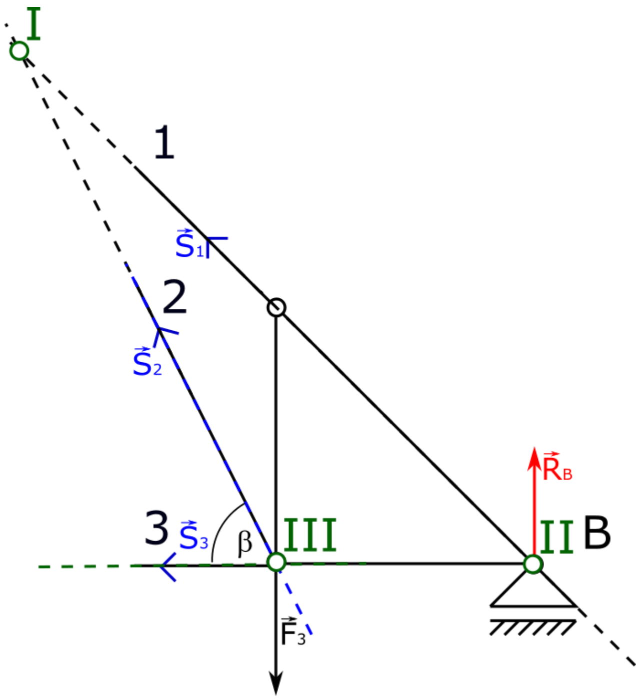
9. First pole. Directions of S_1 and S_2 are extended, and at the point where they are crossing we are introducing first pole (green point I).



10. Second pole. Directions of S_1 and S_3 are extended, and at the point where they are crossing we are introducing second pole (green point II).



11. Third pole. Directions of S_1 and S_3 are extended, and at the point where they are crossing we are introducing third pole (green point III).



12. After establishing three poles, we need to check if these points are not lying in one line. If this condition is met, we can write equations of equilibrium for our system.
13. ATTENTION

    We will use other set of equations of equilibrium the one where we are calculation moments related to three different points, this is why these points cannot lay in one line.

$$\sum_{i=1}^{n} M_A = 0; \quad \sum_{i=1}^{n} M_B = 0; \quad \sum_{i=1}^{n} M_C = 0$$

In our case we will use established poles (I, II, III).

$$\sum_{i=1}^{n} M_I = 0 = -S_3 * 2a - F_3 * a + R_B * 2a \rightarrow S_3 = 25{,}125kN$$

$$\sum_{i=1}^{n} M_{II} = 0 = F_3 * a - S_2 \sin\beta * a \rightarrow S_2 = 33{,}56kN$$

$$\sum_{i=1}^{n} M_{III} = 0 = S_1 * \frac{a\sqrt{2}}{2} + R_B * a \rightarrow S_1 = -56{,}91$$

14. ATTENTION 2

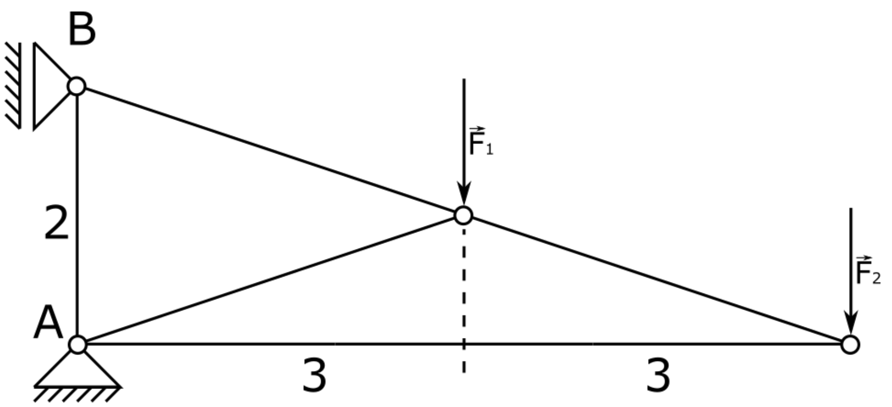
    When situation with two parallel rod will occur, then you are looking for two Ritter's poles. In order to write equations of equilibrium you are using second set of these, where the sum of moments relative to two points on the plane must be equal to zero, and the sum of projections on any axis not perpendicular to the segment connecting the points on which the moments were calculated must be equal to zero.

$$\sum_{i=1}^{n} F_{xi} = 0; \quad \sum_{i=1}^{n} M_A = 0; \quad \sum_{i=1}^{n} M_B = 0$$

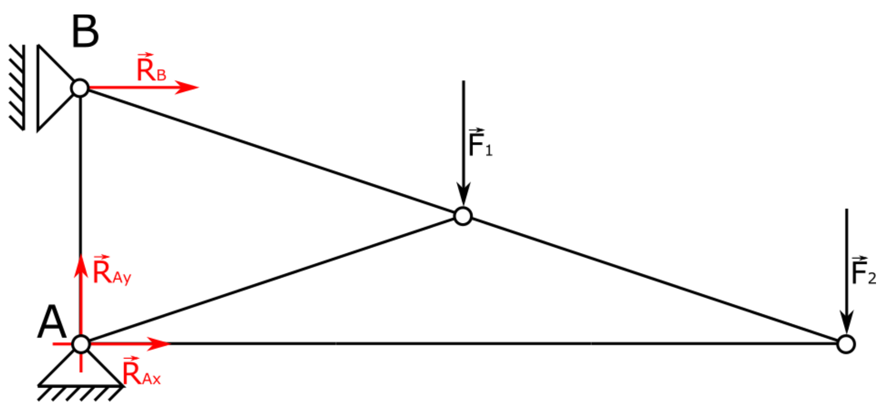
Finally graphical method. Bow's-Cremona's method.
In this method, we are actually reading values of inner forces from the plan of forces. The easiest way to explain this method is to use an example. This method seems to be complicated, and usually is use for smaller trusses.

Example. Solve this truss, with given data: Distance AB=2m, Distance A to directions of F_1=3m, distance between directions of forces F_1 and F_2=3m, F_1=F_2=1kN.

We have given such truss, where we know loads, and some distances. First of all we still need to do the same thing that we did for previous examples. We need to find values of unknown reactions in supports.



1. In order to start solving truss we will mark and count number of unknown reactions. In this example, we have 3 unknown reactions.



Of course we also need to calculate number of rods and joints. We can see that there are 5 rods and 4 joints, so this truss might be solved with static methods. Because

$$5=4*2-3$$

$$5=8-3$$

$$5=5$$

$$L=R$$

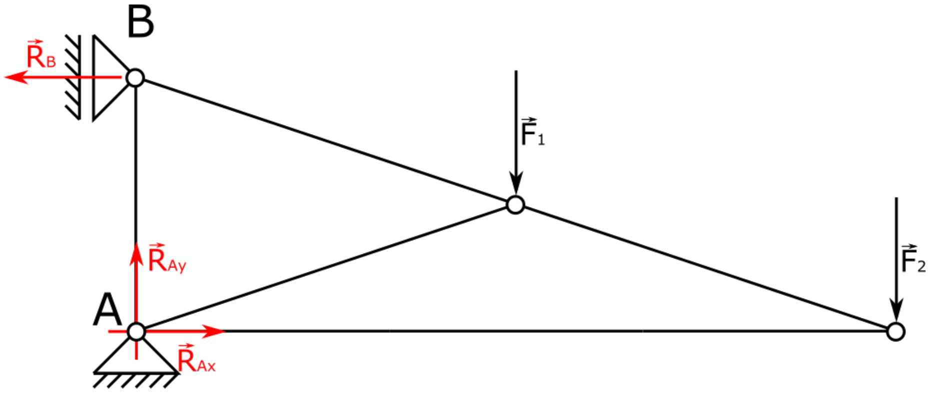
Let's write equations of balance for this truss.

$$\sum_{i=1}^{n} F_{xi} = 0 = R_{Ax} + R_B \rightarrow R_{Ax} = -R_B = 4{,}5kN$$

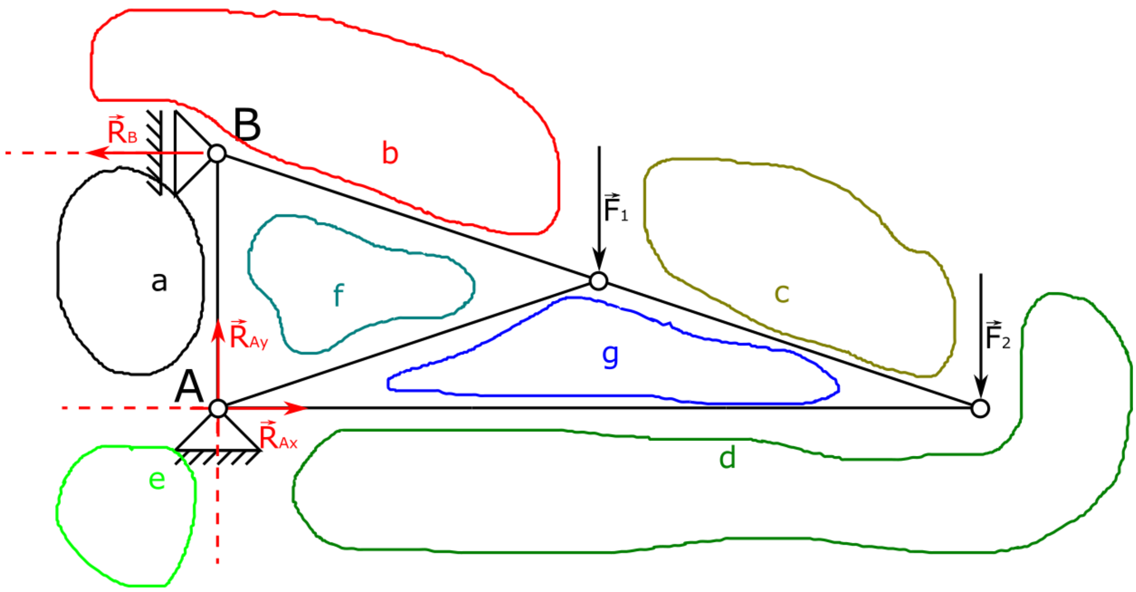
$$\sum_{i=1}^{n} F_{yi} = 0 = R_{Ay} - F_1 - F_2 \rightarrow R_{Ay} = F_1 + F_2 = 2kN$$

$$\sum_{i=1}^{n} M_A = 0 = -R_B * 2 - F_1 * 3 - F_2 * 6 \rightarrow R_B = \frac{-F_1 * 3 - F_2 * 6}{2} = -4{,}5kN$$

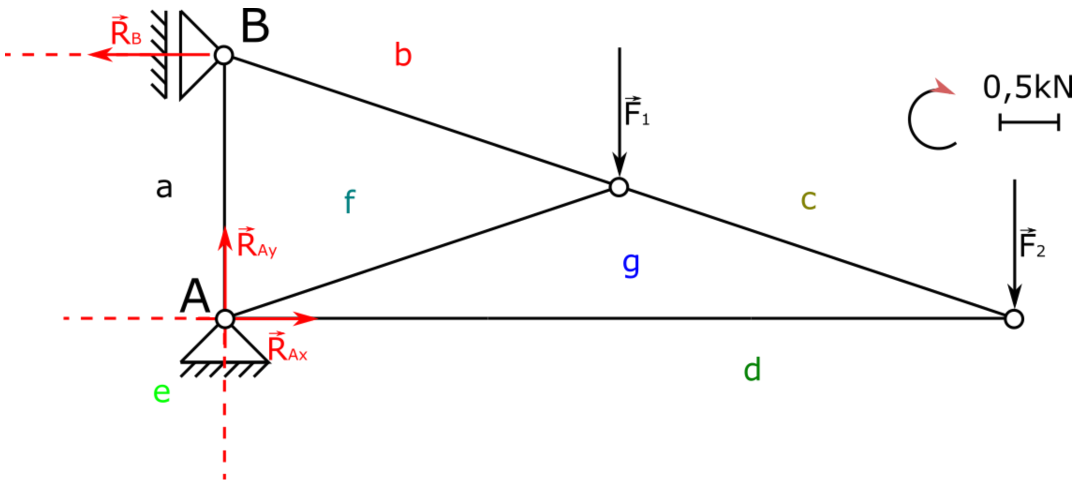
2. Next , what is important in this method you need to draw reaction as they are in reality. It means that sense of reaction R_B must be changed as it is shown in the figure below.



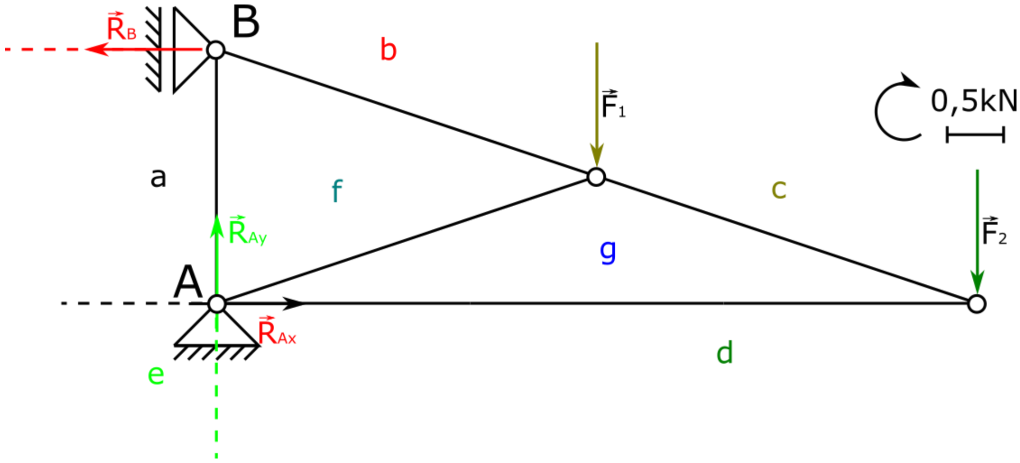
3. Next, we need to divide our truss to areas, as it was shown in the figure. Each area is between directions of forces. The best way is to start from the outside, and then move around our truss. After marking area on the outside we can mark inner areas.
For example: area "a" is between directions of R_{Ax}, R_B, and rod between points A and B.
In order to make it easier to understand I marked each area with different color.



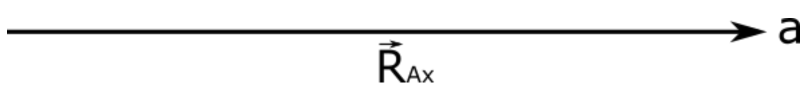
4. Now we have only letters to make everything more clear. At this stage we need to introduce two addition information.
   - First, how we are going to "walk" around truss. As you can see we have included rotation marker which show us that we will be rotating clockwise.
   - Second, we need to establish scale. As you can notice, there is a scale, which shows us 0,5kN.



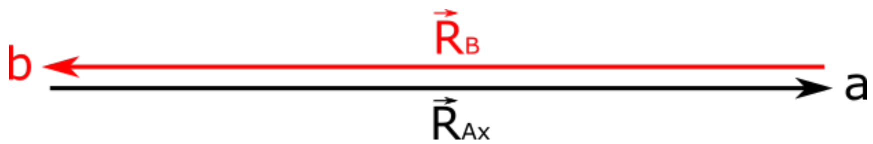
5. In order to clarify everything even more I also change color of outer forces, because we will be using these at this stage.



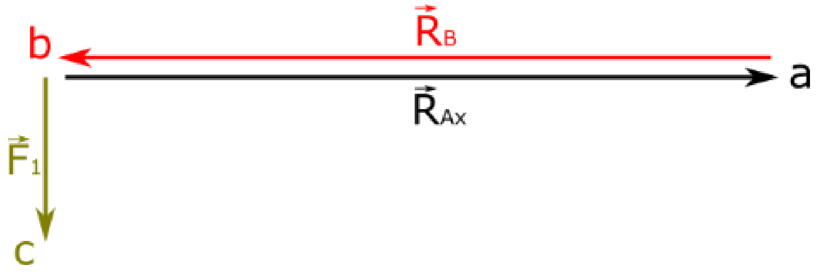
6. First of all, we will "walk around" our truss from the outside. We need to start at some point. Let's start at point A (area "a"). We can see that at this point we have force R_{Ax}. We need to draw this force according to the established scale. At the end of this force we mark that it is touching area "a".



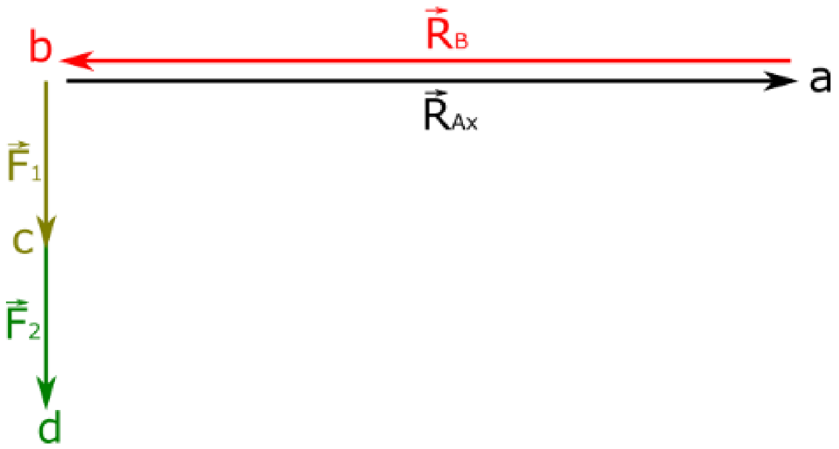
7. Now, we are rotating clockwise. This is why, we are going through the area "a" and reaching force R_B. Now we need to draw this force according to the established scale. At the end of this force we mark that it is touching area "b".



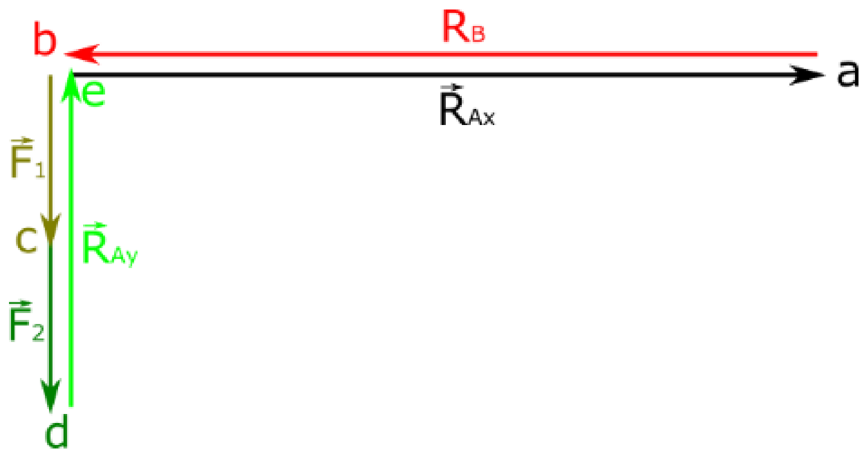
8. We are still rotating clockwise. This is why, we are going through the area "b" and reaching force F_1. Now we need to draw this force according to the established scale. At the end of this force we mark that it is touching area "c".



9. We keep rotating clockwise. This is why, we are going through the area "c" and reaching force F_2. Now we need to draw this force according to the established scale. At the end of this force we mark that it is touching area "d".



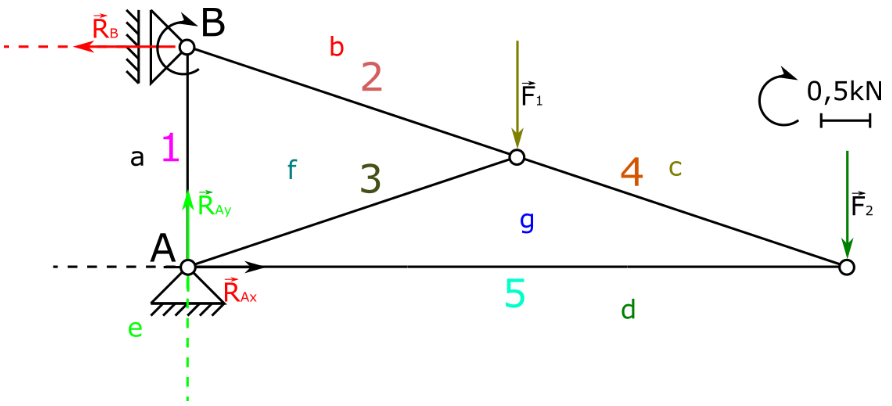
10. We keep rotating clockwise. This is why, we are going through the area "d" and reaching force R_{Ay}. Now we need to draw this force according to the established scale. At the end of this force we mark that it is touching area "e".



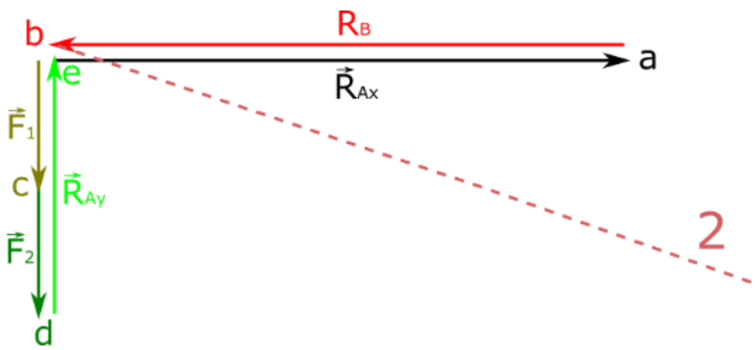
At this point we can see that we have made a whole circle around our truss, because if we will keep rotating we will start from the beginning (force R_{Ax}).

The chain of forces was created. It must be closed chain, because we know that our truss is in the balance, both outside and inside.

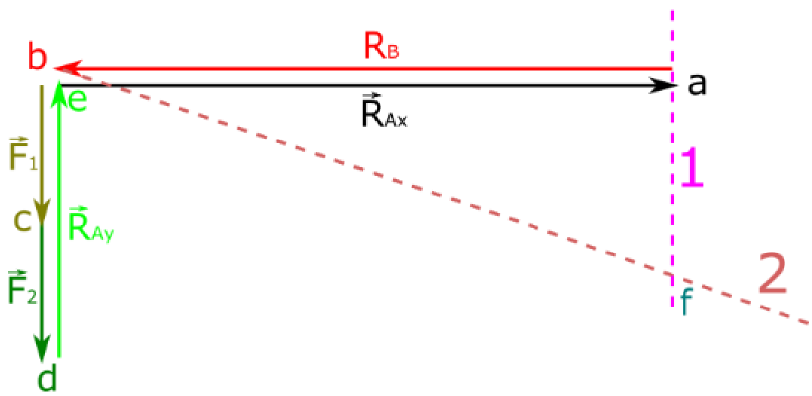
11. Now we can start with next step. We need to know what forces are in each of rods (from 1 to 5). Each digit has a different color corresponding to the force in the rod above which it is located.
    We need to pick one joint around which we will be walking around. You need to pick this point in a way that you will be able to draw chain of forces at this point (maximum two unknown forces). Let's take point B. One can notice that rotation was introduced at this point. It was done to show that we will be rotating around point the same way as we did for the whole truss.



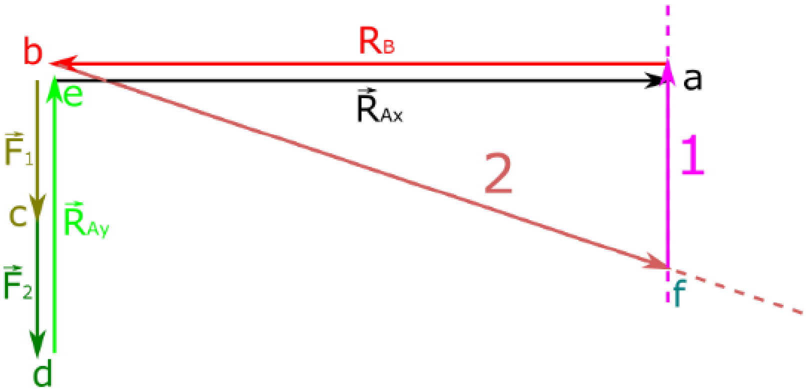
12. At point B we have force R_B which is already drawn. Now, we are rotating clockwise and we are reaching rod 2. We do not know the value of force in this rod but we know the direction of this force. This is why we are drawing it, as it is shown in the figure. We can see that between force R_B and rod 2 is area "b".



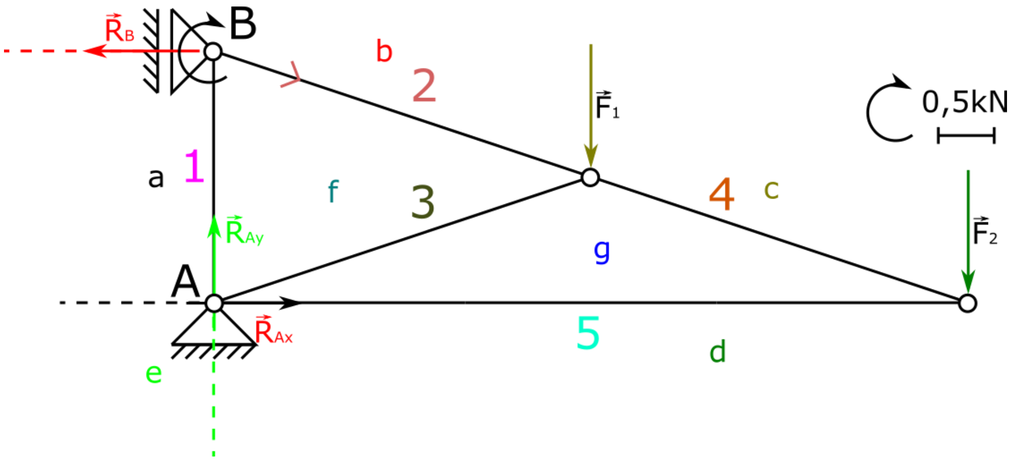
13. Next we are still rotation, and reaching rod 1. There is the same situation as with rod 2, so we do not know value of force but we know direction. We are drawing direction of this force, as it is shown in the figure. We can see that between rod 2 and rod 1 is area "f", and between rod 1 and force R_B is area "a".



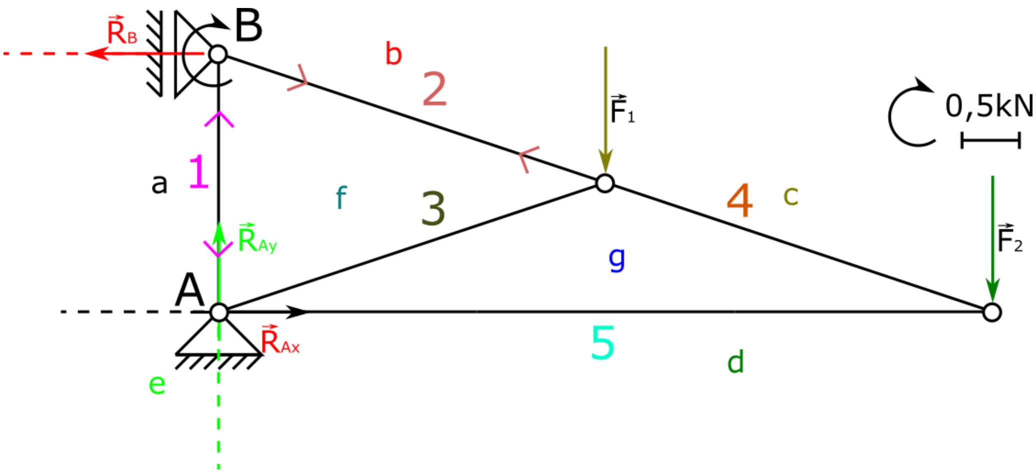
14. We can see that, there are no other forces at point B. It means that force R_B, and directions of these two other forces must crate closed chain of forces. This is why we know what are the senses of each of these forces, and we can draw them in the figure.



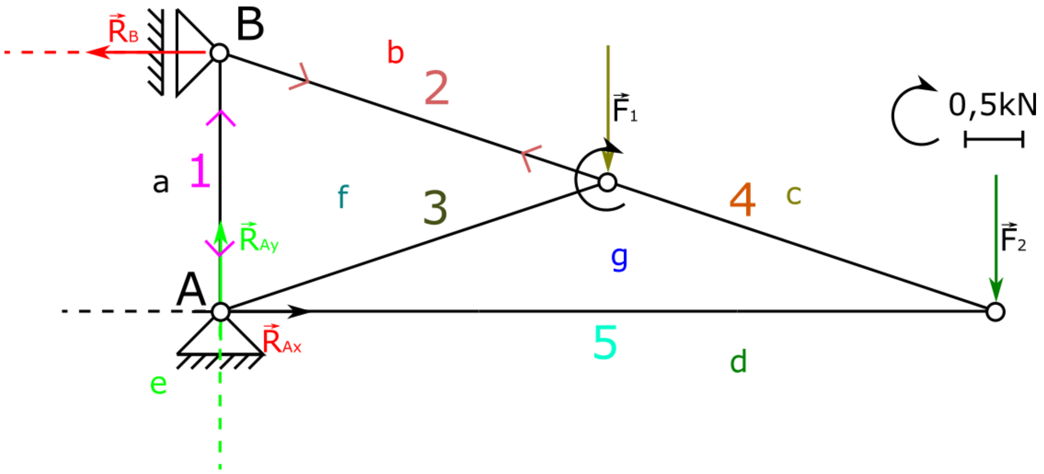
15. Now, we know senses of forces in rods 1 and 2 next to the point B, and we can draw them. In order to make it simpler I will start from force in rod 2. We need to take sense of force 2 from our chain of forces and place it next to the point B, as it was done in figure (arrow on rod 2)



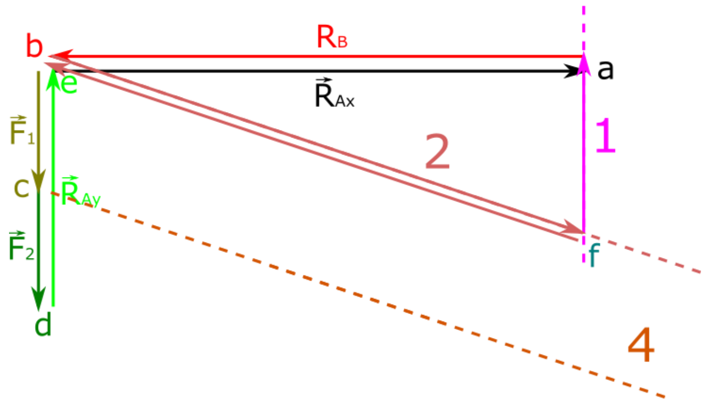
16. Due to the fact that rods are symmetrically loaded we need to draw second arrow next to the second end of rod 2 however, with opposite sense to the first one, as it was done in figure. Now, based on this arrows (inner forces), we can say that this rod is stretched. Now we need to do the same thig for force in rod 1. As you can see this rod is compressed.



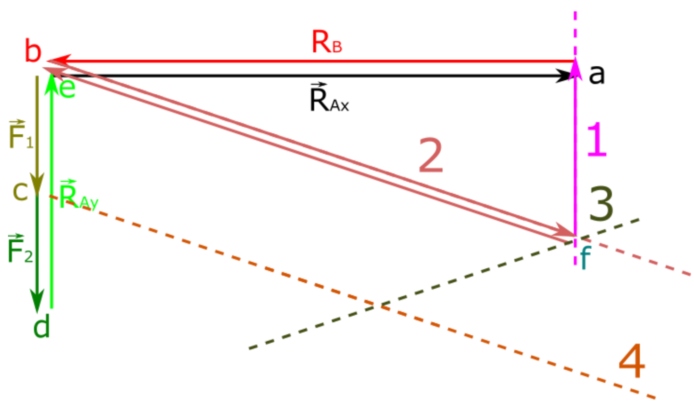
17. Now we need to pick another point and repeat the same thing. Next point is a point where force F_1 is applied (you can see rotation marker).



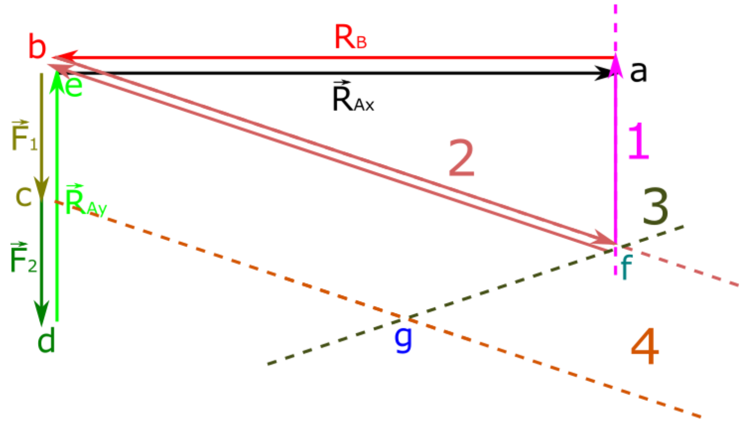
18. At this point we will start with force which is in rod 2. Because next to this point sense of force in rod 2 will be opposite to the previously drawn, we need to draw this force once again with proper sense, as it was shown in figure. Next, we have fore F_1 and it is already drawn. After that we are reaching rod 4, where we know only direction, which we are drawing, as it was shown in figure.



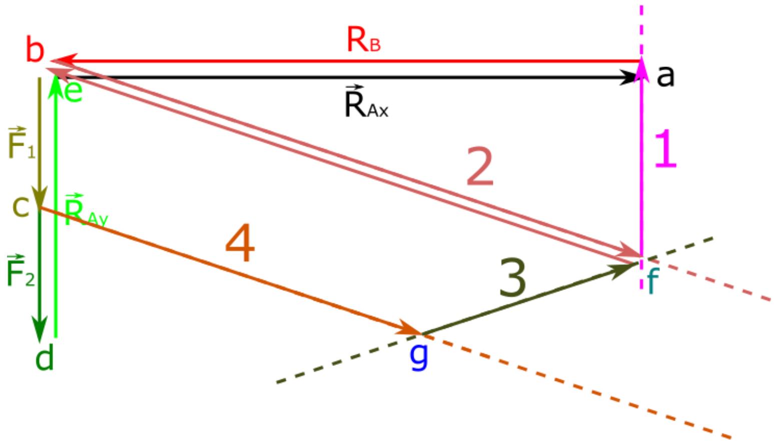
19. Next we are still rotation, and reaching rod 3. There is the same situation as with rod 4, so we do not know value of force but we know direction. We are drawing direction of this force, as it is shown in the figure.



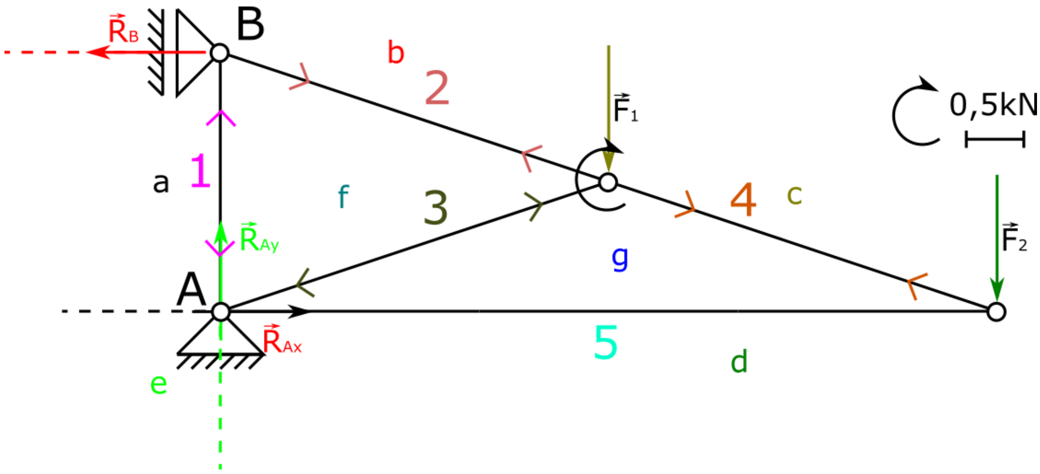
20. We can see that between rod 4 and rod 3 is area “g”.



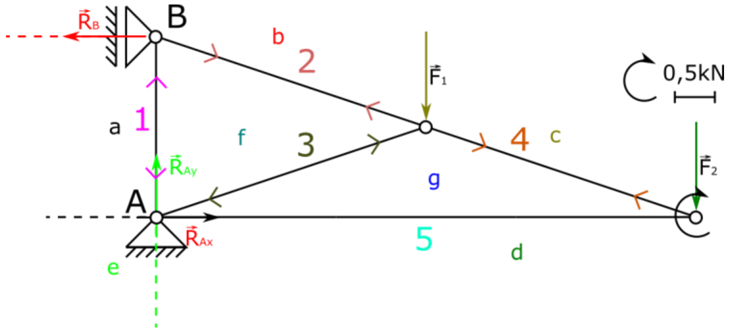
21. We can see that, there are no other forces at this point. It means that forces in rod 2 and F_1, and directions of these two other forces must crate closed chain of forces. This is why we know what are the senses of each of these forces, and we can draw them in the figure.



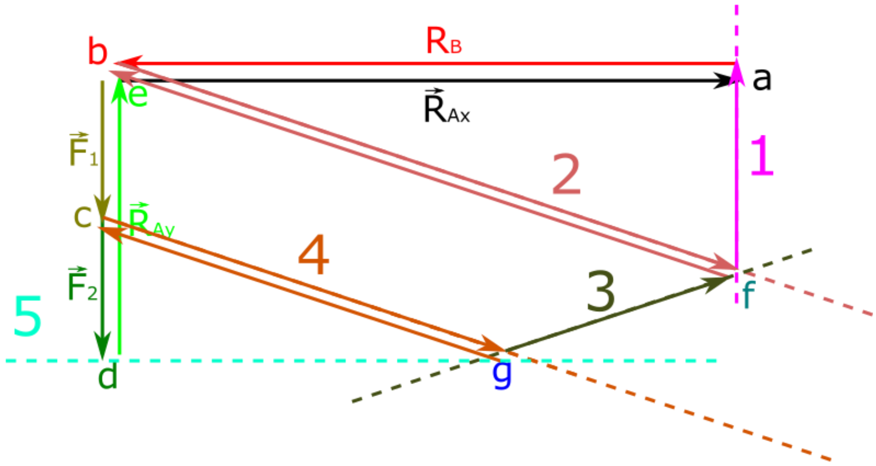
22. Now, we know senses of forces in rods 3 and 4 next to this point, and we can draw them. In a way it was done above for rods 1 and 2. Base on the figure we can say that rod 4 is stretched and rod 3 is compressed.



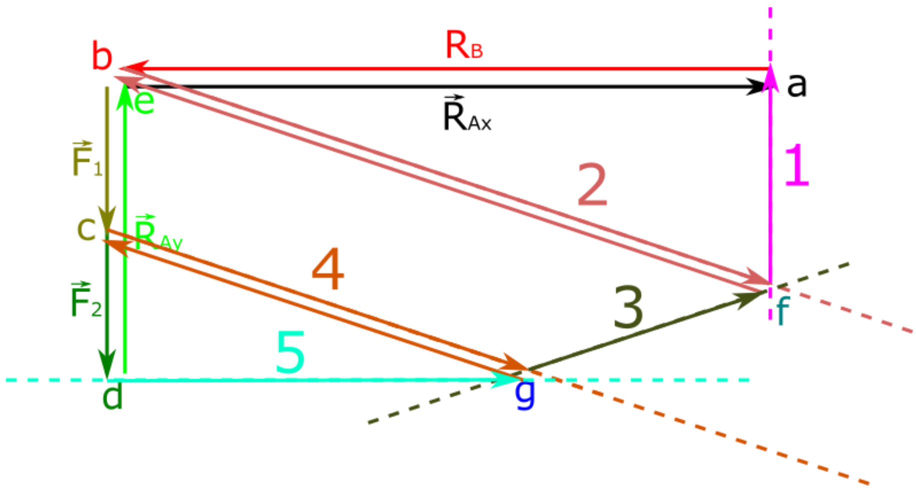
23. Next point is a point where force F_2 is applied (you can see rotation marker). This will be our final point, because based on this point we can establish last unknown force it is force in rod 5.



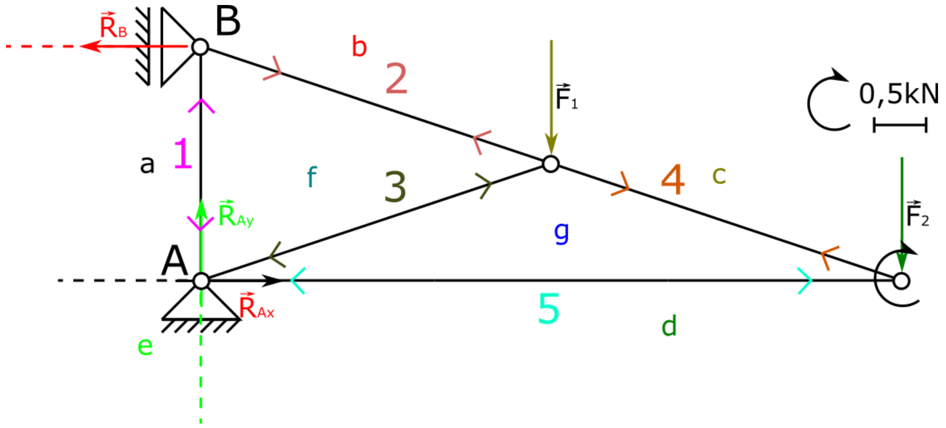
24. At this point we will start with force which is in rod 4. Because next to this point sense of force in rod 4 will be opposite to the previously drawn, we need to draw this force once again with proper sense, as it was shown in figure. Next, we have fore F_2 and it is already drawn. After that we are reaching rod 5, where we know only direction, which we are drawing, as it was shown in figure.



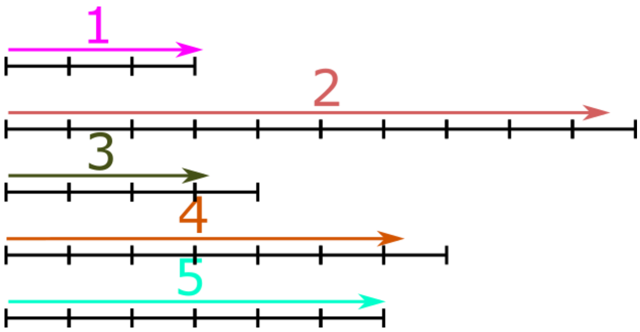
25. We can see that, there are no other forces at this point. It means that forces in rod 4 and F_2, and directions of third force must crate closed chain of forces. This is why we know what is the sense of last force, and we can draw it in the figure.



26. Now, we know sense of force in rod 5 next to this point, and we can draw it, in a way it was done above for other rods. Base on the figure we can say that rod 5 is compressed.



27. Finally we can read values of each of force from the drawing according to established scale.



Rod number	Length according to scale	Force value [N]	Way of load
1	3	1500	compressed
2	9,5	4750	stretched
3	3,2	1600	compressed
4	6,4	3200	stretched
5	6	3000	compressed