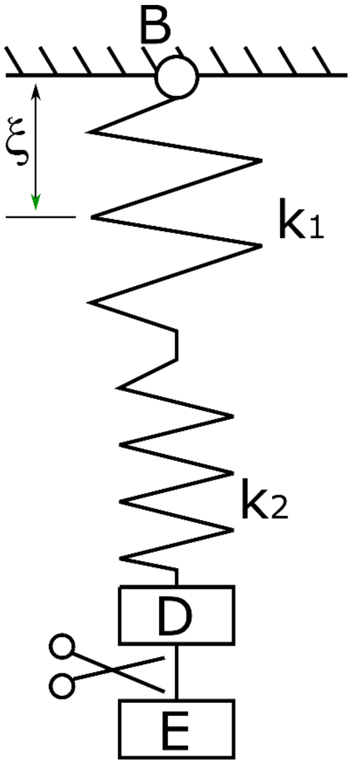
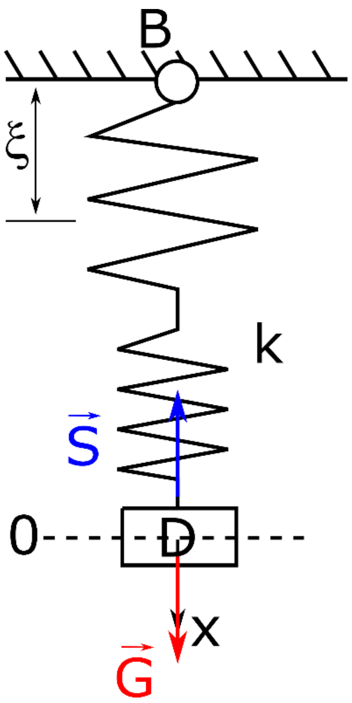
Vibration

Example. Find the equation of motion of the weight D with mass m_D relating the motion to the axis OX. Assume the beginning of the system in the position of the rest of the load D. Consider the rod connecting the loads as weightless and non-deformable. As soon as the bar connecting the loads D and E is cut (t = 0), point B begins to move according to ξ. The initial position of the point on the x-axis corresponds to the mean position of the point B (ξ = 0). Data: $m_D = 1kg$, $m_E = 2kg$, $\xi = 0{,}015\sin(18t)\ m$, $k_1 = 1200\ N/m$, $k_2 = 3600\ N/m$



First, we determine the substitute spring stiffness of the system. In this case, the springs are connected in series.

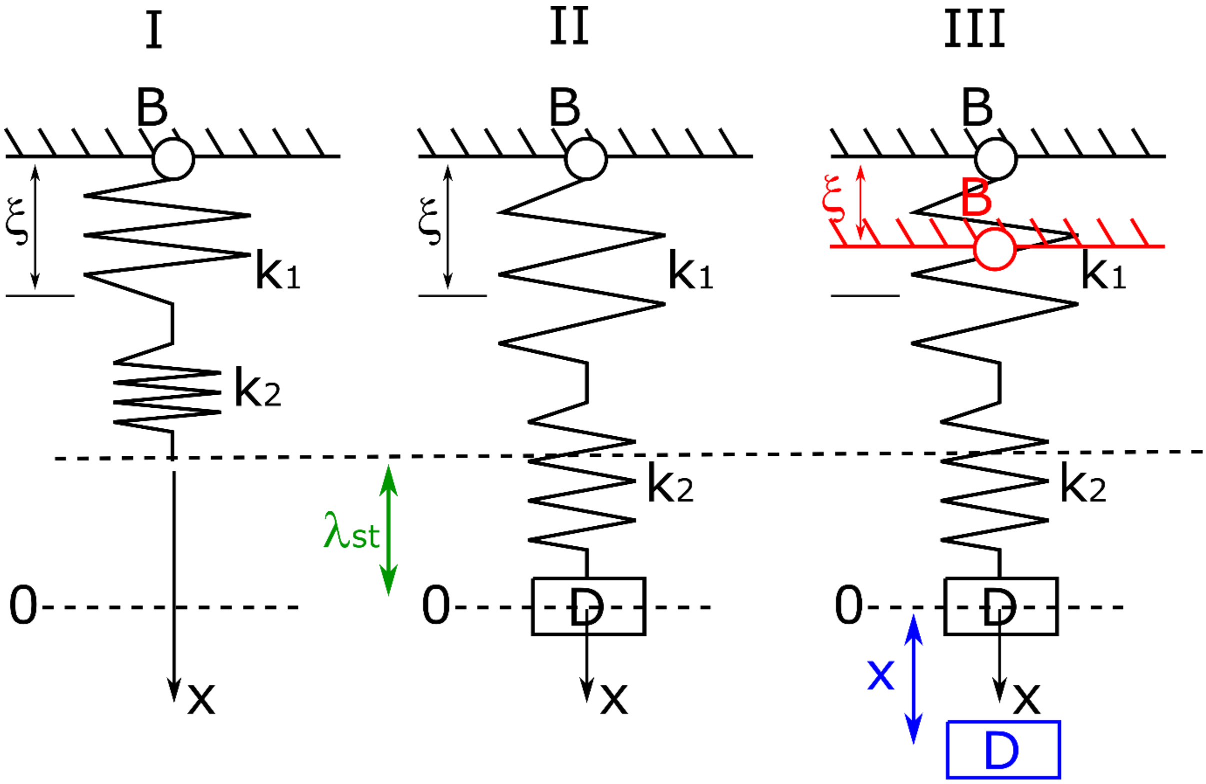
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 * k_2}{k_1 + k_2} = 900\ N/m$$



Then we write the equation of the dynamics of the motion of this system. We can see that the movement is only along the X axis and therefore we have to put all the forces acting along this axis into it.

$$m_D \ddot{x} = G - S$$

We can determine the force in the spring by analyzing the figures below.



$$S = k(x + \lambda_{stD} - \xi)$$

$$m_D \ddot{x} = m_D g - k(x + \lambda_{stD} - \xi)$$

The value of λ_{stD} should be determined. We do this for a situation where the system is in equilibrium under the action of the weight D. The spring is statically stretched, there is no motion, so the acceleration is 0.

$$0 = m_D g - k\lambda_{stD}$$

$$\lambda_{stD} = \frac{m_D g}{k}$$

$$m_D\ddot{x} = m_D g - k\left(x + \frac{m_D g}{k} - \xi\right)$$

$$m_D\ddot{x} = m_D g - kx - k\frac{m_D g}{k} + k\xi$$

$$\ddot{x} = -\frac{kx}{m_D} + \frac{k\xi}{m_D}$$

$$\ddot{x} + \frac{k}{m_D}x = \frac{k\xi}{m_D}$$

$$\ddot{x} + 900x = 13{,}5\sin(18t)$$

We have obtained a heterogeneous differential equation that needs to be solved.

$$x = x^* + x^{**}$$

$$\ddot{x} + 900x = 0$$

$$x = e^{rt};\ \dot{x} = re^{rt};\ \ddot{x} = r^2e^{rt}$$

$$r^2e^{rt} + 900e^{rt} = 0 \setminus e^{rt}$$

$$r^2 + 900 = 0$$

$$\Delta = b^2 - 4ac = -3600 < 0$$

$$\alpha = \frac{-b}{2a} = 0$$

$$\beta = \frac{\sqrt{4ac - b^2}}{2a}30$$

$$x^* = e^{\alpha t}(C_1\cos\beta t + C_2\sin\beta t)$$

$$x^* = C_1\cos 30t + C_2\sin 30t$$

$$\ddot{x} + 900x = 13{,}5\sin(18t)$$

$$x = a\sin(18t) + b\cos(18t)$$

$$\dot{x} = 18a\cos(18t) - 18b\sin(18t)$$

$$\ddot{x} = -324a\sin(18t) - 324b\cos(18t)$$

$$-324a\sin(18t) - 324b\cos(18t) + 900a\sin(18t) + 900b\cos(18t) = 13{,}5\sin(18t)$$

$$576a\sin(18t) + 576b\cos(18t) = 13{,}5\sin(18t)$$

$$b = 0$$

$$576a = 13{,}5 \Rightarrow a = 0{,}023$$

$$x^{**} = 0{,}023\sin(18t)$$

$$x = x^{*} + x^{**}$$

$$x = C_1\cos 30t + C_2\sin 30t + 0{,}023\sin(18t)$$

We determine the constants from the initial conditions

$$x(0) = \lambda_{stDE} - \lambda_{stD}$$

$$x(0) = \lambda_{stE} = \frac{m_E g}{k}$$

$$\dot{x}(0) = 0$$

$$x(0) = C_1 \Rightarrow C_1 = \frac{m_E g}{k} = 0{,}022$$

$$\dot{x} = -30C_1\sin 30t + 30C_2\cos 30t + 0{,}414\cos(18t)$$

$$\dot{x}(0) = 0 \Rightarrow 30C_2 + 0{,}414 = 0 \Rightarrow C_2 = -0{,}014$$

$$x = 0{,}022\cos 30t - 0{,}014\sin 30t + 0{,}023\sin(18t)$$